UNIT-I

MATHEMATICAL MODELLING OF PROCESS

Process controls is a mixture between the statistics and engineering discipline that deals with the mechanism, architectures, and algorithms for controlling a process. A process is the science of automatic control, denotes an operation or series of operation on fluid or solid material during which the materials are placed in more and useful state. The physical and chemical state at the materials is not necessarily altered .many external and internal conditions affect the performance of a process those conditions may be expressed in terms of process variable such as temperature, pressure, flow, liquid level, dimensions, weight, volume etc.

The role of process control has changed throughout the years and is continuously shaped by technology. The traditional role of process control in industrial operations was to contribute to safety, minimized environmental impact, and optimize processes by maintaining process variable near the desired values. Generally, anything that requires continuous monitoring of an operation involve the role of a process engineer. In years past the monitoring of these processes was done at the unit and were maintained locally by operator and engineers. Today many chemical plant have gone to full automation which means that engineers and operators are helped

Benefits of Process Control:

The benefits of controlling or automating process are in a number of distinct area in the operation of a unit or chemical plant. Safety of workers and the community around a plant is probably concern number one or should be for most engineers as they begin to design their processes. Chemical plants have a great potential to do severe damage if something goes wrong and it is inherent the setup of process control to set boundaries on specific unit so that they don’t injure or kill workers or individuals in the community.

Definitions:

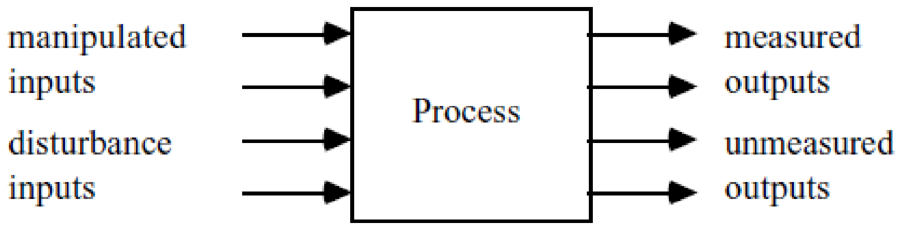
In controlling a process there exist two type of classes of variables.

Classes of process variables

- **Input Variable**

- ✓ **Manipulated inputs**
- ✓ **Disturbances**

- ➢ **Output Variable or Control Variable**
  - ✓ **Measured output variable**
  - ✓ **Unmeasured output variable**
- ➢ **Controlled variable**
- ➢ **Manipulated Variable**
- ➢ **Load variable**



a. Input/Output representation

- ➢ **Input Variable** – This variable shows the effect of the surroundings on the process. It normally refers to those factors that influence the process
  - ✓ *Manipulated inputs:* variable in the surroundings can be control by an operator or the control system in place.
  - ✓ . *Disturbances:* inputs that cannot be controlled by an operator or control system. There exist both measurable and immeasurable disturbances.
- ➢ **Output variable** - Also known as the *control variable* These are the variables that are process outputs that effect the surroundings. These variables may or may not be measured.
  - ✓ Measured output variable: Measurements can be made continuously or discrete interval of time.
  - ✓ Unmeasured output variable:The variables cannot be determined.
- ➢ **Controlled variable-** The controlled variable of the process should be that variable which most directly indicates the desired for or state of the product.
- ➢ **Manipulated Variable-** the Manipulated variable of the process should be that variable which most directly indicates the desired form or state of the product.
- ➢ **Load Variable**- The load variables of the process are all other independent variables except the controlled variable and manipulated variable.

As we consider a controls problem. We are able to look at two major control structures.

- Single input-Single Output (SISO): for one control(output) variable there exist one manipulate (input) variable that is used to affect the process

- Multiple input-multiple output(MIMO): There are several control (output) variable that are affected by several manipulated (input) variables used in a given process.

Design Procedure for process control

- **Understand the process:** Before attempting to control a process it is necessary to understand how the process works and what it does.
- **Identify the operating parameters:** Once the process is well understood, operating parameters such as temperatures, pressures, flow rates, and other variables specific to the process must be identified for its control.
- **Identify the hazardous conditions:** In order to maintain a safe and hazard-free facility, variables that may cause safety concerns must be identified and may require additional control.
- **Identify the measurables:** It is important to identify the measurables that correspond with the operating parameters in order to control the process.
- **Identify the points of measurement:** Once the measurables are identified, it is important locate where they will be measured so that the system can be accurately controlled.
- **Select measurement methods:** Selecting the proper type of measurement device specific to the process will ensure that the most accurate, stable, and cost-effective method is chosen. There are several different signal types that can detect different things.
- **Select control method:** In order to control the operating parameters, the proper control method is vital to control the process effectively. On/off is one control method and the other is continuous control. Continuous control involves Proportional (P), Integral (I), and Derivative (D) methods or some combination of those three.
- **Select control system:** Choosing between a local or distributed control system that fits well with the process effects both the cost and efficacy of the overall control.
- **Set control limits:** Understanding the operating parameters allows the ability to define the limits of the measurable parameters in the control system.
- **Define control logic:** Choosing between feed-forward, feed-backward, cascade, ratio, or other control logic is a necessary decision based on the specific design and safety parameters of the system.

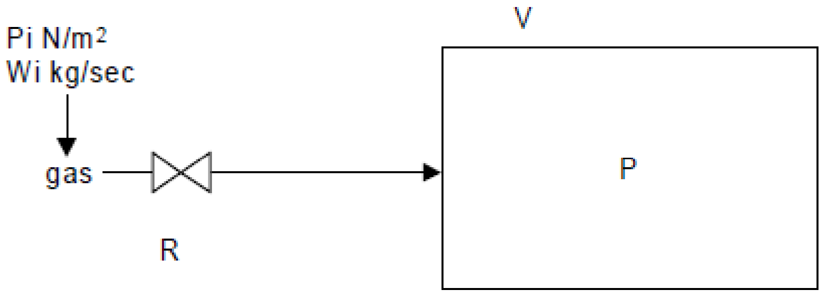
DYNAMIC OF SIMPLE PRESSURE, FLOW, LEVEL AND TEMPERATURE PROCESS

Dynamics of Simple pressure process

In the pressure processs it is of two types

- ✓ Gas storage tank
- ✓ Process with inlet and outlet resistances

Gas Storage Tank:



Parameters are:

- Inlet pressure p_i (N/m^2)
- Volume of Storage tank V(m^3)
- Rate of volume W_i (Kg/sec)
- Resistance of the inlet pipe R(N sec/KgKm^2)
- Density of the gas ρ (Kg/m^3)

Mass Balance Equation:

$$W_i = \frac{d(V_\rho)}{dt} = V\frac{d\rho}{dt}$$

If the gas is an ideal gas $\rho = \frac{P}{R_g T}$

Where R_g is the gas constant (Nm/KgK)

$$W_i = \frac{P_i - P}{R}$$

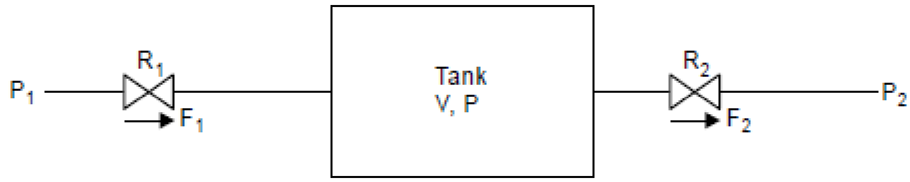
$$P_i - P = \left(\frac{RV}{R_g T}\right)\frac{dP}{dT} = \tau_P \frac{dP}{dT}$$

Taking Laplace transform of equation

$$P_i(s) = P(s)[\tau_P s + 1]$$

$$G(s) = \frac{P(s)}{P_i(s)} = \frac{1}{\tau_P s + 1}$$

Pressure system with two resistance



Mass balance equation:

Accumulation in the tank = Input flow rate (F_1) – Output flow rate (F_2)

$$V\frac{dP}{dt} = \frac{P_1 - P}{R_1} - \frac{P - P_2}{R_2}$$

Where $R_1 = \frac{d(P_1 - P)}{dF_1}$ and $R_2 = \frac{d(P - P_2)}{dF_2}$

$$V\frac{dP}{dt} + P\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{P_1}{R_1} + \frac{P_2}{R_2}$$

Simplifying

$$\tau_P\frac{dP}{dt} + P = K_1P_1 + K_2P_2$$

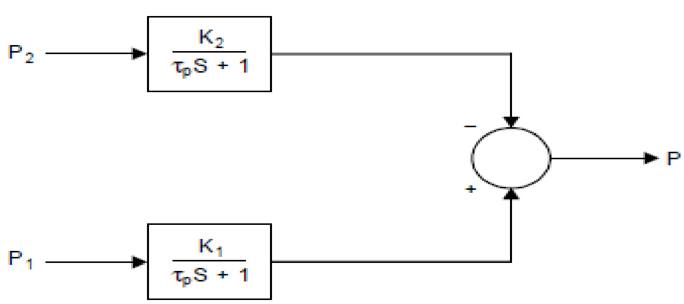
Where $K_1 = \frac{R_2}{R_1+R_2}$, $K_2 = \frac{R_1}{R_1+R_2}$, $\tau_P = \frac{VR_1R_2}{R_1+R_2}$

Taking laplace transform

$$\tau_P\, s\, P(s) + P(s) = K_1P_1(s) + K_2P_2(s)$$

$$P(s) = \frac{K_1}{1 + \tau_P\, s}P_1(s) + \frac{K_2}{1 + \tau_P\, s}P_2(s)$$

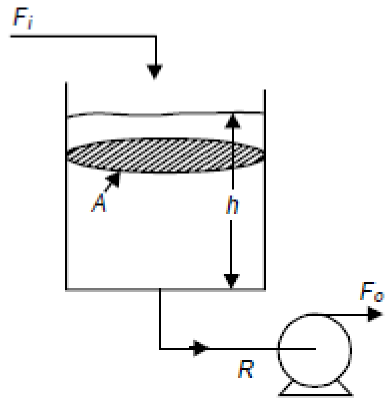
It can be represented in a block diagram



Dynamics of Simple Flow process: Liquid level process with constant flow outlet

Parameters are

- Flow rate F_o
- Liquid level h



Mass balance equation

$$A\frac{dh}{dt} = F_i - F_o$$

At steady state $0 = F_{is} - F_o$

Subtracting the function and taking the differentiation

$$A\frac{d\bar{h}}{dt} = \bar{F}_i$$

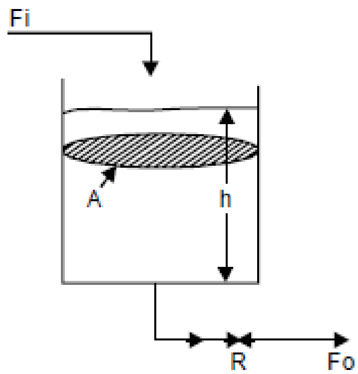
The transfer function is given as

$$G(s) = \frac{\bar{y}(s)}{\bar{F}_i(s)} = \frac{1}{As}$$

Dynamics of Simple Level process

Parameters are

- Inlet volumetric flow rate F_i (m^3/sec)
- Outlet volumetric flow rate F_o (m^3/sec)
- Cross sectional area A(m^2)
- Resistance R
- Liquid head in the tank h



Mass balance equation is

Rate of accumulation =Inflow- outflow

$$A\frac{dh}{dt} = F_i - F_o$$

Flow head equation is

$$F_o = \frac{h}{R}$$

$$A\frac{dh}{dt} = F_i - \frac{h}{R}$$

At steady state

$$A\frac{d\bar{h}}{dt} = \bar{F}_i - \frac{\bar{h}}{R}$$

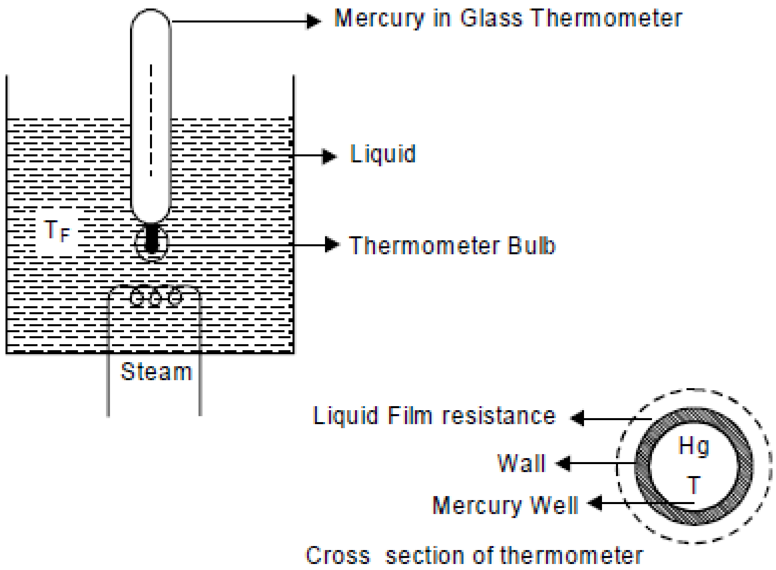
Where $\bar{h} = h - h_s$ and $\bar{F} = F_i - F_{is}$

$$\boldsymbol{G(s)} = \frac{\bar{h}(s)}{\bar{F}(s)} = \frac{\boldsymbol{K_p}}{1+\tau_{PS}}$$

Dynamics of Simple Temperature process

Parameters are

- Temperature of the liquid T_F
- Thermometer reading T
- Surface area of the bulb for heat transfer A m^2
- Mass of mercury in the bulb M (kg)
- Heat Capacity CP (kJ/KgK)
- Heat transfer coefficient U (kW/m^2K)



Mass balance equation is

Rate of accumulation =Inflow- outflow

$$UA(T_F - T) - 0 = MC_p \frac{dT}{dt}$$

At steady state

$$UA(T_{Fs} - T_s) = 0$$

Subtracting and taking derivation

$$UA(\bar{T}_F - \bar{T}) = MC_p \frac{d\bar{T}}{dt}$$

$$\tau_p = \frac{MC_p}{UA}$$

The transfer function is

$$G(s) = \frac{\bar{T}(s)}{\bar{T}_F(s)} = \frac{1}{1 + \tau_P s}$$

Transient Response Of First Order System For Thermal & Level Process

A first order system is one whose output is modeled by a first order differential equation. The transfer function of a first

$$G(s) = \frac{\bar{y}(s)}{\bar{f}(s)} = \frac{1}{1 + \tau_P s}$$

Transient response of Process

For he given transfer function

$$G(s) = \frac{\bar{y}(s)}{\bar{f}(s)} = \frac{K_p}{1 + \tau_p s}$$

Unit Step Function

$$\bar{f}_s = \frac{1}{s}$$

Applying the unit step function

$$\bar{y}(s) = \frac{K_p}{s(\tau_p s + 1)}$$

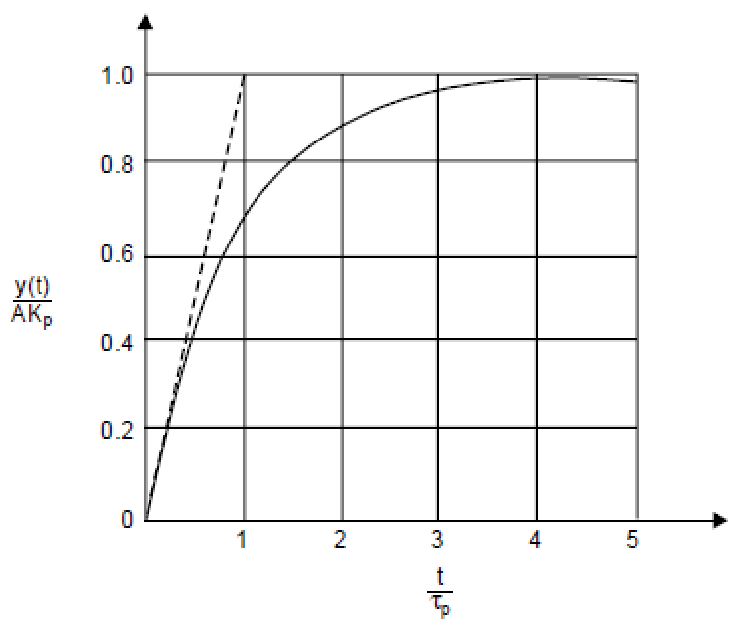
Taking inverse laplace transform

$$y(t) = K_p \left[1 - e^{-\frac{t}{\tau_p}}\right]$$

Taking magnitude A for step change

$$y(t) = A\, K_p \left[1 - e^{-\frac{t}{\tau_p}}\right]$$

Response is given as

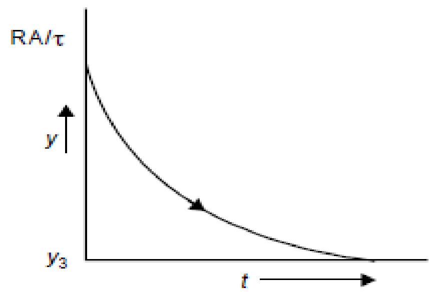


Impulse response:

$$G(s) = \frac{\bar{y}(s)}{\bar{f}(s)} = \frac{1}{1 + \tau_P s}$$

$$y(s) = \frac{A}{1 + \tau_P}$$

$$y(t) = \frac{A}{\tau_P} e^{-\frac{t}{\tau_P}}$$



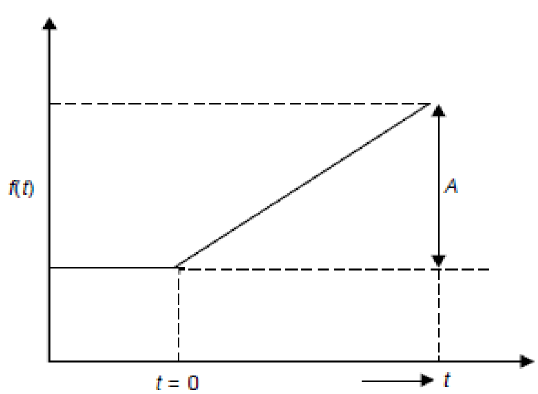
Ramp Function:

$$G(s) = \frac{\bar{y}(s)}{\bar{f}(s)} = \frac{1}{1 + \tau_p s}$$

$$M = \frac{A}{t}$$

$$f(t) = \frac{A}{s^2}$$

$$y(t) = A(t - \tau_p)$$



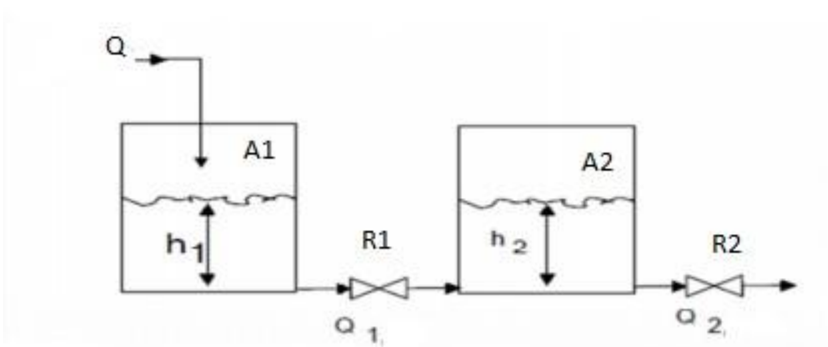
MATHEMATICAL MODELING OF PROCESSES

To analyze the behavior of a process, a mathematical representation of the physical and chemical phenomenon taking place in it is essential and this representation constitutes the mathematical model. The activities leading to the construction of the model is called modeling. The main uses of mathematical modeling are

1. To improve understanding of the process
2. To optimize process design and hence operating conditions.
3. To design a control strategy for the process
4. To train operating personnel.
5. The model-based control action is 'intelligent' and helps in achieving uniformity, disturbance rejection, and set point tracking, all of which translate into better process economics.

INTERACTING AND NONINTERACTING SYSTEM

Interacting system



Mass balance equation : Inflow-outflow= Rate of accumulation

For tank 1

$$A_1 \frac{dh_1}{dt} = Q - Q_1 \qquad (1)$$

Assume linear resistance to flow

$$Q_1 = \left(\frac{h_1 - h_2}{R_1} \right)$$

$$A_1 \frac{dh_1}{dt} = Q - \left(\frac{h_1 - h_2}{R_1} \right) \qquad (2)$$

$$A_1 R_1 \frac{dh_1}{dt} = R_1 Q \ \ - h_1 + h_2 \qquad (3)$$

Time constant of tank 1, $\tau_1 = A_1 R_1$

$$\tau_1 \frac{dh_1}{dt} + h_1 - h_2 = R_1 Q \qquad (4)$$

Taking laplace transform on both sides

$$\tau_1 s h_1(s) + h_1(s) - h_2(s) = R_1 Q\ (s)$$

$$h_1(s)(\tau_1 s + 1) - h_2(s) = R_1\ Q\ (s) \qquad (5)$$

For tank 2

$$A_2 \frac{dh_2}{dt} = Q_1 - Q_2 \qquad (6)$$

Assume linear resistance to flow

$$Q_2 = \left(\frac{h_2}{R_2}\right)$$

$$A_2 \frac{dh_2}{dt} = \left(\frac{h_1 - h_2}{R_1}\right) - \left(\frac{h_2}{R_2}\right) \qquad (7)$$

$$A_2 R_2 \frac{dh_2}{dt} + h_2 + \frac{R_2}{R_1} h_2 = \frac{R_2}{R_1} h_1$$

Time constant of tank2 $\tau_2 = A_2 R_2$

$$\tau_2 \frac{dh_2}{dt} + h_2 + \frac{R_2}{R_1} h_2 = \frac{R_2}{R_1} h_1 \qquad (8)$$

Taking laplace transform on both sides

$$\tau_2 s h_2(s) + h_2(s) + \frac{R_2}{R_1} h_2(s) = \frac{R_2}{R_1} h_1(s) \qquad (9)$$

$$h_2(s)\left(\tau_2 s+1+\frac{R_2}{R_1}\right)=\frac{R_2}{R_1}h_1(s) \tag{10}$$

To obtain $\frac{h_2(s)}{q_{in}(s)}$, cancel $h_1(s)$ in equations (5) & (10)

$$h_2(s)\left(\tau_2 s+1+\frac{R_2}{R_1}\right)(\tau_1 s+1)-(\tau_1 s+1)\frac{R_2}{R_1}h_1(s)=0 \tag{11}$$

$$-h_2(s)\frac{R_2}{R_1}+h_1(s)(\tau_1 s+1)\frac{R_2}{R_1}= Q(s)R_2 \tag{12}$$

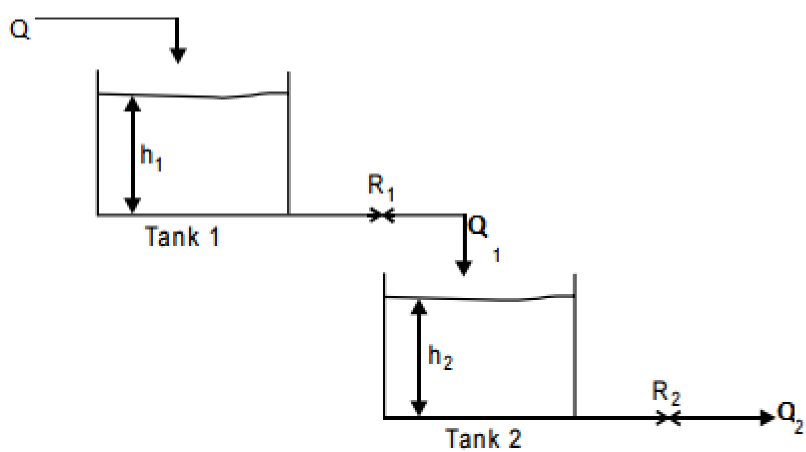
$$h_2(s)\left(\tau_1\tau_2 s^2+\tau_1 s+\frac{R_2}{R_1}\tau_1 s+\tau_2 s+1\right)=R_2 \ Q(s) \tag{13}$$

where, Time constant of tank1 , $\tau_1 = A_1R_1$

$$h_2(s)(\tau_1\tau_2 s^2+\tau_1 s+\tau_2 s+A_1R_2 s+1)=R_2 \ Q(s)$$

$$\frac{h_2(s)}{Q(s)}=\frac{R_2}{\tau_1\tau_2 s^2+s(\tau_1+A_1R_2+\tau_2)+1} \tag{14}$$

Non interacting system



For tank 1

$$A_1 \frac{dh_1}{dt} = Q - Q_1 \qquad (1)$$

Assume linear resistance to flow

$$Q_1 = \frac{h_1}{R_1}$$

$$A_1 \frac{dh_1}{dt} = Q - \left(\frac{h_1}{R_1}\right) \qquad (2)$$

$$A_1 R_1 \frac{dh_1}{dt} = R_1 Q - h_1 \qquad (3)$$

Time constant of tank 1, $\tau_1 = A_1 R_1$

$$\tau_1 \frac{dh_1}{dt} + h_1 = R_1 Q \qquad (4)$$

Taking laplace transform on both sides

$$\tau_1 s h_1(s) + h_1(s) = R_1 Q(s)$$

$$h_1(s)(\tau_1 s + 1) = R_1 Q(s) \qquad (5)$$

For tank 2

$$A_2 \frac{dh_2}{dt} = Q_1 - Q_2 \qquad (6)$$

Assume linear resistance to flow

$$Q_2 = \frac{h_2}{R_2}$$

$$A_2 \frac{dh_2}{dt} = \left(\frac{h_1}{R_1}\right) - \left(\frac{h_2}{R_2}\right) \qquad (7)$$

$$A_2R_2\frac{dh_2}{dt}=R_2Q_1-h_2 \quad (8)$$

Time constant of tank 2 $\tau_2 = A_2R_2$

$$\tau_2\frac{dh_2}{dt}+h_2=R_2Q_1 \quad (9)$$

Taking laplace transform on both sides

$$\tau_2 sh_2(s)+h_2(s)=R_2Q_1(s) \quad (10)$$

$$h_2(s)(\tau_2 s+1)=R_2Q_1(s)$$

$$\frac{h_2(s)}{Q_1(s)}=\frac{R_2}{(\tau_1 s+1)\ (\tau_2 s+1)} \quad (11)$$

Degrees of Freedom:

In control engineering, a degree of freedom analysis is necessary to determine the regulatable variables within the chemical process. These variables include descriptions of state such as pressure or temperature as well as compositions and flow rates of streams.

Definition:

The state of process or the configuration of the system is determined when each of its degrees of freedom is specified.

The number of degrees of freedom is defined as

$$n = n_v - n_e$$

Where n -number of degrees of freedom of a system

n_v -number of variables that describe the system

n_e -number of defining equation of the system

Example: A ball placed on the Billiard table

number of variables, n_v = 3 (1.north south coordinate, 2.east west coordinate,3.height)

number of defining equation, n_e =1 (height of the table) number of degrees of freedom, $n = n_v - n_e$

=3-1 =2

The number of process variables over which the operator or designer may exert control. Specifically, control degrees of freedom include:

1. The number of process variables that may be manipulated once design specifications are set
2. The number of said manipulated variables used in control loops
3. The number of single-input, single-output control loops
4. The number of regulated variables contained in control loops

The degrees of freedom of a process represents the maximum number of independently acting controllers that can be placed on the process.

The chemical processes involving separation, distillation or fractionation where heterogeneous equilibrium exists and where each component is present in each phase, the modification of the rule may be derived. It is known as Gibb's phase rule.

$$n = n_c - n_p + 2$$

Where n -number of Chemical degrees of freedom

n_c -number of components

n_p -number of Phases

The number 2 represents temperature and pressure

Example: steam boiler producing saturated steam

number of components , n_c =1 (water)

number of Phases, n_p =2 (liquid ,gas)

$$n = n_c - n_p + 2$$

=1-2+2 =1

For isothermal process

$$n = n_c - n_p + 1$$

For a constant pressure process.

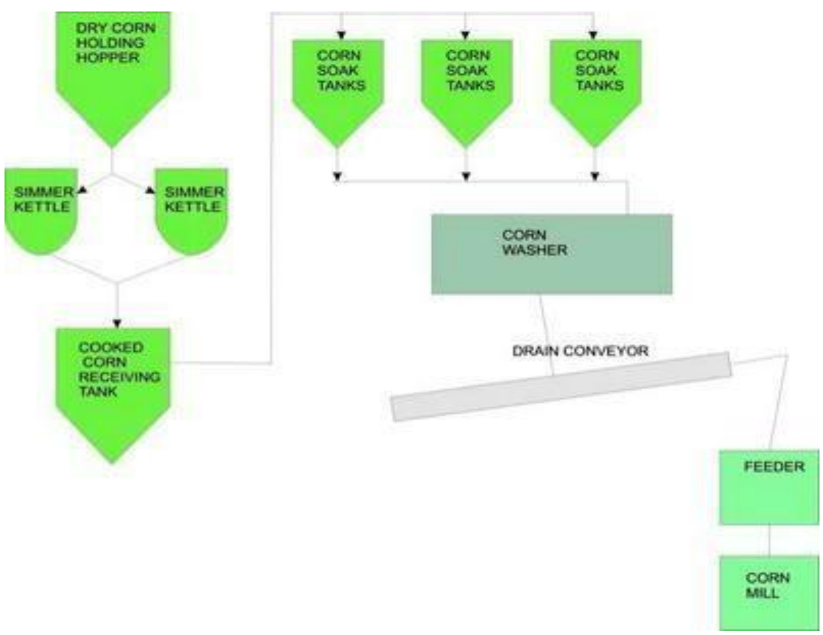
BATCH PROCESS AND CONTINUOUS PROCESS:

Batch process: A process in which the materials or work are stationary at one physical location while being treated is termed as batch process. eg: annealing of steel, coke making in coke ovens, furnaces in foundries, batch reactor in chemical plants.

Batch process are most often of the thermal type where materials are placed in a vessel or furnace and the system is controlled for a cycle of temperatures under controlled pressure for a period of time. It is always defined by temperature, pressure or associated conditions such as compositions. Its degree of freedom is well defined .The purpose of such processes is to produce one or more products at

- ✓ A given composition
- ✓ A maximum amount
- ✓ Best economy(least materials, energy and time)

Example for batch Process



The batch process has the following advantages versus the continuous process:

- Flexibility when the feed water quality changes
- System recovery can be maximized batch by batch
- Cleaning is easily implemented

• Simple automatic controls

• Permeate quality can be controlled by termination of the process

• Permeate quality can be improved by total or partial second-pass treatment

• Favorable operating conditions for single (or low number) element systems, because the membranes are only in contact with the final concentrate for a short time

• Expansion is rather easy

• Lower investment costs

The disadvantages are:

• No continuous permeate flow

• No constant permeate quality

• Large feed tank required

• Larger pump required

• Larger power consumption

• Longer residence time for feed/concentrate

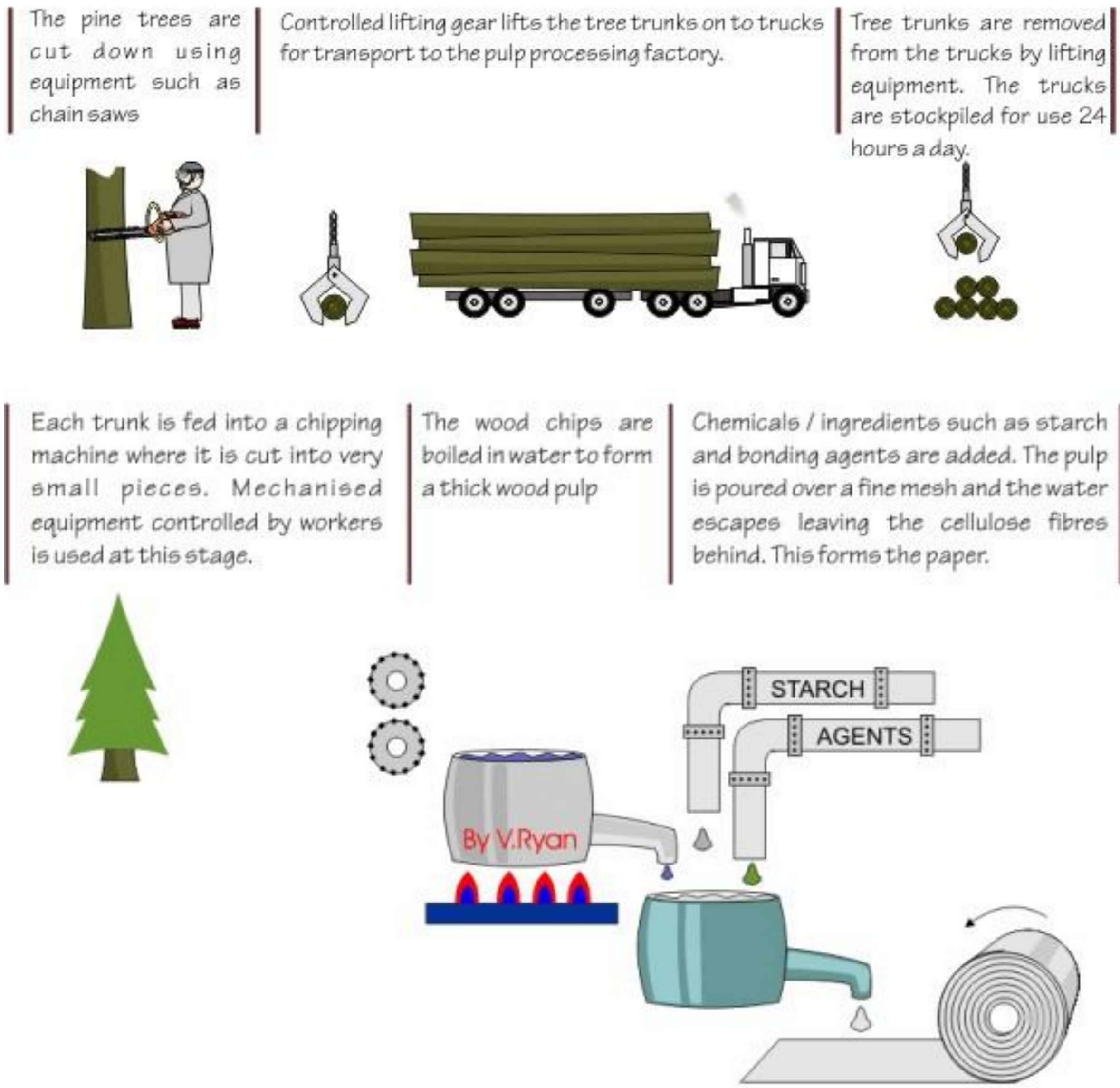
• Higher total running costs

Continuous Process: A process in which the materials or work flows more or less continuously through a plant apparatus while being manufactured or treated is termed a continuous process. eg: production of sinter, continuous annealing of metal sheets, production of steam, continuously stirred tank reactors(CSTR).

The purpose of such processes is to produce one or more products at

- ✓ A given composition
- ✓ A given maximum flow rate
- ✓ Best economy(employing least materials, energy and time)

Examples for continuous process



Comparision between Batch & continuous process

	Batch Process	Continuous Process
Types of materials	Can be used with all types of materials (with non-flow materials, it is easier to use the batch process).	Easier for use with flowing materials (today, almost any material can be produced with the continuous process; investment cost is the decisive factor).
Installation size	Relatively large installations. Very big investment in land and installations.	Relatively small installations. Significant savings in land and installations.
Reactor	Changes occur in the concentrations of materials over time.	At all locations, conditions are constant over time (durable conditions).
Feeding raw materials	Raw materials are fed before the start of the reaction.	Constant feeding of raw materials during the entire reaction process.
Control of the set of actions in the system	Simple control. It is easier to control reaction conditions (pH, pressure, temperature). Manual control can also be done.	Complex control. Automatic control must be used. Control of reactor conditions is more difficult. Control must be exercised over the rate of flow of the materials.
Product(s)	Extraction of materials only after all the actions are finished with the conclusion of the reaction.	Continuous extraction of products at all times during the reaction.
Trouble shooting	A fault or dealing with a batch requiring "repair" does not cause problems in the other stages. Appropriate tests are conducted after each stage.	The installations are interconnected, so a fault in one causes a stoppage in all the others. Material that has been damaged cannot be repaired under the same working conditions. It must be isolated and the process restarted.
Quantities	Preferable when production of small	Preferable for large scale production.

produced	quantities of a specific material are planned.	
Variety of products in the plant	Preferable when the plant produces a wide variety of materials and when the product is likely to be changed now and again, while using the same reactor.	Preferable for a central and permanent product.
Product development stage	Preferable when the process is relatively new and still unfamiliar. In this case the initial investment is in a smaller batch reactor, and thus the economic risk is smaller.	Preferable after the conclusion of all the stages of grossing-up and economic feasibility tests.

SELF REGULATOR:

A significant characteristic of some processes to adopt a specific value or stable value of controlled variable under nominal load without regulation via process control loop is called self-regulation. The control operations are significantly affected by the selfregulation. (or)

Self regulation of a process is defined as the process is one in which either inflow and outflow is dependent to the controlled variable. Most of the causes the flow is self regulating because of its steady state is increased by increasing the outflow.

An example of a self regulating process is a tank of water with an input of water entering the tank and an output of water leaving the tank. Let the water level in the tank is constant at 10 inches. Water enters the tank at a rate of 20 gallons per minute. As long as this balance is maintained water level in the tank will remain constant at 10 inches.

Problem: What happens if the outlet valve is opened an 1/8 of a turn and water leaving the tank changes toa rate of 25 gallons per minute.

Since this is a self regulating process, the level will actually stabilize at a new position and maintain that position. Flow out of the tank is proportional to the square root of the differential pressure across the output valve. As level decrease, the differential pressure will also decrease, causing the rate of drainage to decrease. At some point , the drainage rate will once again equal the fill rate, and the tank will reach a new equilibrium point.

Time constants

Every self regulated process has a time constant associated with it. The time constant is the amount of time it takes the process 63.2% of the final value of the process. In this example, the process changes by 10%. The time it takes to change 6.32 inches(63.2% of 10 inches), is the time constant. It takes five time constants in order for the process to complete the total change

Process Gain

The time constant is affected by the capacity of the process and the process resistance to change. The larger the process capacity, the longer the time constant, and the more resistive elements in the process the longer the time constant.

Dead Time:

Dead time is defined as the time difference between when a change occurs in a process and when the change is detected. Dead time exists in all processes and is a factor in the control loop control, which must be addressed when tuning the loop.

SERVO AND REGULATOR OPERATION

Servomechanisms and Regulators are used to control the process either via automatic controllers or as a self contained unit. They are physically doing the job of adjusting the manipulated variable to have the controlled variable at around set point. A controller automatically adjusts one of the inputs to the process in response to a signal fed back from the process output.

Servo Operation: If the purpose of the control system is to make the process follow changes in the set point as closely as possible, such an operation is called servo operation. Changes in load variables such as uncontrolled flows, temperature and pressure cause large errors than the set point changes (normally in batch processes). In such cases servo operation is necessary. Though the set point changes quite slowly and steadily, the errors from load changes may be as large as the errors caused by the change of set point. Eg: ship steering mechanism

Regulator Operation: In many of the process control applications, the purpose of control system is to keep the output (controlled variable) almost constant in spite of changes in load. Mostly in continuous processes the set point remains constant for longer time. Such an operation is called 'Regulator Operation'. The set point generated and the actual value from sensors is given to a controller. The controller compares both the signals, generates error signal which is utilized to generate a final signal as controller output. Eg. Process water heater

Part A Questions

1. Distinguish between batch process and continuous process.
2. Define degrees of freedom.
3. What is meant by self-regulation
4. What is non-self regulation.
5. Distinguish between servo and regulator operation of control system
6. Write any two characteristics of first order process
7. Define interacting system
8. Define non interacting system
9. What is the need for mathematical model
10. What is the significance of "degree of freedom"?
11. What is the need for servo operation
12. Define process variable,
13. Define load variable
14. Define manipulated variable

Part B Questions

1. Derive a mathematical model of a first order thermal process.
2. Differentiate servo and regulatory operation with the help of suitable example
3. Derive the mathematical model for the interacting system
4. Derive the mathematical model for the non interacting system
5. Bring out the difference between the continuous and batch process with the help of neat diagrams.
6. Derive a mathematical model for the dynamics of simple Pressure process

UNIT II

VARIOUS CONTROLLERS AND ITS CHARACTERSTICS

A controller is a device, using mechanical, hydraulic, pneumatic or electronic techniques often in combination, which monitors and physically alters the operating conditions of a given dynamical system. Broad classifications of different controller modes used in process control are as follows:

Controller Modes

Controller modes refer to the methods to generate different types of control signals to final control element to control the process variable.

Broad classifications of different controller modes used in process control are as follows:

(1) Discontinuous Controller Modes
  - (a) Two-position (ON/OFF) Mode
  - (b) Multiposition Mode
  - (c) Floating Control Mode: Single Speed and Multiple Speed

(2) Continuous Controller Modes
  - (a) Proportional Control Mode
  - (b) Integral Control Mode
  - (c) Derivative Control Mode

(3) Composite Controller Modes
  - (a) Proportional-Integral Control (PI Mode)
  - (b) Proportional-Derivative Control (PD Mode)
  - (c) Proportional-Integral-Derivative Control (PID or Three Mode Control)

Based on the controller action on the control element, there are two modes:

(1) *Direct Action*: If the controller output increases with increase in controlled variable then it is called direct action.
(2) *Reverse Action*: If the controller output decreases with increase in controlled variable then it is called reverse action

The choice operating mode for any given process control system is complicated decision. It involves not only process characteristics but cost analysis, product rate, and other industrial factors. The process control technologist should have good understanding of the operational mechanism of each mode and its advantages and disadvantages.

In general, the controller operation for the error e_p is expressed as a relation:

$$p = F(e_p) \qquad (2.1)$$

where $F(e_p)$ represents the relation by which the appropriate controller output is determined.

Discontinuous Controller Modes

In these controller modes the controller output will be discontinuous with respect to controlled variable error.

Two-Position (ON/OFF) Mode

The most elementary controller mode is the two-position or ON/OFF controller mode. It is the simplest, cheapest, and suffices when its disadvantages are tolerable. The most general form can be given by

$$P = \begin{cases} 0\ \% & e_p < 0 \\ 100\ \% & e_p > 0 \end{cases} \qquad (2.2)$$

The relation shows that when the measured value is less than the setpoint (i.e. $e_p > 0$), the controller output will be full (i.e. 100%), and when the measured value is more than the setpoint (i.e. $e_p < 0$), the controller output will be zero (i.e. 0%).

Neutral Zone: In practical implementation of the two-position controller, there is an overlap as e_p increases through zero or decreases through zero. In this span, no change in the controller output occurs which is illustrated in Fig. 2.1

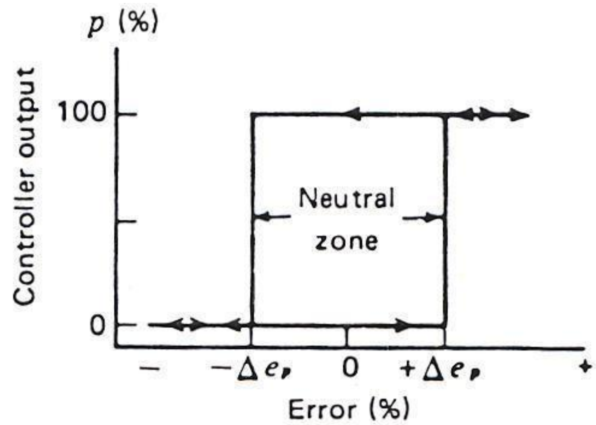


Fig. 2.1 Two-position controller action with neutral zone.

It can be observed that, until an increasing error changes by Δe_p above zero, the controller output will not change state. In decreasing, it must fall Δe_p below zero before the controller changes to 0%. The range $2\Delta e_p$ is referred to as *neutral zone or differential gap*. Two-position controllers are purposely designed with neutral zone to prevent excessive cycling. The existence of such a neutral zone is an example of desirable hysteresis in a system.

Applications: Generally the two-position control mode is best adapted to:

- Large-scale systems with relatively slow process rates

Example: Room heating systems, air-conditioning systems.

Systems in which large-scale changes are not common
Examples: Liquid bath temperature control, level control in large-volume tanks.

Proble m 2.2

A liquid- level control system linearly converts a displacement of 2 to 3 m into a 4 to 20 mA control signal. A relay serves as the two-position controller to open and close the inlet valve. The relay closes at 12 mA and opens at 10 mA. Find (a) the relation between displacement level and current, and (b) the neutral zone or displacement gap in meters.

Solution

Given data: Liquid- level range = 2 to 3 m i.e. H_{min} = 2m & H_{max} = 3m

Control signal range = 4 to 20 mA i.e. I_{min} = 4mA & I_{max} = 20mA

(a) Relation between displacement level (H) and current (I)

(b) Neutral zone (NZ) in meters.

(a) The linear relationship between level and current is given by

$$H = K\,I + Ho$$

The simultaneous equations for the above range are:

For low range signal 2 = K x 4 + Ho

For higher range signal 3 = K x 20 + Ho Solving

the above simultaneous equations we get:

K = 0.0625 m/mA, & Ho = 1.75 m

Therefore, the relation between displacement level (H) and current (I) is given by

$$\mathbf{H = 0.0625\ I + 1.75}$$

(b) The relay closes at 12 mA, which is high level, H_H

$$H_H = 0.0625 \times 12 + 1.75 = 2.5$$

m The relay opens at 10 mA, which is low level, H_L

$$H_L = 0.0625 \times 12 + 1.75 = 2.375 \text{ m}$$

Therefore, the neutral zone, NZ = (H_H - H_L) = (2.5 - 2.375) = 0.125 m

Proble m 2.3

As a water tank loses heat, the temperature drops by 2 K/min when a heater is on, the system gains temperature at 4 K/min. A two- position controller has a 0.5 min control lag and a neutral zone of ± 4% of the setpoint about a setpoint of 323 K. Plot the heater temperature versus time. Find the oscillation period.

Solution

Given data:

Temperature drops = 2 K/min

Temperature rises = 4 K/min

Control Lag = 0.5 min

Neutral zone = ± 4%

Setpoint = 323 K

± 4% of 323 = 13 K. Therefore, the temperature will vary from 310 to 336 K (without considering the lag)

Initially we start at setpoint value. The temperature will drop linearly, which can be expressed by

$$T_1(t) = T(ts) - 2\,(t - ts)$$

where ts = time at which we start the observation

T(ts) = temperature when we start observation i.e. 323.

The temperature will drop till - 4% of setpoint (323K), which is 310 K.

Time taken by the system to drop temperature value 310 K is

$$310 = 323 - 2\,(t - 0)$$

$$t = 6.5 \text{ min}$$

Undershoot due to control lag = (control lag) x (drop rate) = 0.5 min x 2 K/min = 1 K Due control lag temperature will reach 309 instead of 310 K.

From this point the temperature will rise at 4 K/min linearly till +4% of set point i.e.336K which can be expressed by

$$T_2(t) = T(t_h) + 2\ (t - t_h)$$

where t_h = time at which heater goes on

$T(t_h)$ = temperature at which heater goes on

$$336 = (310\text{-}1) + 4\ [t - (6.5 + 0.5)]$$

$$t = 13.75 \text{ min}$$

Overshoot due to control lag = (control lag) x (rise rate) = 0.5 min x 4 K/min = 2 K Due control lag temperature will reach 338 instead of 336 K.

The oscillation period is = 13.75 + 0.5 +0.5 + 6.5 = 21.25 ≈ 21.5 min
The system response is plotted as shown in Fig. 2.2 with undershoot and overshoot values

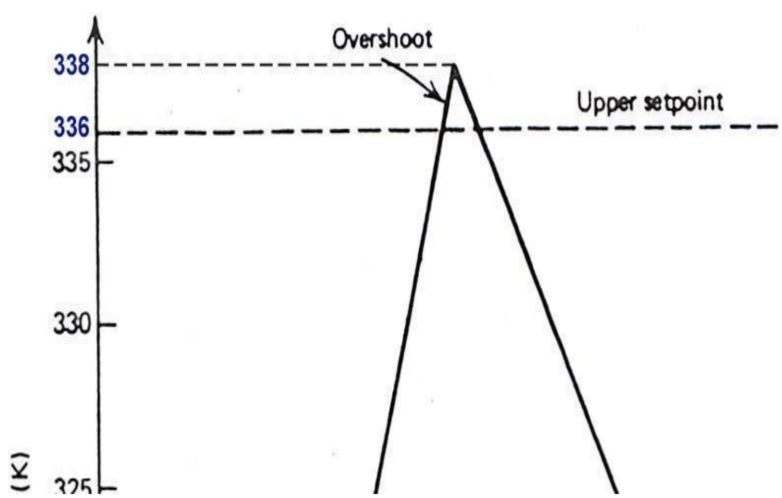


Fig. 2.2 Plot of heater temperature versus time for Problem 2.3

Proble m 2.4

A 5m diameter cylindrical tank is emptied by a constant outflow of 1.0 m^3/min. A two position controller is used to open and close a fill valve with an open flow of 2.0 m^3/min. For level control, the neutral zone is 1 m and the setpoint is 12 m. (a) Calculate the cycling period (b) Plot the level vs time.

Solution

Given data: Diameter cylindrical tank = 5 m, therefore radius, r = 2.5 m

Output flow rate (Q_{out}) = 1.0 m^3/min

Input flow rate (Q_{in}) = 2.0 m^3/min

Neutral Zone (h) = 1 m

Setpoint = 12 m

(a) The volume of the tank about the neutral zone is

$$V = \Pi r^2 h$$

$$V = 3.142 \text{ x } (2.5)^2 \text{ x } 1 = 19.635 \text{ m}^3$$

$Q_{in} = 2.0 \text{ m}^3/\text{min}$, and $Q_{out} = 1.0 \text{ m}^3/\text{min}$

Therefore, net inflow into the tank = $Q = Q_{in} - Q_{out} = 2-1 = 1 \text{ m}^3/\text{min}$

To fill 1 m^3 of tank it requires 1 min, therefore to fill 19.635 m^3 of tank requires 19.63min

Similarly it takes same time for the tank to get emptied by 19.635 m^3 i.e. 19.635 min.

Therefore, Cycling period = 19.635 + 19.635 = 39.27 ≈ 39.3 min

(b) Plot the level vs time

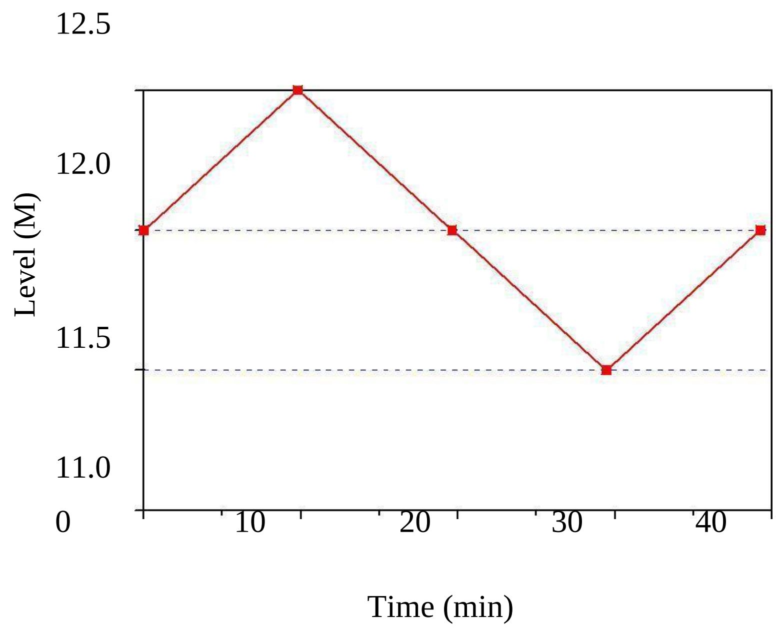


Fig. 2.3 Plot of level versus time for Problem 2.4

Multiposition Control Mode

It is the logical extension of two-position control mode to provide several intermediate settings of the controller output. This discontinuous control mode is used in an attempt to reduce the cycling behaviour and overshoot and undershoot inherent in the two-position mode. This control mode can be preferred whenever the performance of two-position control mode is not satisfactory.

The general form of multiposition mode is represented by

$$p = p_i \qquad e_p > |e_i| \quad i = 1, 2, \ldots, n \tag{2.3}$$

As the error exceeds certain set limits $\pm e_i$, the controller output is adjusted to present values p_i.

Three-position Control Mode: One of the best example for multiposition control mode is three-position control mode, which can be expressed in the following analytical form:

$$p = \begin{cases} 100\% & e_p > -e_1 \\ 50\% & -e_1 < e_p < +e_1 \\ 0\% & e_p < -e_1 \end{cases}$$

As long as the error is between $+e_1$ *and* $-e_1$ of the set point, the controller stays at some nominal setting indicated by a controller output of 50%. If the error exceeds the set point by $+e_1$ or more, then the output is increased to 100%. If it is less than the set point by $-e_1$ or more, the controller output is reduced to zero. Figure 2.4 illustrates three-position mode graphically.

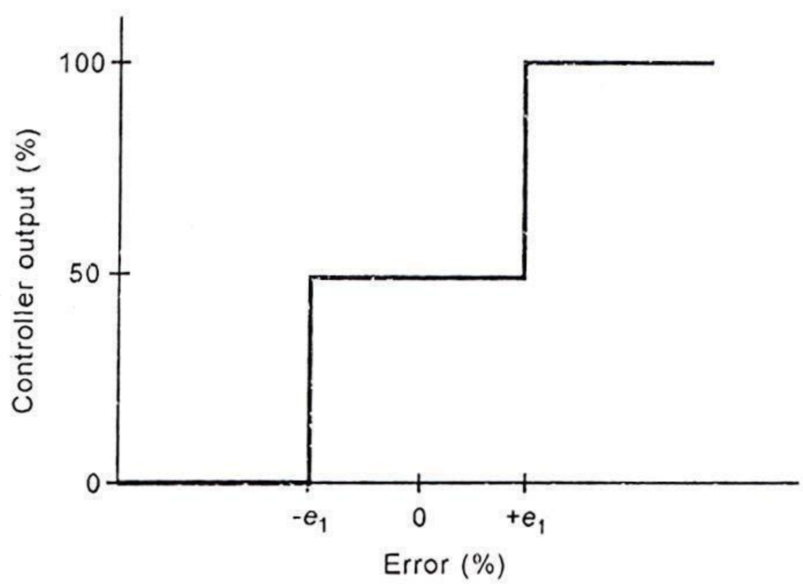


Fig. 2.4 Three-position controller action

The three-position control mode usually requires a more complicated final control element, because it must have more than two settings. Fig. 2.5 shows the relationship between the error and controller output for a three-position control. The finite time required for final control element to change from one position to another is also shown. The graph shows the overshoot and undershoots of error around the upper and lower setpoints. This is due to both the process lag time and controller lag time, indicated by the finite time required for control element to reach new setting.

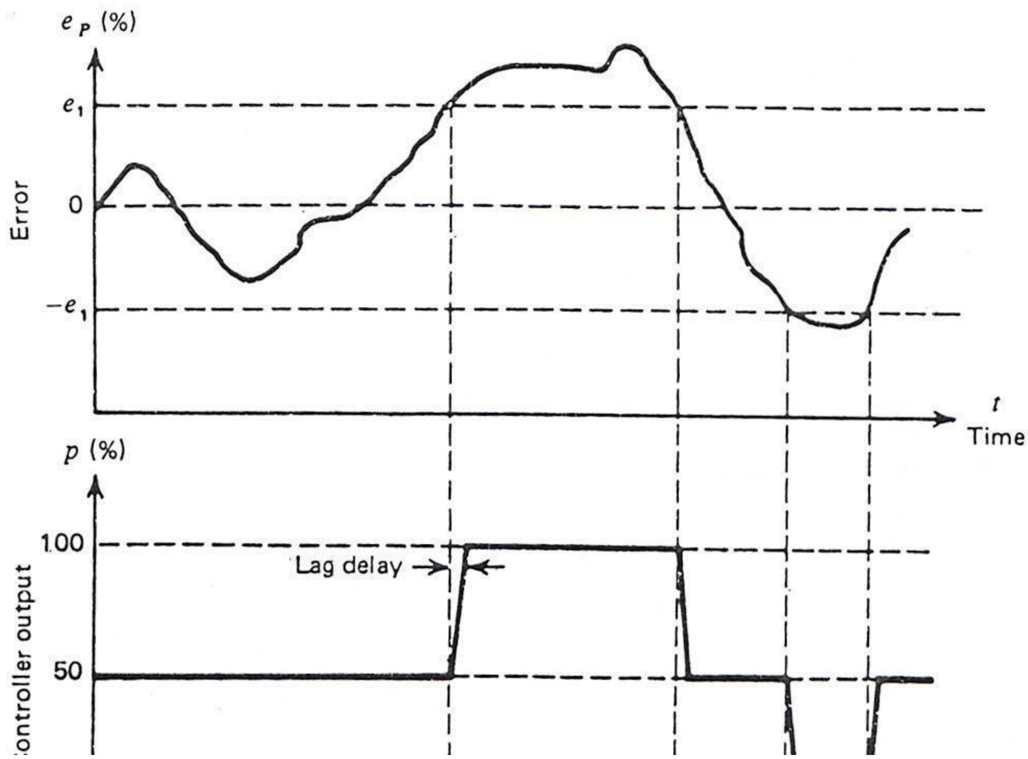


Fig. 2.5 Relationship between error and three-position controller action, including the effectsoflag.

Floating Control Mode

In floating control, the specific output of the controller is not uniquely determined by error. If the error is zero, the output does no t change but remains (floats) at whatever setting it was when error went to zero. When error moves of zero, the controller output again begins to change. Similar to two-position mode, there will be a neutral zone around zero error where no change in controller output occurs. Popularly there are two types:

(a) Single Speed

(b) Multiple Speed

(a) Single Speed: In this mode, the output of the control element changes at a fixed rate when the error exceeds the neutral zone. The equation for single speed floating mode is:

$$\frac{dp}{dt} = \pm K_F \qquad |e_p| > \Delta e_p \tag{2.5}$$

where dp/dt = rate of change of controller output with time
K_F = rate constant $(\% / s)$
Δe_p = half the neutral zone

If the equation (5) is integrated for actual controller output, we get

$$p = \pm K_F \, t + p(0) \qquad |e_p| > \Delta e_p \tag{2.6}$$

where $p(0)$ = controller output at $t = 0$

The equation shows that the present output depends on the time history of errors that have previously occurred. Because such a history is usually not known, the actual value of p floats at an undetermined value. If the deviation persists, then equation (6) shows that the controller saturates at 100% or 0% and remains there until an error drives it toward the opposite extreme. A graph of single speed floating control is shown in Fig.2.6

The single- speed controller action as output rate of change to input error is shown in Fig.2.6 (a). The graph in Fig.2.6 (b) shows a reverse acting controller, which means the controller output decreases when error exceeds neutral zone, which corresponds to negative K_F in equation (5). The graph shows that the controller starts at some output p(0). At time t_1, the error exceeds the neutral zone, and the controller output decreases at a constant rate until t_2 , when the error again falls below the neutral zone limit. At t_3 , the error falls below the lower limit of neutral zone, causing controller output to change until the error again moves within the allowable band.

(b) Multiple Speed: In this mode several possible speeds (rate) are changed by controller output. Usually, the rate increases as the deviation exceeds certain limits. For speed change point e_{pi} error there will be corresponding output rate change *Ki*. The expression can be given by

$$\frac{dp}{dt} = \pm K_{Fi} \qquad \left|e_p\right| > e_{pi} \qquad (2.7)$$

If the error exceeds e_{pi} , then the speed is K_{Fi} . If the error rises to exceed e_{p2}, the speed is increased to K_{F2} , and so on. The graph of multiple-speed mode is shown in Fig. 2.7

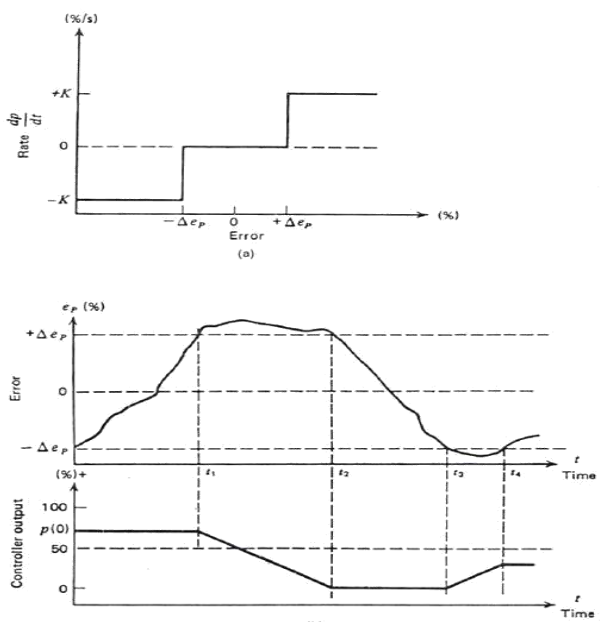


Fig. 2.6 Single speed floating controller (a) Controller action as output rate of change to input error, and (b) Error versus controller response.

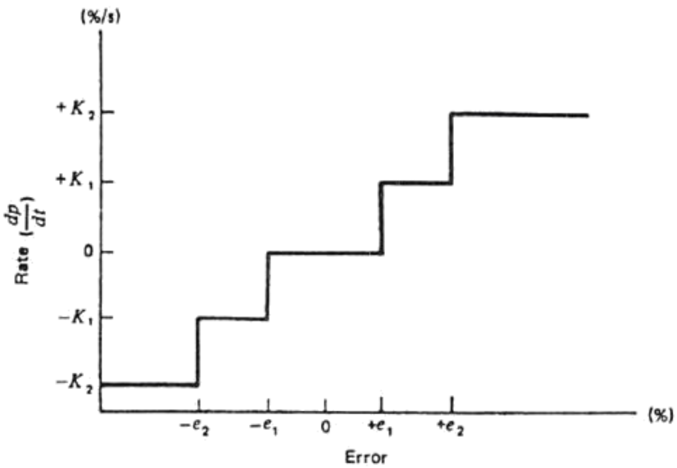


Fig. 2.7 Multiple-speed floating control mode action.

Applications:

- Primary applications are in single-speed controllers with neutral zone
- This mode is well suited to self-regulation processes with very small lag or dead time, which implies small capacity processes. When used for large capacity systems, cycling must be considered.

The rate of controller output has a strong effect on the error recovery in floating control mode.In continuous controller modes the controller output changes smoothly in response to the error or rate of change of error. These modes are an extension of discontinuous controller modes. In most of the industrial processes one or combination of continuous controllers are preferred.

Proportional Control Mode

In this mode a linear relationship exits between the controller output and the error. For some range of errors about the setpoint, each value of error has unique value of controller output in one-to-one correspondence. The range of error to cover the 0% to 100% controller output is called proportional band, because the one-to-one correspondence exits only for errors in this range. The analytical expression for this mode is given by:

$$p = K_p e_p + p_0 \qquad (2.8)$$

where K_p = proportional gain (% per %)

p_0 = controller output with no error or zero error (%)

The equation (8) represents reverse action, because the term $K_p e_p$ will be subtracted from p_0 whenever the measured value increases the above setpoint which leads negative error. The equation for the direct action can be given by putting the negative sign in front of correction term i.e. - $K_p e_p$. A plot of the proportional mode output vs. error for equation (8) is shown in Fig.2.9

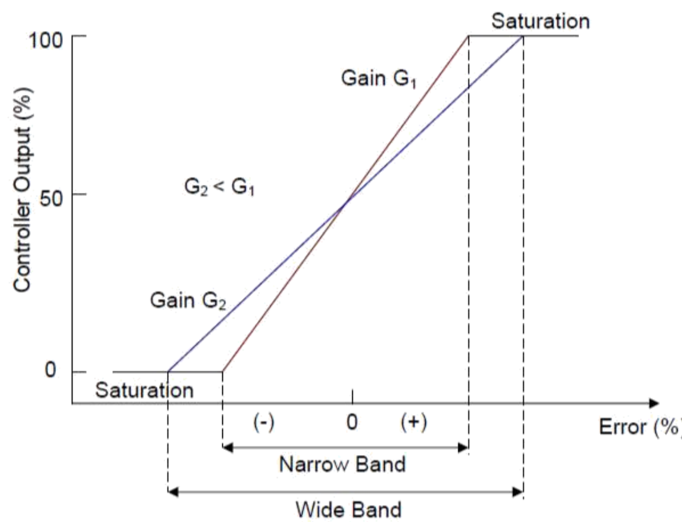


Fig. 2.9 Proportional controller mode output vs. error.

In Fig.2.9, p_0 has been set to 50% and two different gains have been used. It can be observed that proportional band is dependent on the gain. A high gain (G_1) leads to large or fast response, but narrow band of errors within which output is not saturated. On the other side a low gain (G_2) leads to small or slow response, but wide band of errors within which output is not saturated. In general, the proportiona l band is defined by the equation:

$$PB = \frac{100}{K_p} \tag{2.9}$$

The summary of characteristics of proportional control mode are as follows:

1. If error is zero, output is constant and equal to p_0.

2. If there is error, for every 1% error, a correction of K_p percent is added or subtracted from p_0, depending on sign of error.

3. There is a band of errors about zero magnitude PB within which the output is not saturated at 0% or 100%.

Offset: An important characteristic of the proportional control mode is that it produces a permanent residual error in the operating point of the controlled variable when a load change occurs and is referred to as offset. It can be minimized by larger constant K_p which also reduces the proportional band. Figure 2.10 shows the occurrence of offset in proportional control mode.

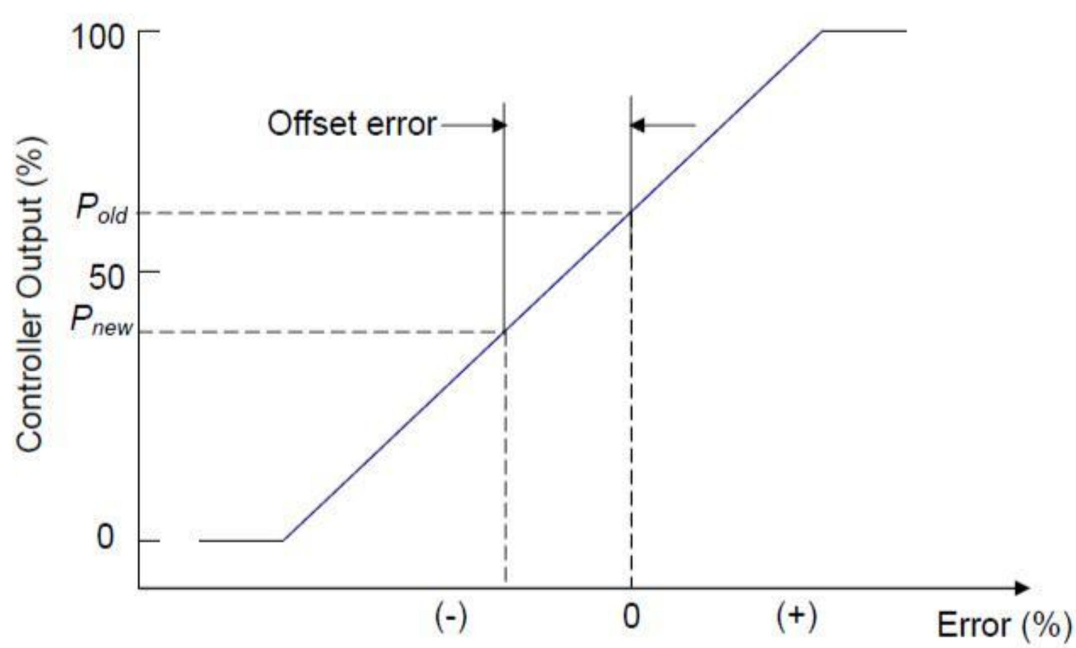


Fig. 2.10 Occurrence of offset error in proportional controller for a load change.

Consider a system under nominal load with the controller output at 50% and error zero as shown in Fig.2.10 If a transient error occurs, the system responds by changing controller output in correspondence with the transient to effect a return-to-zero error. Suppose, however, a load change occurs that requires a permanent change in controller output to produce the zero error state. Because a one-to-one correspondence exists between controller output and error, it is clear that a new zero-error controller output can never be achieved. Instead, the system produces a small permanent offset in reaching compromise position of controller output under new loads.

Applications:

- Whenever there is one-to-one correspondence of controller output is required with respect to error change proportional mode will be ideal choice.

- The offset error limits the use of proportional mode, but it can be used effectively wherever it is possible to eliminate the offset by resetting the operating point.
- Proportional control is generally used in processes where large load changes are unlikely or with moderate to small process lag times.

- If the process lag time is small, the PB can be made very small with large K_p, which reduces offset error.

- If K_p is made very large, the PB becomes very small, and proportional controller is going to work as an ON/OFF mode, i.e. high gain in proportional mode causes oscillations of the error.

Proble m 2.5

For a proportional controller, the controlled variable is a process temperature with a range of 50 to 130 ^{o}C and a setpoint of 73.5 ^{o}C. Under nominal conditions, the setpoint is maintained with an output of 50%. Find the proportional offset resulting from a load change that requires a 55% output if the proportional gain is (a) 0.1 (b) 0.7 (c) 2.0 and (d) 5.0.

Solution:

Given data: Temperature Range = 50 to 130 ^{o}C

Setpoint (Sp)	= 73.5 ^oC
Po	= 50%
P	= 55%
e_p	= ?
Offset error	= ? for Kp=0. 1, 0. 7, 2. 0 & 5. 0

For proportional controller: P = Kp e_p + Po

e_p = [p-Po] / Kp = [55 – 50] / Kp = 5 / Kp %

(a) when Kp = 0.1 Offset error, e_p = 5/0.1 = 50%

(b) when Kp = 0.7 Offset error, e_p = 5/0.7 = 7.1%

(c) when Kp = 2.0 Offset error, e_p = 5/2.0 = 2.5%

(d) when Kp = 5.0 Offset error, e_p = 5/5.0 = 1%

[It can be observed from the results that as proportional gain Kp increases the offset error decreases.]

Proble m 2.6

A proportional controller has a gain of Kp = 2.0 and Po = 50%. Plot the controller output for the error given by Fig.2.11.

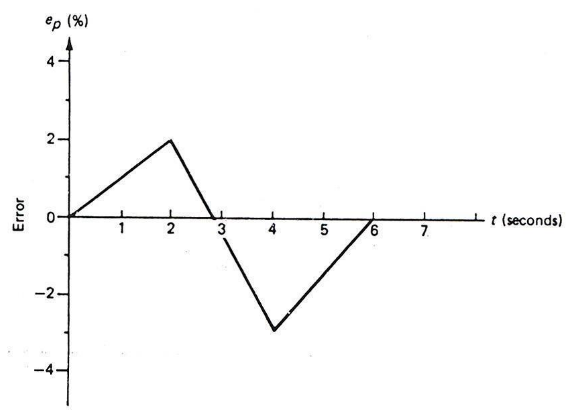


Fig. 2.11 Error graph

Solution:

Given data: Kp = 2.0 Po

= 50%

Error graph as in Fig.2.11

To find the controller output and plot the response, first of all we need to find the error which is changing with time and express the error as function of time. The error need to be found in three time regions: (a) 0-2 sec (b) 2-4 sec (c) 4-6 sec.

Since, the error is linear, using the equation for straight line we find the error equation

i.e. Ep = mt + c (i.e. Y = mX + c)

(a) *For error segment 0-2 sec:*

Slope of the line, m = $[Y_2-Y_1] / [X_2-X_1]$ = [2-0]/[2-0] = 1

Y = mX + c

2 = 1 x t + c, 2 = 1x 2 + c, c = 0

Therefore, error equation, Ep = t

Controller output P = Kp Ep + Po = 2 t + 50

Therefore, at **t = 0 sec, P = 50%** and **at t = 2 sec, P = 54%**

(b) *For error segment 2-4 sec:*

Slope of the line, m = $[Y_2-Y_1] / [X_2-X_1]$ = [-3-2]/[4-2] = -2.5

Y = mX + c

2 = (-2.5) x 2 + c, c = 7

Therefore, error equation, Ep = -2.5t + 7

Controller output P = Kp Ep + Po = 2 (-2.5t +7) + 50

Therefore, at t = 2 sec, P = 54% and, at **t = 4 sec, P = 44%**

(b) *For error segment 4-6 sec:*

Slope of the line, m = $[Y_2-Y_1] / [X_2-X_1]$ = [0+3]/[6-4] = 1.5

$$Y = mX + c$$

$$-3 = 1.5 \times 4 + c, \qquad c = -9$$

Therefore, error equation, $E_p = 1.5t - 9$

Controller output $\quad P = K_p E_p + P_o = 2\,(1.5t - 9) + 50$

Therefore, $\quad$ at t = 4 sec, P = 44% and, at **t = 6 sec, P = 50%**

Therefore, the controller output for the error shown in Fig. 2.11 is given by Fig.2.12.

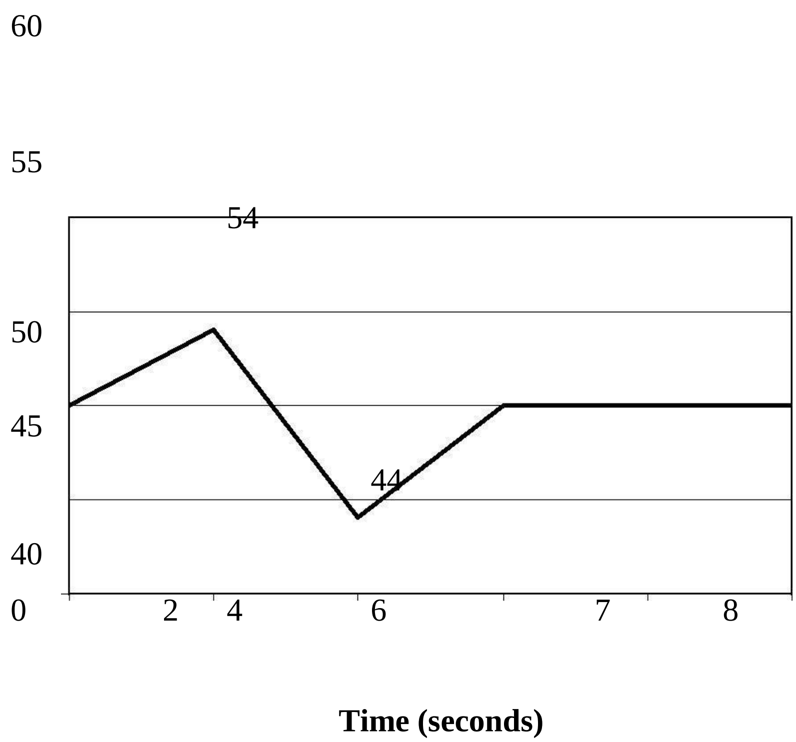


Fig. 2.12 Controller output for the error shown in Fig. 2.11

Integral Control Mode

The integral control eliminates the offset error problem by allowing the controller to adapt to changing external conditions by changing the zero-error output.

Integral action is provided by summing the error over time, multiplying that sum by a gain, and adding the result to the present controller output. If the error makes random excursions above and below zero, the net sum will be zero, so the integral action will not contribute. But if the error becomes positive or negative for an extended period of time, the integral action will begin to accumulate and make changes to the controller output. The analytical expression for integral mode is given by the equation

$$p(t) = K_I \int_0^t e_p \, dt + p(0) \qquad (2.10)$$

where $p(0)$ = controller output when the integral action starts (%)

K_I = Integral gain (s^{-1})

Another way of expressing the integral action is by taking derivative of equation (10), which gives the relation for the rate of change of controller output with error.

$$\frac{dp}{dt} = K_I \, e_p \qquad (2.12)$$

The equation (12) shows that when an error occurs, the controller begins to increase (or decrease) its output at a rate that depends upon the size of the error and the gain. If the error is zero, controller output is not changed. If there is positive error, the controller output begins to ramp up at a rate determined by Equation (12). This is shown in Fig.2.13 for two different values of gain. It can be observed that the rate of change of controller output depends upon the value of error and the size of the gain. Figure 2.14 shows how controller output will vary for a constant error & gain.

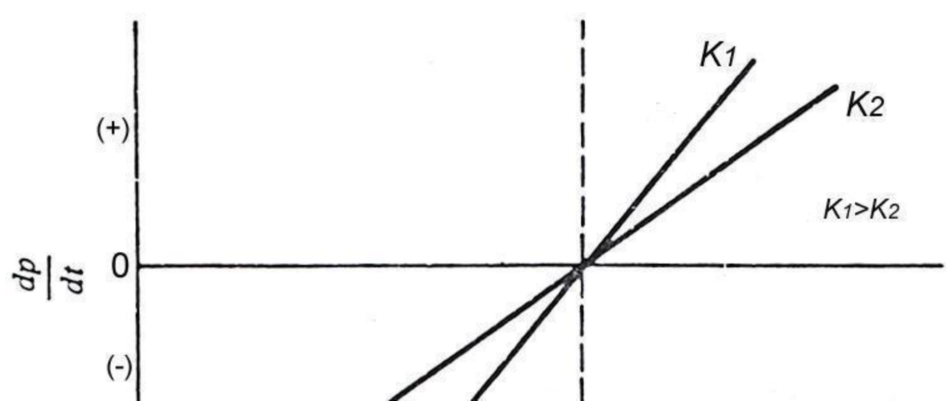


Fig. 2.13 Integral control action showing the rate of output change with error & gain

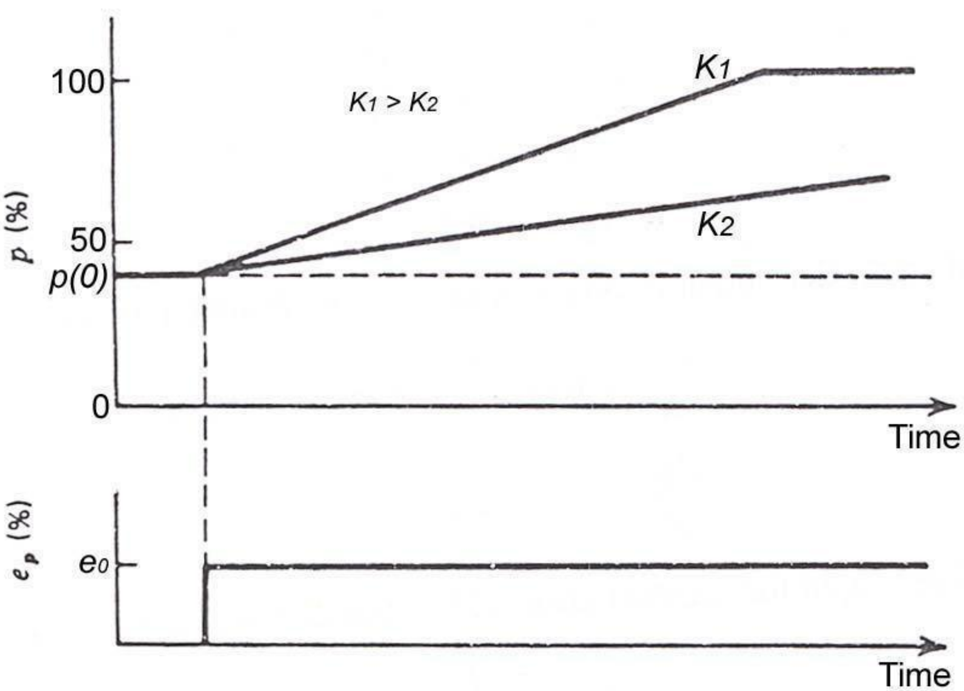


Fig. 2.14 Integral controller output for a constant error

It can be observed that the controller output begins to ramp up at a rate determined by the gain. In case of gain K_1, the output finally saturates at 100%, and no further action can occur.

The summary of characteristics of integral control mode are as follows:

1. If the error is zero, the output stays fixed at a value equal to what it was when the error went to zero (i.e. *p(0)*)
2. If the error is not zero, the output will begin to increase or decrease at a rate of K_I %/sec for every 1% of error.

Area Accumulation: It is well known fact that integral determines the area of the function being integrated. The equation (1.12) provides controller output equal to the net area under error- time curve multiplied by $K_{I.}$ It can be said that the integral term accumulates error as function of time. Thus, for every 1%-sec of accumulated error-time area, the output will be K_1 percent.The integral gain is often represented by the inverse, which is called the integral time or reset action, i.e. $\boldsymbol{T_I = 1 / K_I}$, which is expressed in minutes instead of seconds because this unit is more typical of many industrial process speeds. The integral operation can be better understood by the Fig. 2.15

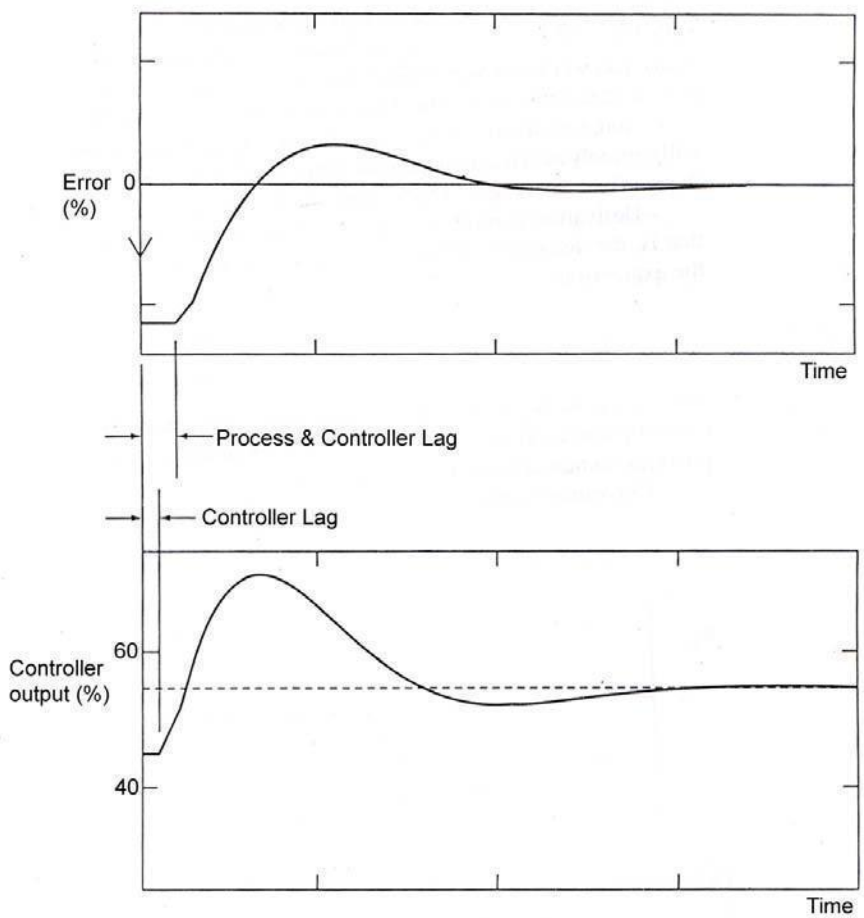


Fig. 2.15 Integral mode output and error, showing the effect of process and control lag.

A load change induced error occurs at t = 0. Dashed line is the controller output required to maintain constant output for new load. In the integral control mode, the controller output value initially begins to change rapidly as per Equation (12). As the control element responds and error decreases, the co ntroller output rate also decreases. Ultimately the system drives the error to zero at a slowing controller rate. The effect of process and control system lag is shown as simple delays in the controller output change and in the error reduction when the controller action occurs. If the process lag is too large, the error can oscillate about zero or even be cyclic.

Applications: In general, integral control mode is not used alone, but can be used for systems with small process lags and correspondingly small capacities.

Proble m 2.7

An integral controller has a reset action of 2.2 minutes. Express the integral controller constant in s^{-1}. Find the output of this controller to a constant error of 2.2%.

Solution:

Given Data: Reset action time = T_I = 2.2 min = 132 Seconds

Error = e_p = 2.2%

Asked: Integral controller constant = K_I = ?

Controller output = p = ?

$$K_I = 1 / T_I = 1 / 132 = 0.0076\ s^{-1}$$

$$p(t) = K_I \int_0^t e_p\, dt + p(0)$$

$$p = 0.0076 \int_0^t (2.2)\, dt + 0$$

$$p = 0.0167\, t$$

Derivative Control Mode

The need for derivative control mode can be explained with the error graph shown in Fig.2.16

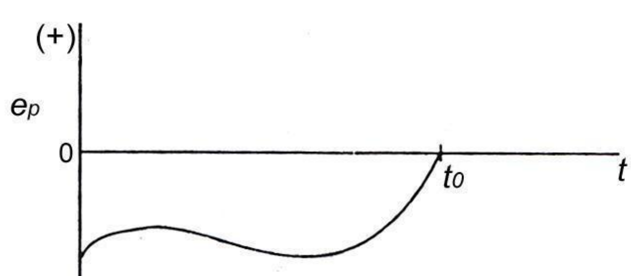


Fig. 2.16 Error graph with zero error and large rate of change.

It can be observed that even though the error at t_0 is zero, it is changing in time and will certainly not be zero in the following time. Under such situations some action should be taken even though the error is zero. Such scenario describes the nature and need for derivative action.

Derivative controller action responds to the rate at which the error is changing- that is, derivative of the error. The analytical expression for derivative control mode is given by;

$$p(t) = K_D \frac{de_p}{dt} \qquad (2.13)$$

where K_D = Derivative gain (s)

Derivative action is not used alone because it provides no output when the error is constant. Derivative controller action is also called *rate action* and *anticipatory control*. Figure 2.17 illustrates how derivative action changes the controller output for various rates of change of error. For this example, it is assumed that the controller output with no error or rate of change of error is 50%. When the error changes very rapidly with a positive slope, the output jumps to a large value, and when the error is not changing, the output returns to 50%. Finally, when error is decreasing - that is negative slope - the output discontinuously changes to a lower value.

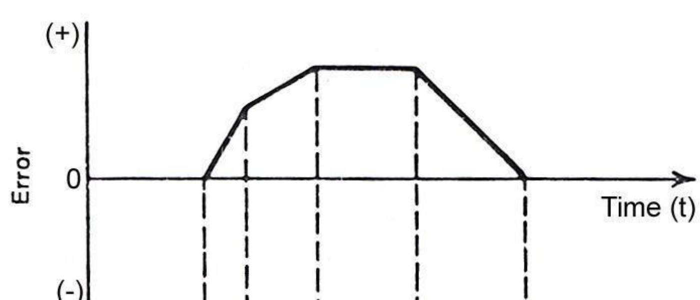


Fig.2.17 Derivative controller output for different rate of error.

The derivative mode must be used with great care and usually with a small gain, because a rapid rate of change of error can cause very large, sudden changes of controller output and lead to instability.

The summary of characteristics of derivative control mode are as follows:

1. If the error is zero, the mode provides no output.
2. If the error is constant in time, the mode provides no output
3. If the error is changing with time, the mode contributes an output of KD percent for every 1% per second rate of change of error.
4. For direct action, positive rate of change of error produces a positive derivative mode output.

Proble m 2.8

How would a derivative controller with K_D = 4 s respond to an error that varies as e_p=2.2 Sin(0.04t)?

Solution

Given: K_D = 4 s e_p = 2.2 Sin(0.04t) Asked: Derivative controller o/p=? For derivative mode, $\mathbf{p(t) = K_D (de_p/dt)}$

p(t) = 4 x d/dt(2.2 Sin(0.04t))

$= 4 \times 2.2 \times \text{Cos}(0.04t) \times 0.04$

$= 0.352 \text{ Cos}(0.04t)$

Composite Control Modes

It is found from the discontinuous and continuous controller modes, that each mode has its own advantages and disadvantages. In complex industrial processes most of these control modes do not fit the control requirements. It is both possible and expedient to combine several basic modes, thereby gaining the advantages of each mode. In some cases, an added advantage is that the modes tend to eliminate some limitations they individually posses. The most commonly used composite controller modes are: Proportional-Integral (PI), Proportional- Derivative (PD) and Proportional- Integral-Derivative (PID) control modes.

Proportional-Integral Control Mode (PI Mode):

This control mode results from combination of proportional and integral mode. The analytical expression for the PI mode is given by:

$$p = K_p e_p + K_p K_I \int_0^t e_p \, dt + p_I(0) \qquad (2.14)$$

where $p_I(0)$ = integral term value at t = 0 (initial value)

The main advantage of this composite control mode is that one-to-one correspondence of the proportional control mode is available and integral mode eliminates the inherent offset. It can be observed from the equation (2.14) that the proportional gain also changes the net integration mode gain, but the integration gain, through K_I, can be independently adjusted. The proportional mode when used alone produces offset error whenever load change occurs and nominal controller output will not provide zero error. But in PI mode, integral function provides the required new controller output, thereby allowing the error to be zero after a load change. The integral feature effectively provides a reset of the zero error output after a load change occurs. Figure 2.18 shows the PI mode response for changing error. At time t_1 , a load change occurs that produces the error shown. Accommodation of the new load condition requires a new controller output. It can be observed that the controller output is provided through a sum of proportional plus integral action that finally brings the error back to zero value.

The summary of characte ristics of PI mode are as follows:

1. When the error is zero, the controller output is fixed at the value that the integral term had when the error went to zero, i.e. output will be $p_I(0)$ when e_p=0 at t = 0.

2. If the error is not zero, the proportional term contributes a correction, and the integral term begins to increase or decrease the accumulated value [i.e. initial value $p_I(0)$], depending on the sign or the error and direct or reverse action.

The integral term cannot become negative. Thus, it will saturate at zero if the error and action try to drive the area to a net negative value.

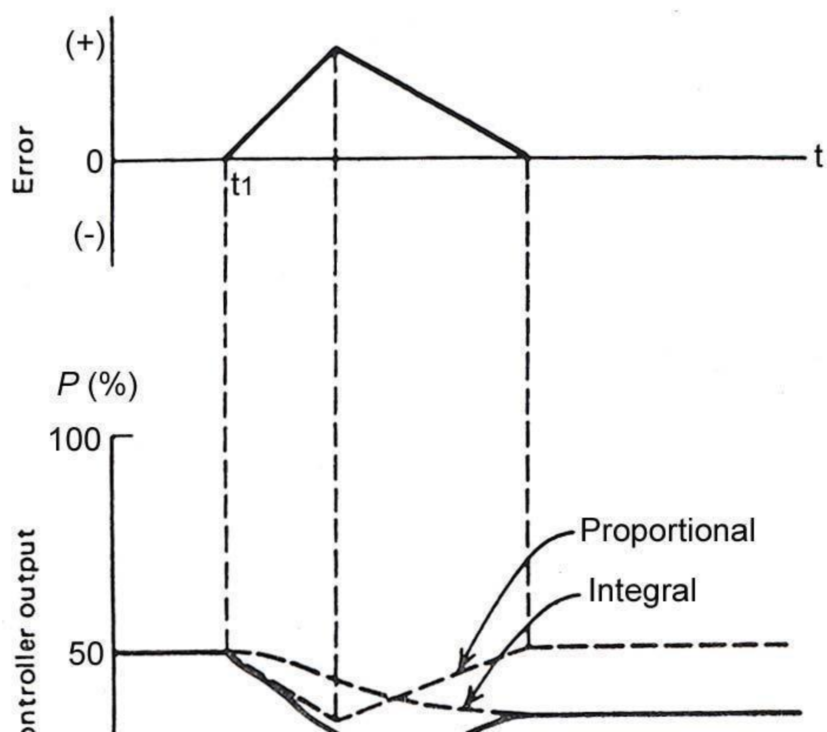


Fig. 2.18 PI mode action for changing error (for reverse acting system)

Application, Advantages and Disadvantages:

- This composite PI mode eliminates the offset problem of proportional controller.
- The mode can be used in systems with frequent or large load changes
- Because of integration time the process must have relatively slow changes in load to prevent oscillations induced by the integral overshoot.
- During start-up of a batch process, the integral action causes a considerable overshoot of the error and output before settling to the operation point. This is shown in Fig.1.20, the dashed band is proportional band (PB). The PB is defined as hat positive and negative error for which the output will be driven to 0% and 100%. Therefore, the presence of an integral accumulation changes the amount of error that will bring about such saturation by the proportional term. In Fig. 2.19, the output saturates whenever the error exceeds the PB limits. The PB is constant, but its location is shifted as the integral term changes.

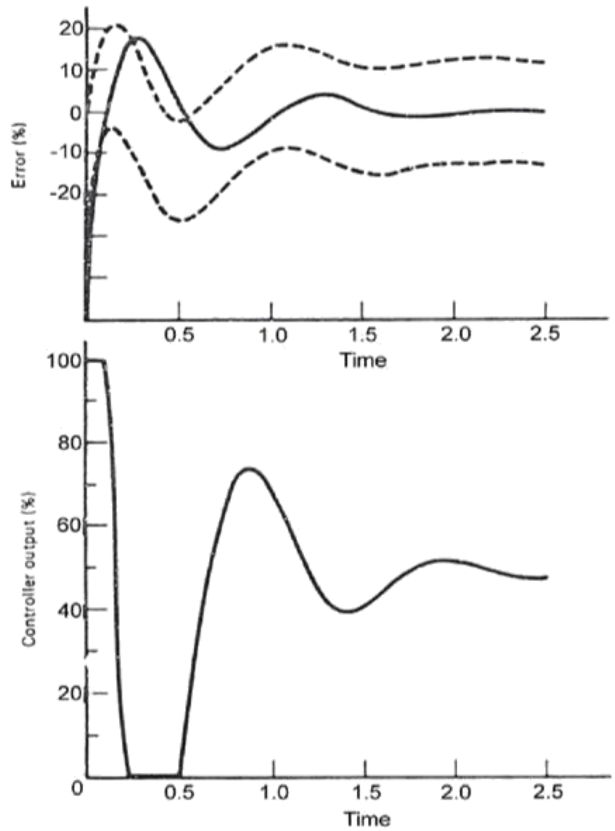


Fig. 2.19 Overshoot and cycling when PI mode control is used in start-up of batch processes. The dashed lines show PB.

Proportional-Derivative Control Mode (PD Mode):

The PD mode involves the serial (cascaded) use of proportional and derivative modes and this mode has many industrial applications. The analytical expression for PD mode is given by

$$p = K_p e_p + K_p K_D \frac{de_p}{dt} + p_0 \qquad 2.15$$

This system will not eliminate the offset of proportional controller, however, handle fast process load changes as long as the load change offset error is acceptable. Figure 2.20 shows a typical PD response for load changes. It can be observed that the derivative action moves the controller output in relation to the error rate change.

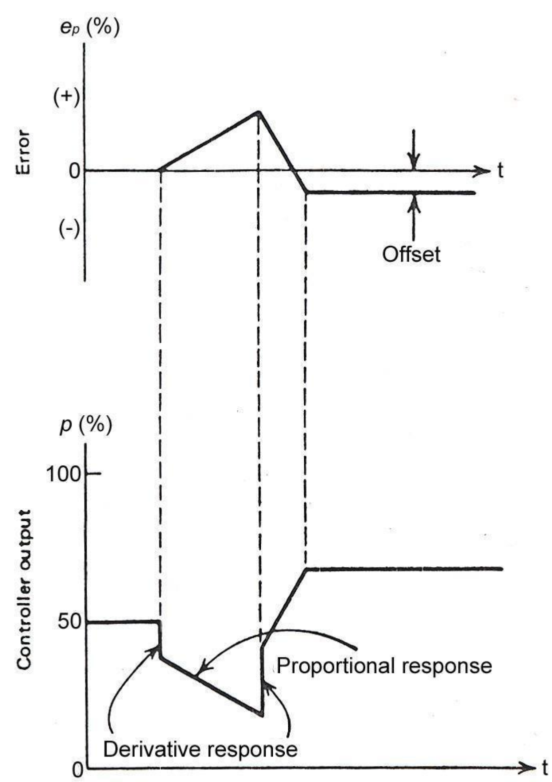


Fig. 2.20 PD control mode response, showing offset error from proportional mode and derivative action for changing load, for reverse acting system.

Proportional-Integral-Derivative Control Mode (PID or Three Mode):

One of the most powerful but complex controller mode operations combines the proportional, integral, and derivative modes. This PID mode can be used for virtually any process condition. The analytical expression is given by

$$p = K_p e_p + K_p K_I \int_0^t e_p \, dt + K_p K_D \frac{de_p}{dt} + p_I(0) \qquad (2.16)$$

This mode eliminates the offset of the proportional mode and still provides fast response for changing loads. A typical PID response is shown in Fig. 2.21

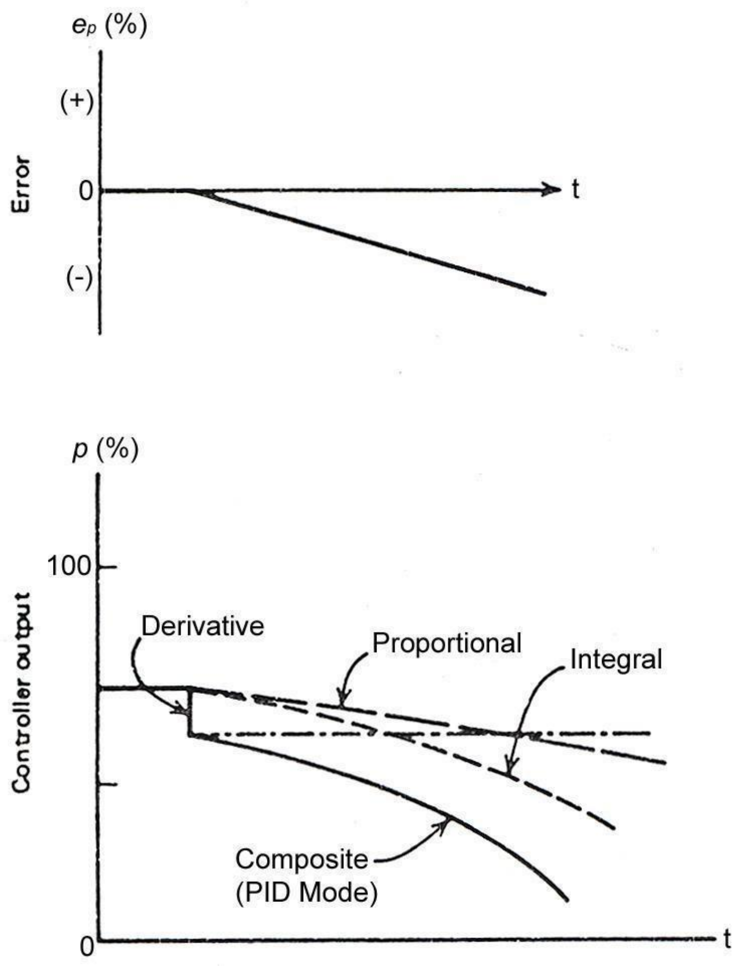


Fig. 2.21 Three mode (PID) controller action, exhibiting proportional, integral and derivative action.

Proble m 2.9

A PI controller is reverse acting, PB=20, 12 repeats per minute. Find (a) Proportional gain (b) Integral gain, and (c) Time that the controller output will reach 0% after a constant error of 1.5% starts. The controller output when the error occurred was 72%.

Solution:

Given : PB = 20

Integral time = T_I = 1/12 min = 60/12 s = 5 s

$P_I(0)$ = 72%, e_p = 1.5%

Asked : (a) Kp = ? (b) K_I = ? t =? when P = 0%

(a) Kp = 100 / PB = 100 / 20 = 5

(b) $K_I = 1 / T_I = 1/5 = 0.2\ s^{-1}$

(c) For PI mode, $p = -\{ K_p e_p + K_p K_I \int_0^t e_p\, dt \} + p_I(0)$

-ve sign is for reverse acting

$$p = -\{ 5 \times (1.5) + 5 \times 0.2 \int_0^t 1.5\, dt \} + 72$$

$$p = -\{ 7.5 + 1.5t \} + 72$$

When P = 0% $1.5t = 64.5$ **t = 43 s = 0.72 min**

Proble m 2.10

A PD controller has Kp = 2.0, K_D = 2 s, and P_0 = 40%. Plot the controller output for the error input shown in Fig.2.22

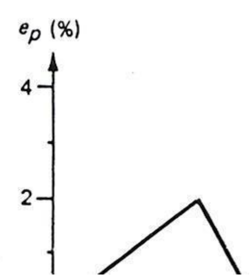


Fig. 2.22 Error graph

Solution: Given data: $K_p = 2.0$ $K_D = 2$ s $P_O = 40\%$

To find the controller output and plot the response, first of all we need to find the error which is changing with time and express the error as function of time. The error need to be found in three time regions: (a) 0-2 sec (b) 2-4 sec (c) 4-6 sec.

Since, the error is linear, using the equation for straight line we find the error equation

i.e. $E_p = mt + c$ (i.e. $Y = mX + c$)

(a) *For error segment 0-2 sec:*

$Y = mX + c$, $2 = 1 \times t + c$, $2 = 1 \times 2 + c$, $c = 0$

Therefore, error equation, $E_p = t$

Controller output $P = K_p E_p + K_p K_D [dE_p/dt] + P_o$

$= 2\,t + 2 \times 2\,[d/dt\,(t)] + 40$

$= 2\,t + 4 + 40$

Therefore, at **t = 0 sec, P = 44%** and **at t = 2 sec, P = 48%**

(b) *For error segment 2-4 sec:*

$Y = mX + c$, $2 = (-2.5) \times 2 + c$, $c = 7$

Therefore, error equation, $E_p = -2.5t + 7$

Controller output $P = 2\,[-2.5t+7] + 2 \times 2\,[d/dt\,(-2.5t+7)] + 40$

$= -5t + 14 - 10 + 40$

Therefore, **at t = 2 sec, P = 34%** and, at **t = 4 sec, P = 24%**

(c) *For error segment 4-6 sec:*

Slope of the line, $m = [Y_2-Y_1] / [X_2-X_1] = [0+3]/[6-4] = 1.5$

$Y = mX + c$, $-3 = 1.5 \times 4 + c$, $c = -9$

Therefore, error equation, $E_p = 1.5t - 9$

Controller output $P = 2 [1.5t - 9] + 2 \times 2 [d/dt (1.5t-9)] + 40$

$= 3t - 18 + 6 + 40$

Therefore, **at t = 4 sec, P = 40%** and, at **t = 6 sec, P = 46%**

Therefore, the controller output for the error shown in Fig. 2.22 is given by Fig.2.23.

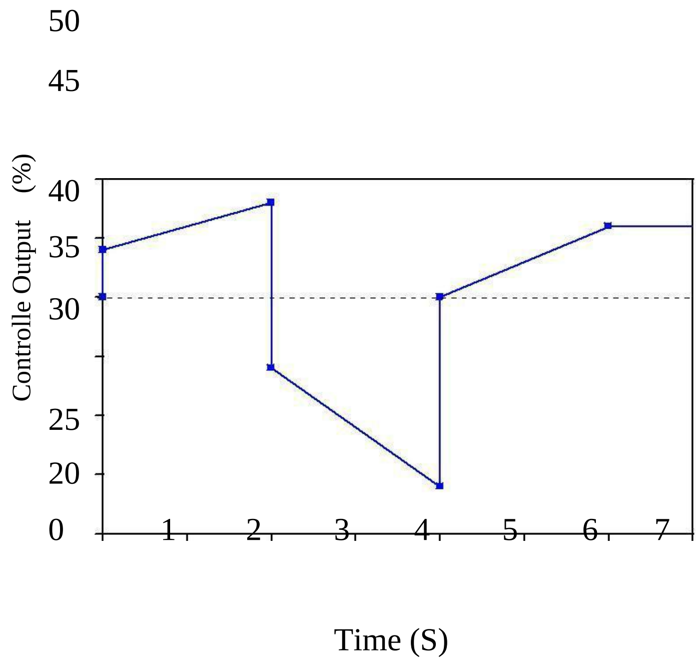


Fig. 2.23 Controller output for the error shown in Fig.2.22

Summary:

In this chapter general characteristics of controller operating modes without considering implementation of these modes are discussed. The terms that are important to understand the process control and controller operations are defined. The important points which are discussed in this chapter are as follows:

1. In considering the controller operating mode for industrial process control, it is important to know all the *process characteristics* and *control system parameters* which may influence the process and controller operations.
2. Discontinuous controller modes refer to instances where the controller output does not change smoothly for input error. The examples are two-position, multiposition, and floating control modes.
3. Continuous controller modes are modes where the controller output is a smooth function of the error input or rate of change. Examples are proportional, integral and derivative control modes.
4. The continuous controller modes, such as proportional, integral and derivative modes have their own advantages and disadvantages. In complex industrial processes most of these control modes do not fit the control requirements. It is both possible and expedient to combine several basic modes, thereby gaining the advanta ges of each mode. In some cases, an added advantage is that the modes tend to eliminate some limitations they individually posses. Examples are proportional- integral (PI), proportional-derivative (PD) and proportional- integral-derivative (PID) control modes.

Proble ms:

1. A floating controller with a rate gain of 6%/min and $p(0) = 50\%$ has a ±5 gal/min deadband. Plot the controller output for an input given by Fig. 1. The setpoint is 60 gal/min.

80

70

60

50

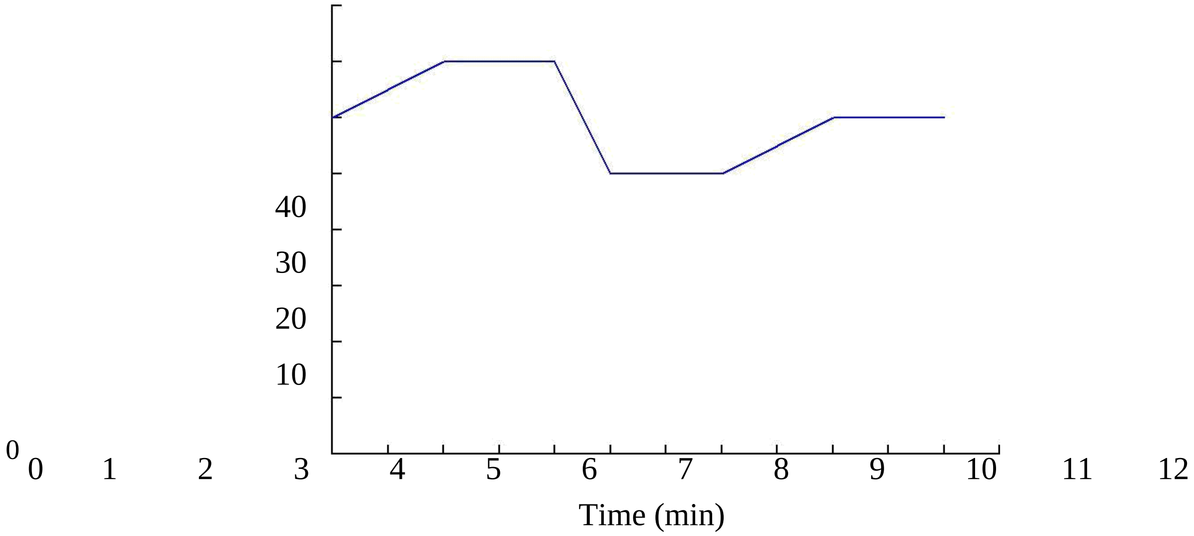


Fig. 1

2. A PI controller has Kp = 2.0, $K_I = 2.2\ s^{-1}$, and $P_I(0) = 40\%$. Plot the controller output for the error input shown in Fig.2.

3. A PID controller has Kp = 2.0, $K_I = 2.2\ s^{-1}$, and $K_D = 2$ s $P_I(0) = 40\%$. Plot the controller output for the error input shown in Fig.2.

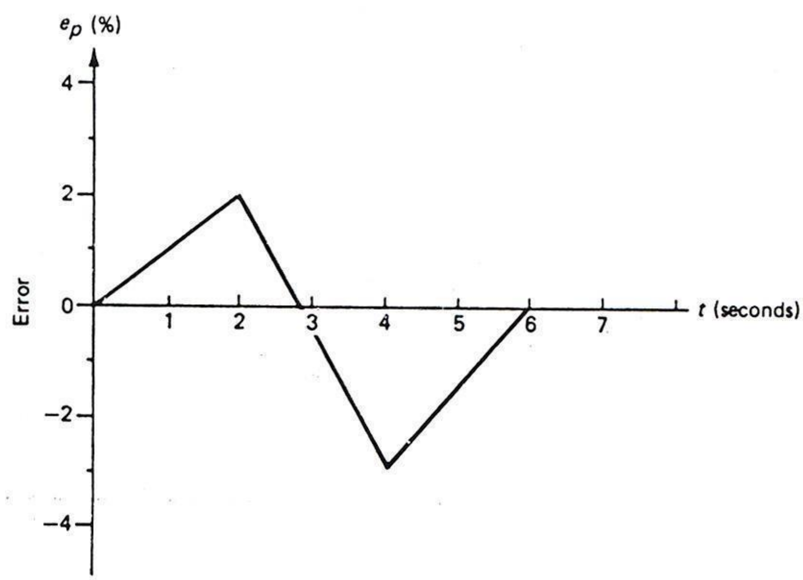


Fig. 2

4. A PD controller has Kp = 5.0, $K_D = 0.5$ s and $P_0 = 20\%$. Plot the controller output for the error input shown in Fig.3.

5. A PID controller has $K_P = 5.0$, $K_I = 0.7\ s^{-1}$, and $K_D = 0.5$ s and $P_I(0) = 20\%$. Plot the controller output for the error input shown in Fig.3.

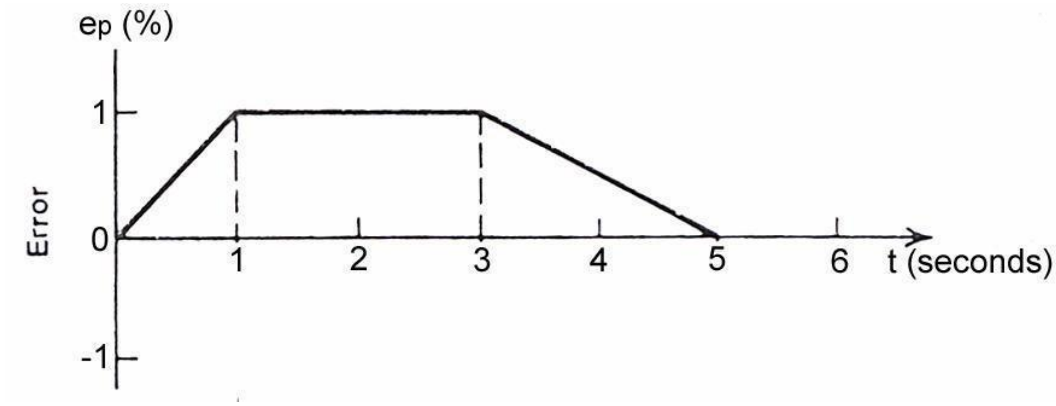


Fig. 3

6. A PI controller has $K_P = 5.0$, $K_I = 1.0\ s^{-1}$, and $K_D = 0.5$ s and $P_I(0) = 20\%$. Plot the controller output for the error input shown in Fig.1.283.

7. A PD controller has $K_P = 5.0$, $K_D = 0.5$ s and $P_0 = 20\%$. Plot the controller output for the error input shown in Fig.4

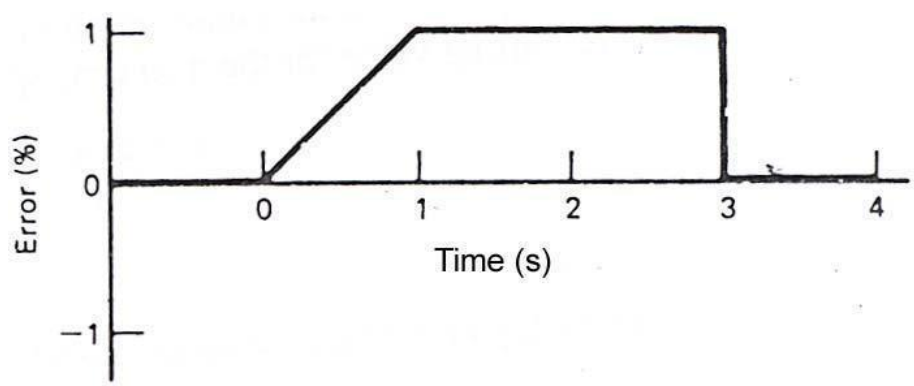


Fig. 4

8. A PI controller has $K_P = 4.5$, $K_I = 7.0\ s^{-1}$. Find the controller output for an error given by ep = 3 Sin (Πt). What is the phase shift between error and controller output?

The word „Pneuma" means breath or air . Pneumatics is application of compressed air in automation. In Pneumatic control, compressed air is used as the working medium, normally at a pressure from 6 bar to 8 bar. Using Pneumatic Control, maximum force up to 50 kN can be developed. Actuation of the controls can be manual, Pneumatic or Electrical actuation. Signal medium such as compressed air at pressure of 1-2 bar can be used [Pilot operated Pneumatics] or Electrical signals [D.C or A.C source- 24V – 230V] can be used [Electro pneumatics]

Characteristics of Compressed Air

The following characteristics of Compressed air speak for the application of Pneumatics

- Abundance of supply of air
- Transportation
- Storage
- Temperature
- Explosion Proof
- Cleanliness
- Speed
- Regulation
- Overload Proof

Selection Criteria for Pneumatic Control System

- Stroke
- Force
- Type of motion [Linear or Angular motion]
-
- Size
- Service
- Sensitivity
- Safety and Reliability
- Energy Cost
- Controllability
- Handling
- Storage

Advantages of Pneumatic Control

- Unlimited Supply
- Storage
- Easily Transportable
- Clean
- Explosion Proof
- Controllable (Speed, Force)
- Overload Safe
- Speed of Working Elements

Disadvantages

- Cost
- Preparation
- Noise Pollution
- Limited Range of Force (only economical up to 25 kN)

General Applications of Pneumatic Control

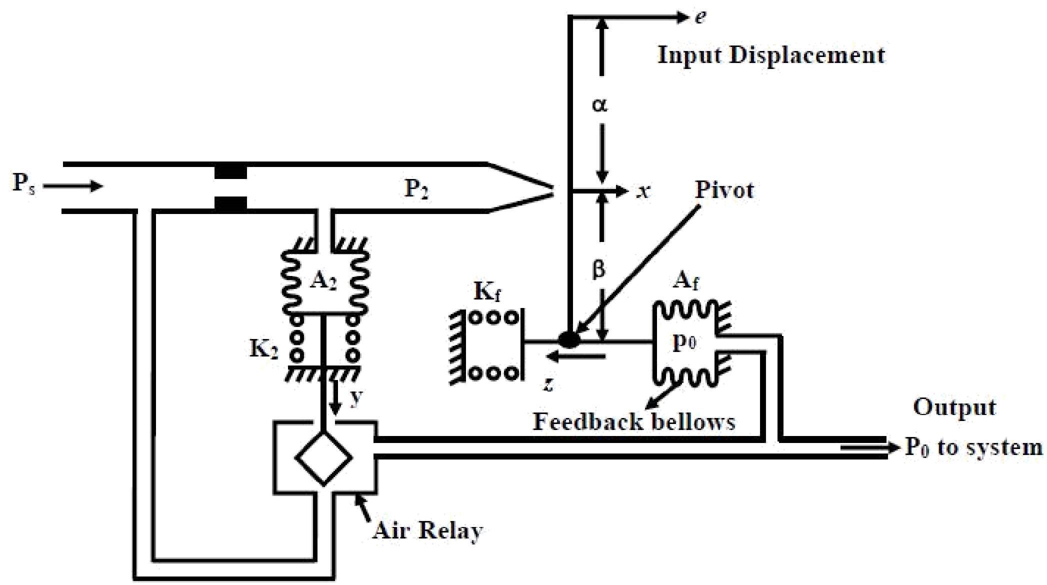
- Clamping
- Shifting
- Metering
- Orienting
- Feeding
- Ejection
- Braking
- Bonding
- Locking
- Packaging
- Feeding
- Door or Chute Control
- Transfer of Material
- Turning or Inverting of Parts
- Sorting of Parts
- Stacking of Components
- Stamping and Embossing of components

Applications in Manufacturing

- Drilling Operation
- Turning
- Milling
- Sawing
- Finishing
- Forming
- Quality Control

Pneumatic Proportional Controller

Consider the pneumatic system shown in Fig 2.24. It consists of several pneumatic components The components, which can be easily identified, are: flapper nozzle amplifier, air relay, bellows and springs, feedback arrangement etc. The overall arrangement is known as a pneumatic proportional controller. It acts as a controller in a pneumatic system generating output pressure proportional to the displacement e at one end of the link. The input to the system is a small linear displacement e and the output is pressure Po. The input displacement may be caused by a small differential pressure to a pair of bellows, or by a small current driving an electromagnetic unit. There are two springs K_2 and K_f those exert forces against the movements of the bellows A_2 and A_f. For a positive displacement of e (towards right) will cause decrease of pressure in the flapper nozzle. This will cause an upward movement of the bellows A_2 (decrease in y). Consequently the output pressure of the air relay will increase. The increase in output pressure will move the free end of the feedback bellows towards left, bringing in the gap between the flapper and nozzle to almost its original value. We will first develop the closed loop representation of the scheme and from there the input output relationship will be worked out. The air is assumed to be impressible here.



2.24 A pneumatic proportional controller

Pneumatic PID Controller

Many pneumatic PID controllers use the force-balance principle. One or more input signals (in the form of pneumatic pressures) exert a force on a beam by acting through diaphragms, bellows, and/or bourdon tubes, which is then counter-acted by the force exerted on the same beam by an output air pressure acting through a diaphragm, bellows, or bourdon tube. The self-balancing mechanical system "tries" to keep the beam motionless through an exact balancing of forces, the beam‟s position precisely detected by a nozzle/baffle mechanism.

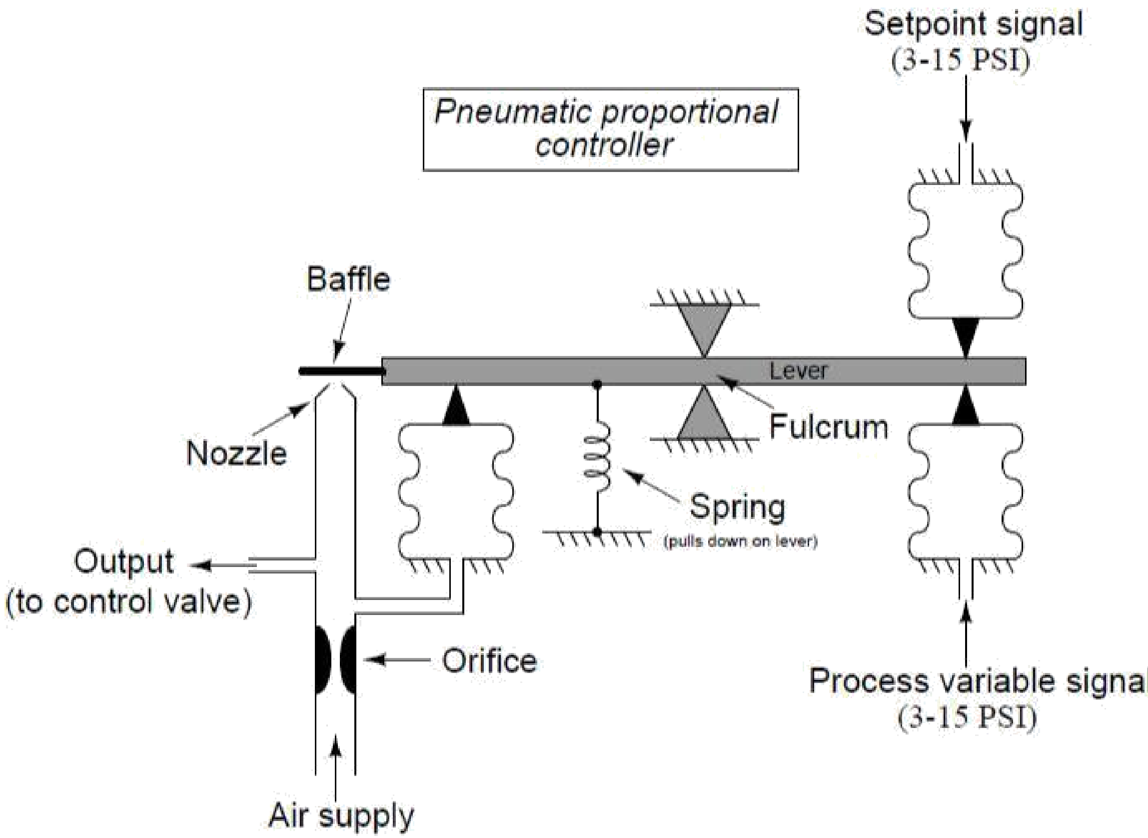


Fig 2.25 Proportional Controllers

The action of this particular controller is direct, since an increase in process variable signal (pressure) results in an increase in output signal (pressure). Increasing process variable (PV) pressure attempts to push the right-hand end of the beam up, causing the baffle to approach the nozzle. This blockage of the nozzle caus es the nozzle‟s pneumatic backpressure to increase, thus increasing the amount of force applied by the output feedback bellows on the left- hand end of the beam and returning the flapper (very nearly) to its original position. If we wished to reverse the co ntroller‟s action, all we would need to do is swap the pneumatic signal connections between the input bellows, so that the PV pressure was applied to the upper bellows and the SP pressure to the lower bellows. Any factor influencing the ratio of input pressure(s) to output pressure may be exploited as a gain (proportional band) adjustment in this mechanism. Changing bellows area (either both the PV and SP bellows equally, or the output bellows by itself) would influence this ratio, as would a change in output bellows position (such that it pressed against the beam at some difference distance from the fulcrum point). Moving the fulcrum left or right is also an option for gain control, and in fact is usually the most convenient to engineer.

Derivative and integral actions

Interestingly enough, derivative (rate) and integral (reset) control modes are relatively easy to add to this pneumatic controller mechanism. To add derivative control action, all we need to do is place a restrictor valve between the nozzle tube and the output feedback bellows, causing the bellows to delay filling or emptying its air pressure over time:

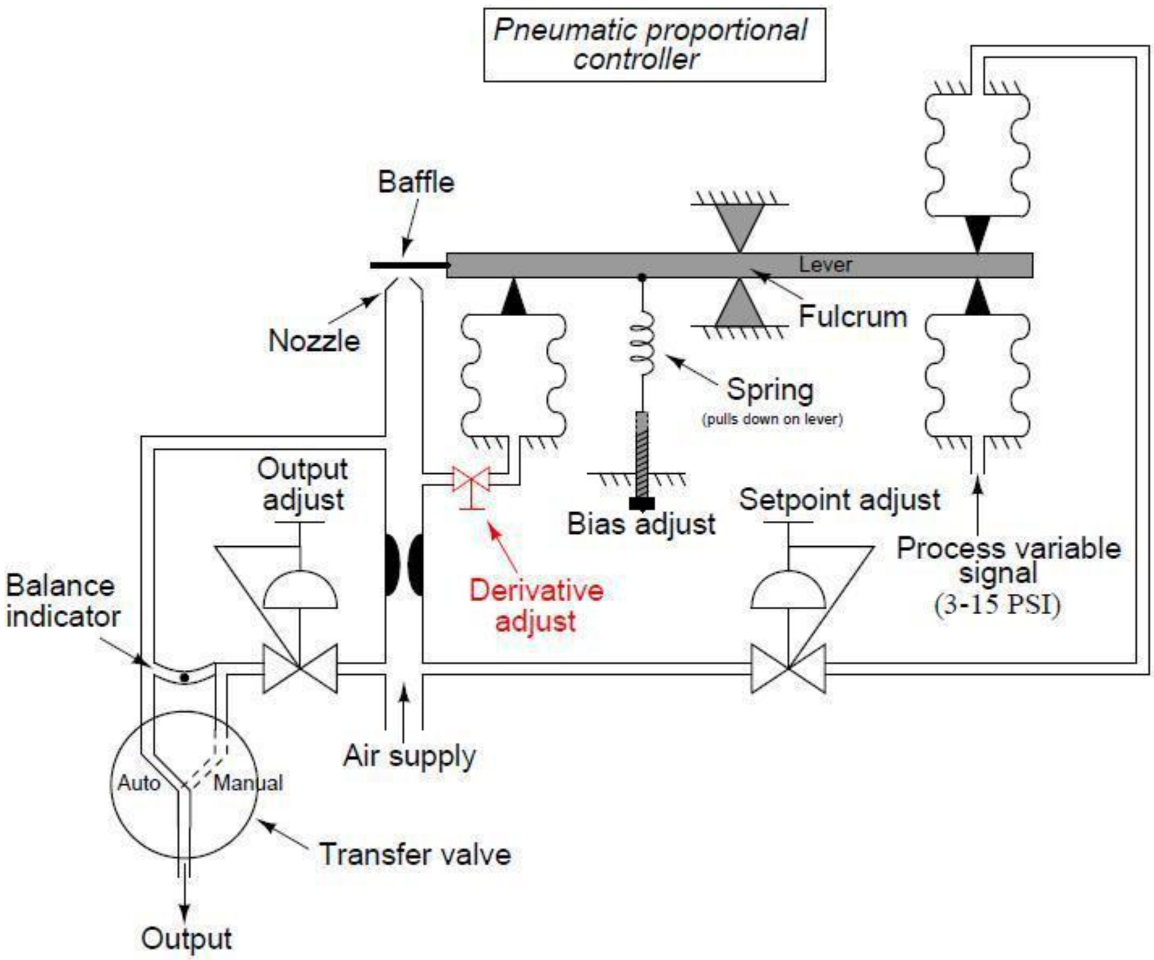


Fig 2.26 Proportional + Derivative Controllers

If any sudden change occurs in PV or SP, the output pressure will saturate before the output bellows has the opportunity to equalize in pressure with the output signal tube. Thus, the output pressure “spikes” with any sudden “step change” in input: exactly what we would expect with derivative control action.

If either the PV or the SP ramps over time, the output signal will ramp in direct proportion (proportional action), but there will also be an added offset of pressure at the output signal in order to keep air flowing either in or out of the output bellows at a constant rate to generate the force necessary to balance the changing input signal. Thus, derivative action causes the output pressure to shift either up or down (depending on the direction of input change) more than it would with just proportional action alone in response to a ramping input: exactly what we would expect from a controller with both proportional and derivative control actions.

Integral action requires the addition of a second bellows (a "reset" bellows, positioned opposite the output feedback bellows) and another restrictor valve to the mechanism

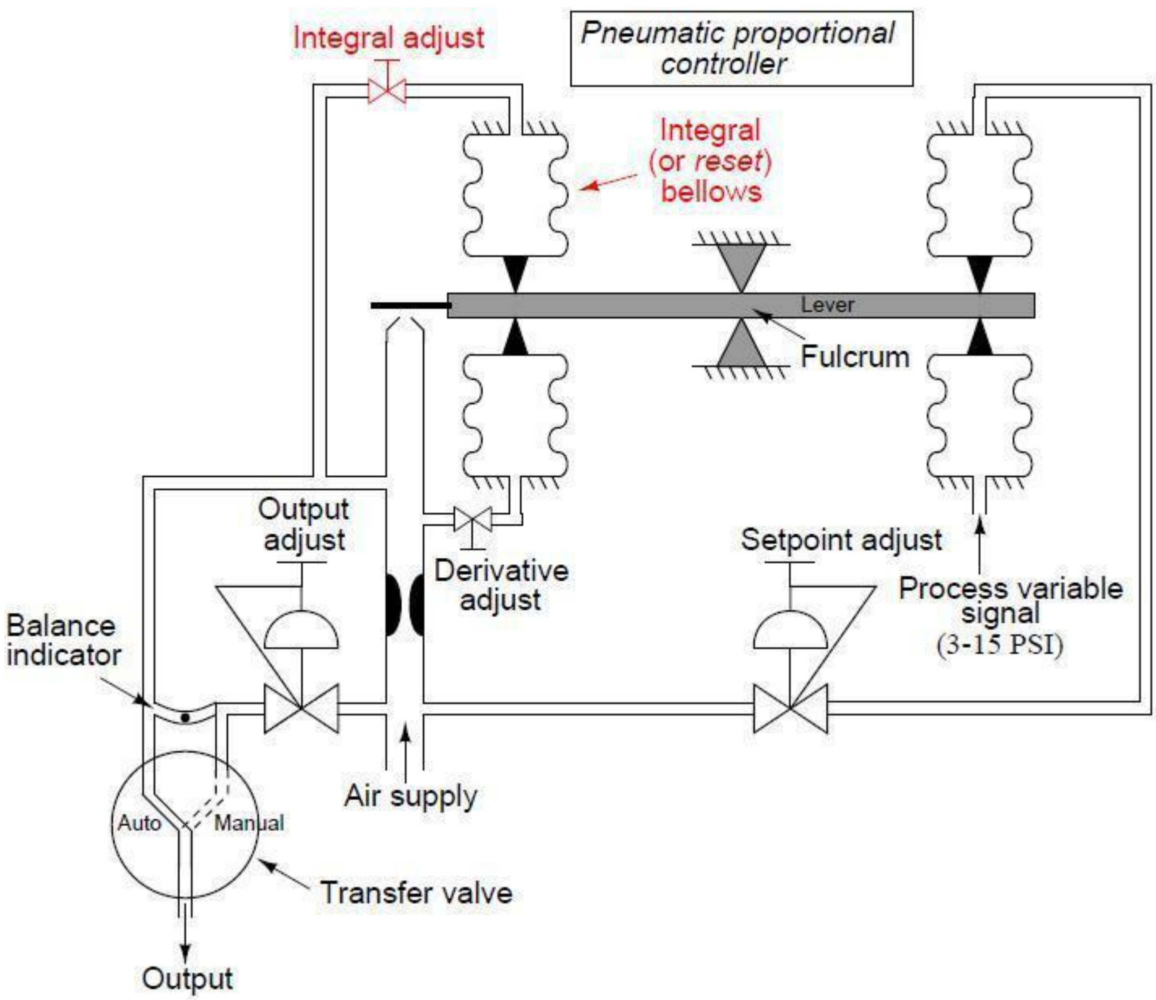


Fig 2.27 Proportional + Derivative + Integral

This second bellows takes air pressure from the output line and translates it into force that opposes the original feedback bellows. At first, this may seem counter-productive, for it nullifies the ability of this mechanism to continuously balance the force generated by the PV and SP bellows. Indeed, it would render the force-balance system completely ineffectual if this new "reset" bellows were allowed to inflate and deflate with no time lag. However, with a time lag provided by the restriction of the integral adjustment valve and the volume of the bellows (a sort of pneumatic "RC time constant"), the nullifying force of this bellows becomes delayed over time. As this bellows slowly fills (or empties) with pressurized air from the nozzle, the change in force on the beam causes the regular output bellows to have to "stay ahead" of the reset bellows action by constantly filling (or emptying) at some rate over time.

Pneumatic Integral Controller

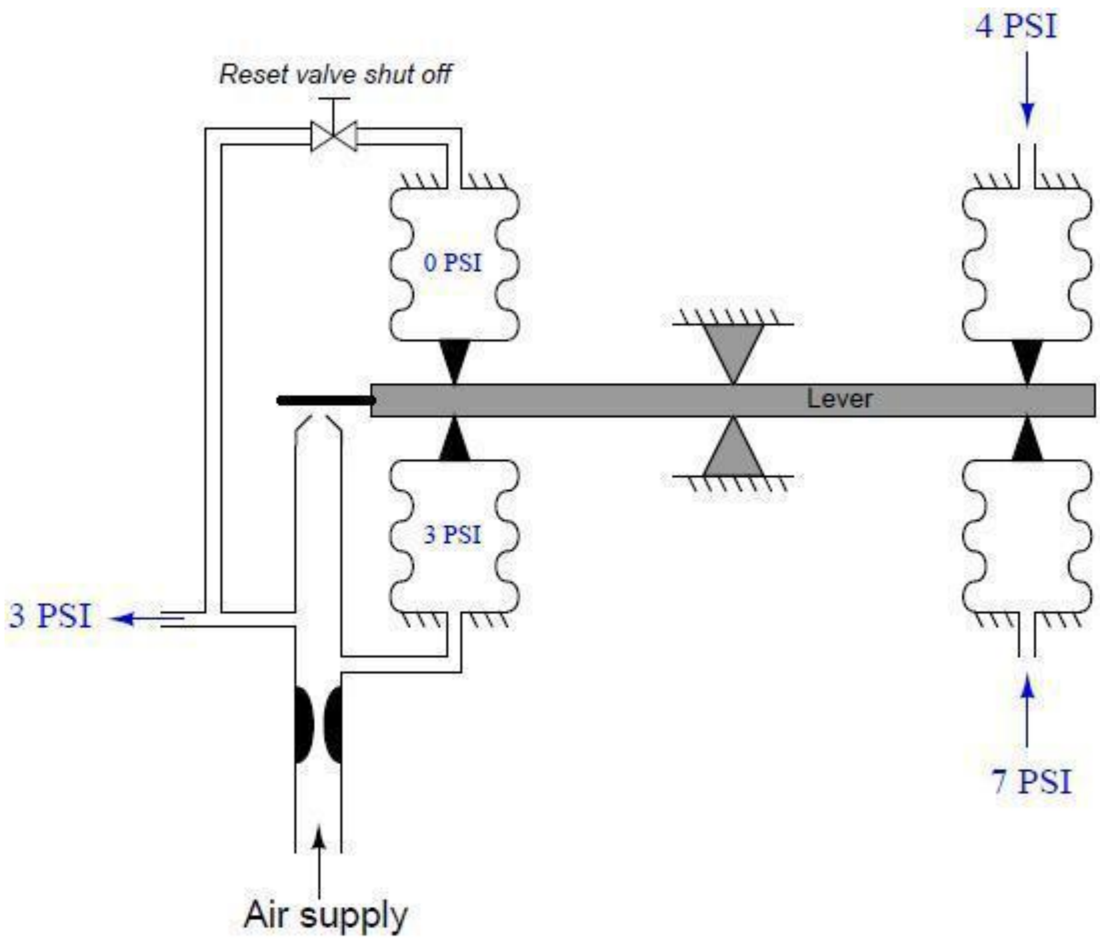


Fig 2.28 Pneuamtic Integral controllers

Here, the PV and SP air pressure signals differ by 3 PSI, causing the force-balance mechanism to instantly respond with a 3 PSI output pressure to the feedback bellows (assuming a central fulcrum location, giving a controller gain of 1). The reset (integra l) valve has been completely shut off to begin our analysis The result of these two bellows‟ opposing forces (one instantaneous, one time-delayed) is that the lower bellows must always stay 3 PSI ahead of the upper bellows in order to maintain a force-balanced condition with the two input bellows whose pressures differ by 3 PSI. This creates a constant 3 PSI differential pressure across the reset restriction valve, resulting in a constant flow of air into the reset bellows at a rate determined by that press ure drop and the opening of the restrictor valve. Eventually this will cause the output pressure to saturate at maximum, but until then the practical importance of this rising pressure action is that the mechanism now exhibits integral control response to the constant error between PV and SP

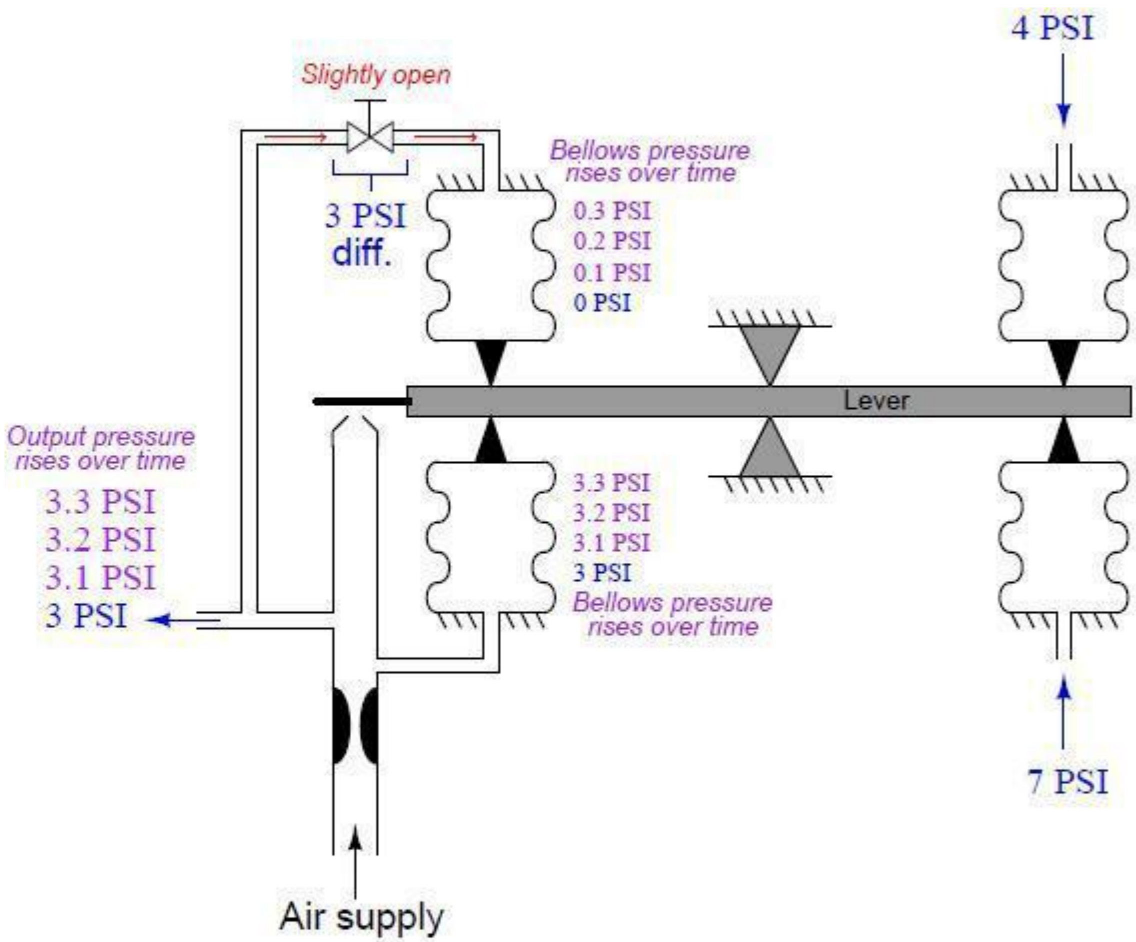


Fig 2.29 Integral Control Action

The greater the difference in pressures between PV and SP (i.e. the greater the error), the more pressure drop will develop across the reset restriction valve, causing the reset bellows to fill (or empty, depending on the sign of the error) with compressed air at a faster rate2, causing the output pressure to change at a faster rate. Thus, we see in this mechanism the defining nature of integral control action: that the magnitude of the error determines the velocity of the output signal (its rate of change over time, or dmdt). The rate of integration may be finely adjusted by changing the opening of the restrictor valve, or adjusted in large steps by connecting capacity tanks to the reset bellows to greatly increase its effective volume.

INTRODUCTION TO ELECTRONIC CONTROLLERS

A controller is a comparative device that receives an input signal from a measured process variable, compares this value with that of a predetermined control point value (set point), and determines the appropriate amount of output signal required by the final control element to provide corrective action within a control loop. An Electronic Controller uses electrical signals to perform its receptive, comparative and corrective functions.

Two Position controller using OPAMP

Fig. 2.30 represents the OPAMP implementation of ON/OFF controller with adjustable neutral zone.

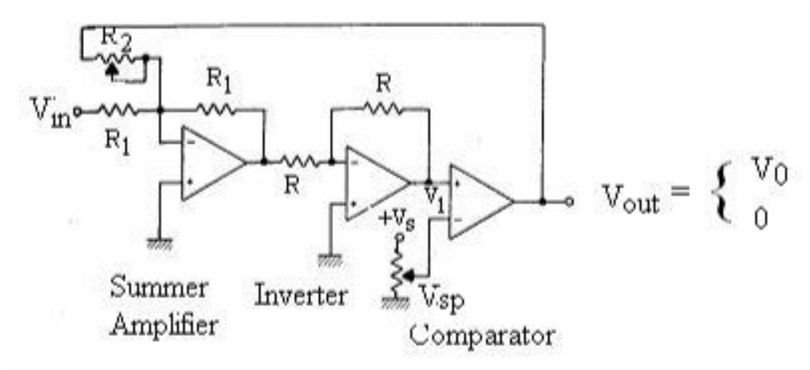


Fig 2.30 A two position controller with neutral zone made from op amps and a Comparator

Assume that, if the controller input voltage, Vin reaches a value VH then the comparator output should go to the ON state, which is defined as some voltage V0. When the input voltage falls bellow a value VL the comparator output should switch to the OFF state, which is defined as 0 V. This defines a two position controller with a neutral zone of NZ = VH - VL as shown in the Fig. 2.31

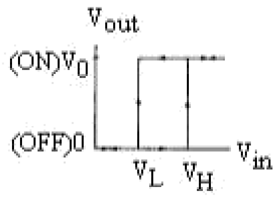


Fig. 2.31. Two position controller response in terms of voltages

Assume that, in the beginning, the comparator is in the OFF state. i.e. the voltage, V1 at the input of the comparator is less than the setpoint voltage, Vsp. Hence,

Vout = 0 (2.17)

The comparator output switches states when the voltage on its input, V1 is equal to the set point value Vsp Analyzing this circuit,

$$V_1 = V_{in} + \frac{R_1}{R_2} V_{out}$$

(2.18)

Substituting Eq. 2.17, in Eq. 2.18, yields

V1 = Vin

The comparator changes to ON state when V1 = Vin = VH. Thus, the high (ON) switch voltage is

VH = Vsp (2.19)

and the corresponding output voltage Vout is

Vout = V0 (2.20)

With this V1 changes to

$$V_1 = V_{in} + \frac{R_1}{R_2} V_0$$

(2.21) If Vin = VL the comparator changes to OFF state, giving the relation,

$$V_1 = V_{sp} = V_L + \frac{R_1}{R_2} V_0$$

(2.22)

This gives the low (OFF) switching voltage of

$$V_L = V_{SP} - \frac{R_1}{R_2} V_o \tag{2.23}$$

As mentioned before, Fig 2.31 shows typical two position relationship between input and output voltage for the circuit. The width of the neutral zone between VL and VH can be adjusted by variation of R2. The relative location of the neutral zone is calculated from the difference between the equations (2.19) and (2.23).

The inverter resistance value in Fig. 2.30 can be chosen as any convenient value. Typically it is in the 1 to 100 K Ω range.

Three position Controllers

Fig. 2.32 shows how a simple three position controller can be realized with op amps and comparators.

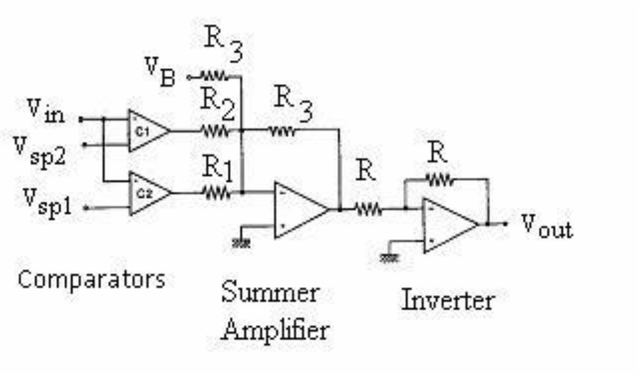


Fig 2.32 Three Position Controller

Fig. 2.32 A three position controller using two comparators and op amps Assume that, the output of the comparators is 0 V for the OFF state and V0 volts for the ON state. The summing amplifier also includes a bias voltage input, VB which allows the three position mode response to be biased up or down in voltage to suit particular needs. The inverter is needed to convert the sign of the inverting action of the summing amplifier.

When Vin < VSP1 ,

Comparator C1 is OFF, C2 is OFF (Because VSP1< VSP2) Outputs of both comparators are 0 V. Thus,

Vout = VB

When VSP1 < Vin< VSP2 , Comparator C1 is OFF, C2 is ON   Outputs of comparator

C1 = 0 and Output of Comparator $C2 = \frac{R_3}{R_1} V_0$ Volts. Thus,

$$V_{out} = V_B + \frac{R_3}{R_1} V_0$$

Outputs of comparator $C1 = \frac{R_3}{R_2} V_0$ and Output of Comparator $C2 = \frac{R_3}{R_1} V_0$ Volts. Thus,

$$V_{out} = V_B + \frac{R_3}{R_1} V_0 + \frac{R_3}{R_2} V_0$$

Thus,

When $V_{in} < V_{SP1}$, $\quad V_{out} = V_B$

$V_{SP1} < V_{in} < V_{SP2}$, $\quad V_{out} = V_B + \frac{R_3}{R_1} V_0$

$V_{in} > V_{SP2}$, $\quad V_{out} = V_B + \frac{R_3}{R_1} V_0 + \frac{R_3}{R_2} V_0$

Here, the output need not be symmetric. (e.g. 0%, 50% and 100%). Fig. 2.33 shows the response of this circuit for a particular case VB = 0

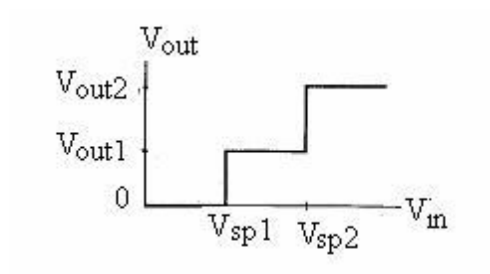


Fig. 2.33. Response of the three position controller with VB = 0

Proportional Mode

Implementation of this mode requires a circuit that has the response given by:

P = Kpep + P0 (2.24)

Where P = controller output 0 – 100 %

Kp = Proportional gain

ep = error in percent of variable range

$P0$ = Controller output with no error

Implementation of P – Mode controller using OPAMP

If both the controller output and error expressed in terms of voltage, then the above Eq. 2.24 is a summing amplifier. Fig.2.34 shows such an electronic proportional controller.

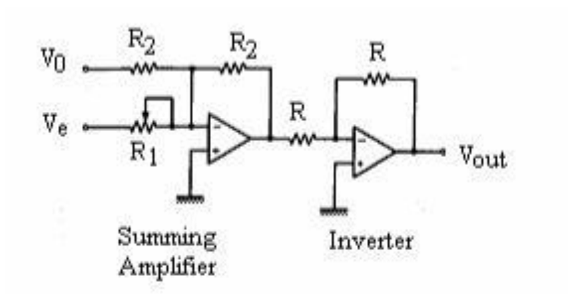


Fig.2.34. An op amp proportional mode controller

Now, the analog electronic equation for the output voltage is:

$$V_{out} = G_p V_e + V_0 \tag{2.25}$$

Where, V_{out} = output voltage

$G_p = R2/R1$ = gain

V_e = Error voltage

V_0 = output with zero error

To use the circuit of Fig.2.34 for proportional mode, a relationship must be established with the characteristics of the mode, defined already, in chapter 1. In Eq. 2.35, the error is expressed as the percent of measurement range, and the output is simply 0 % to 100%. Yet Fig. 2.34 deals with voltage on both the input and output.

Thus, first identify that the output voltage range of the circuit, whatever it is, represents a swing of 0% to 100%. Thus, if a final control element needs 0 to 5 V, then a Zener is added as shown in the Fig.2.35 so that the op amp output can swing only between 0 and 5V.

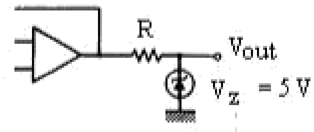


Fig.2.35. A zener diode used to clamp the output swing of an op amp controller

Integral Mode

The general representation of integral controller is

$$P(t) = K_I \int_0^t e_p(t)dt + P_I(0) \tag{2.26}$$

Where, P(t) = controller output in percent of full

scale KI = integration gain (s-1)

ep(t) = deviations in percent of full scale variable

value PI(0) = Controller output at t = 0

Implementation Using OPAMP

Integral controller implemented using OPAMPs is shown in Fig.

2.36 Analysis of the circuit gives,

$$V_{out} = G_I \int_0^t V_e dt + V_{out}(0) \tag{2.27}$$

Where, Vout = Output voltage

GI = 1/RC = Integration gain

Ve = error voltage

Vout(0) = Initial output voltage.

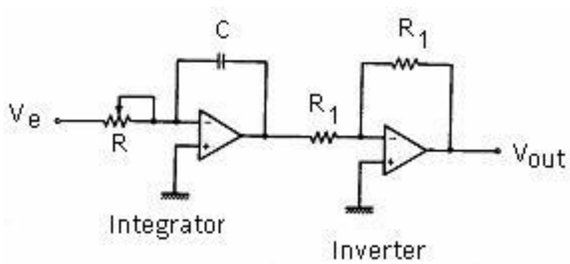


Fig. 2.36. An op amp integral mode controller

The values of R and C can be adjusted to obtain the desired integration time. The initial controller output is the integrator output at t = 0.

If K_I is made too large, the output rises so fast that overshoots of the optimum setting occur and cycling is produced.

Determination of G_I

The actual value of G_I and therefore R and C, is determined from KI and the input and output voltage ranges. Integral gain says that, an input error of 1 % must produce an output that changes as K_I % per second. Or if an error of 1 % lasts for 1 s, the output must change by KI percent.

e.g. Consider an input range of 6 V

Output range of 5V

KI = 3.0 %/(%- min)

Note: Integral gain is often given in minutes because industrial processes are slow, compared to a time of seconds. This gain is often expressed as integration time, TI, which is just the inverse of the gain.

Solution

First convert the time units to seconds. Therefore, [(3 %)/(%- min)][(1min/60s)]

= 0.05%/(%-s) Error of 1 % for 1 sec = (0.01)(6V)(1s) = 0.06 V-s

KI % of the output = (0.0005)(5V) = 0.0025 V

The integral gain GI = (KI % of the output) / (Error of 1 % for 1 sec)

= (0.0025V)/(0.06 V-s)

= 0.0417 s-1

Values of R and C can be selected from this.

Derivative mode

The derivative mode is never used alone because it can not provide a controller output when the error is zero or constant. The control mode equation is given by

$$P(t) = K_D \frac{de_p}{dt} \qquad (2.28)$$

where, P = Controller output in percent of full

output KD = Derivative time constant (s)

ep = error in percent of full scale range

where, P = Controller output in percent of full output

KD = Derivative time constant (s)

ep = error in percent of full scale range

Implementation of derivative controller using OPAMP:

Consider an OPAMP differentiator circuit shown in Fig. 2.37 The theoretical transfer function for this circuit will be given by

$$V_{out} = -RC\frac{dV_e}{dt} \qquad (2.29)$$

where, the input voltage has been set equal to the controller error voltage

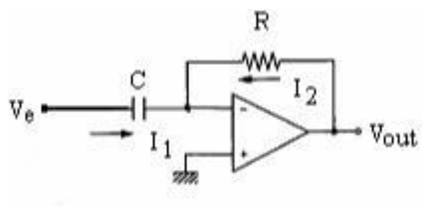


Fig 2.37 OPAMP differentiator circuit

Composite controller modes

Composite modes combine the advantages of each mode and in some cases eliminate the disadvantages. Composite modes are implemented easily using opamp techniques.

Proportional – Integral mode

PI controller is the combination of proportional and integral controller defined by:

$$P = K_p e_p + K_p K_I \int_0^t e_p dt + P_I(0) \qquad (2.30)$$

Where, P = controller output in percent of full

scale ep = process error in percent of the maximum

Kp = Proportional gain

KI= Integral gain

PI(0) = initial controller integral output

Implementation of PI controller using opamps

Figure 2.38 a shows one method of implementation of the PI controller using opamps.

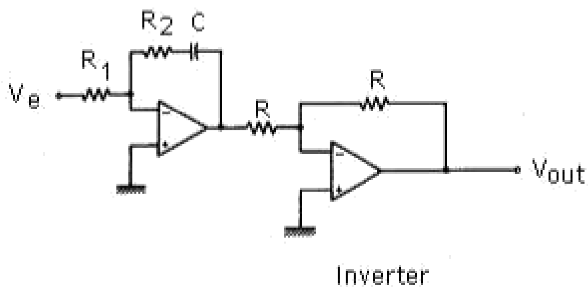


Fig. 2.38 a. An op amp proportional integral (PI) mode controller

To derive an expression for the output voltage of this circuit, first define nodes and currents as shown in the Fig. 2.38 b.

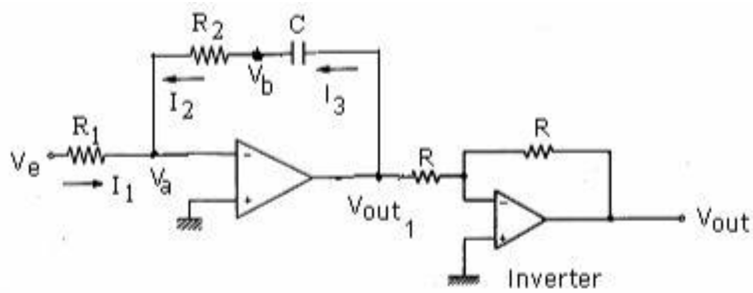


Fig.2.38 b. An op amp proportional integral (PI) mode controller

Note that, there is no current through op amp input terminals and no voltage across the input terminals. Therefore, Va = 0 and

I1 + I2 = 0 (2.31)

I3 – I2 = 0 (2.32)

The relationship between the voltage across the capacitor and current through a capacitor is given by

$$I_c = C\frac{dV_C}{dt} \quad (2.33)$$

Where Vc is the voltage across the capacitor. Combining this with Ohm"s law allows the preceding current equations (2.31 and 2.32) to be written in terms of voltage as

$$\frac{V_e}{R_1} + \frac{V_b}{R_2} = 0 \tag{2.34}$$

$$C\frac{d}{dt}\left[V_{out_1} - V_b\right] - \frac{V_b}{R_2} = 0 \tag{2.35}$$

The Eq. 2.34 can be solved for Vb as:

$$V_b = -\frac{R_2}{R_1}V_e \tag{2.36}$$

Substituting this in to Eq. 2.35

$$C\frac{dV_{out_1}}{dt} - C\frac{d}{dt}\left(-\frac{R_2}{R_1}V_e\right) - \frac{1}{R_2}\left(-\frac{R_2}{R_1}V_e\right) = 0 \tag{2.37}$$

$$C\frac{dV_{out_1}}{dt} + C\frac{R_2}{R_1}\frac{d}{dt}V_e + \frac{1}{R_1}V_e = 0$$

$$\text{Or, } \frac{dV_{out_1}}{dt} + \frac{R_2}{R_1}\frac{d}{dt}V_e + \frac{1}{R_1 C}V_e = 0$$

In order to solve for Vout, integrate this equation to eliminate the derivative on Vout. i.e.:

$$V_{out_1} = -\frac{R_2}{R_1}V_e - \frac{1}{R_1 C}\int_0^t V_e dt + V(0) \tag{2.38}$$

$$V_{out_1} = -\frac{R_2}{R_1}V_e - \frac{R_2}{R_1}\frac{1}{R_2 C}\int_0^t V_e dt + V(0)$$

After inverting

$$V_{out} = \frac{R_2}{R_1}V_e + \frac{R_2}{R_1}\frac{1}{R_2 C}\int_0^t V_e dt + V(0)$$

$$\text{Or, } V_{out} = G_p V_e + G_p G_I \int_0^t V_e dt + V(0) \tag{2.39}$$

Where, Proportional gain, Gp = R2/R1 and integral gain GI = 1/ (R2C)

Proportional Derivative Mode of controller

PD controller is the combination of proportional and derivative mode of controllers. The general definition of PD controller is

$$\mathrm{P(t)} = K_p \mathrm{e_p} + \mathrm{K_p K_D}\frac{\mathrm{de_p}}{\mathrm{dt}} + P(0) \tag{2.40}$$

Where, P = Controller output in percent of full output

Kp = Proportional gain

KD = Derivative time constant (s)

ep = error in percent of full scale range

P(0) = Zero error controller output

Implementation of PD controller using opamps.

Fig. 2.39 a shows how a PD controller can be implemented using op amps. Where the quantities are defined in the figure and the output inverter has been included. This circuit includes the clamp to protect against high gain at high frequency in the derivative term.

$$R = \frac{R_1 R_3}{R_1 + R_3}$$

Then the condition becomes as usual, 2π fmax RC = 0.1. Assuming this criterion has been met, while deriving the equation for the PD response given below

$$V_{out} = \left(\frac{R_2}{R_1 + R_3} \right) V_e + \left(\frac{R_2}{R_1 + R_3} \right) R_3 C \frac{dV_e}{dt} + V_0$$

$$V_{out_1} = G_p V_e + G_p G_D \frac{dV_e}{dt} + V_0$$

Where the proportional gain is

And the derivative gain is GD = R3 C

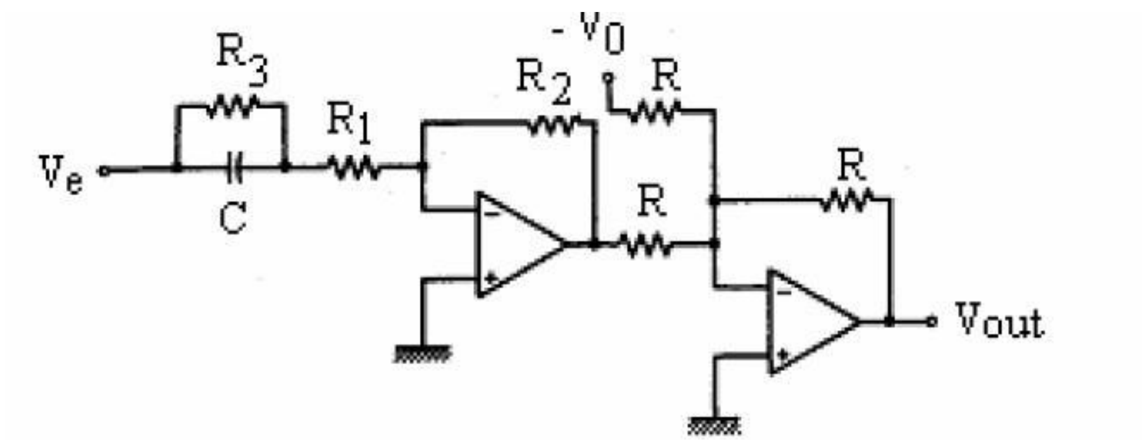


Fig.2.39 a An op amp Proportional Derivative (PD) mode controller

Derivation of PD controller response:

Analysis of PD circuit can be performed using the circuit shown in Fig. 2.39b showing currents and nodes. The voltage across the op amp input terminals, Vb = 0. Also there is no current in to the op amp inputs.

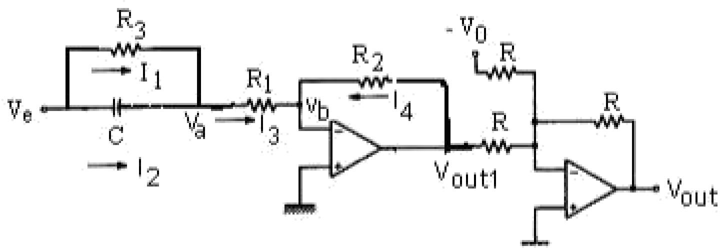


Fig.2.39 b An op amp Proportional Derivative (PD) mode controller

Application of KCL, to the two active nodes provides the equations:

$I_1 + I_2 - I_3 = 0$

$I_4 + I_3 = 0$

Ohm's law and the differential relation between current and voltage for a capacitor can be used to express these equations in terms of voltage.

$$\frac{V_e - V_a}{R_3} + C\frac{d}{dt}\left[V_e - V_a\right] - \frac{V_a}{R_1} = 0$$

$$\frac{V_{out_1}}{R_2} + \frac{V_a}{R_1} = 0$$

$$V_a = -\frac{R_1}{R_2}V_{out_1}$$

$$\frac{V_e}{R_3} + \frac{R_1}{R_2 R_3}V_{out_1} + C\frac{dV_e}{dt} + \frac{R_1}{R_2}C\frac{dV_{out_1}}{dt} + \frac{1}{R_2}V_{out_1} = 0$$

Or, $V_e + \frac{R_1}{R_2}V_{out_1} + R_3 C\frac{dV_e}{dt} + \frac{R_3 R_1}{R_2}C\frac{dV_{out_1}}{dt} + \frac{R_3}{R_2}V_{out_1} = 0$

After rearranging and some more algebra, this reduces to:

$$V_{out_1} + \left(\frac{R_1}{R_1 + R_3}\right)R_3 C\frac{dV_{out_1}}{dt} = -\left(\frac{R_2}{R_1 + R_3}\right)V_e - \left(\frac{R_2}{R_1 + R_3}\right)R_3 C\frac{dV_e}{dt}$$

After inverting

$$V_{out} + \left(\frac{R_1}{R_1 + R_3}\right)R_3 C\frac{dV_{out_1}}{dt} = \left(\frac{R_2}{R_1 + R_3}\right)V_e + \left(\frac{R_2}{R_1 + R_3}\right)R_3 C\frac{dV_e}{dt}$$

Or, $V_{out} = \left(\frac{R_2}{R_1 + R_3}\right)V_e + \left(\frac{R_2}{R_1 + R_3}\right)R_3 C\frac{dV_e}{dt} + V_0$

$$V_{out} = G_p V_e + G_p G_D\frac{dV_e}{dt} + V_0$$

Where the proportional gain is $G_p = \left(\frac{R_2}{R_1 + R_3}\right)$

And the derivative gain is $G_D = R_3 C$

PD controller still has the offset error of a proportional controller because the derivative term cannot provide reset action.

Three Mode Controller

Three mode controllers is the combination of proportional, integral and derivative mode of controllers. Characterized by

$$P = K_p e_p + K_p K_I \int_0^t e_p dt + K_p K_D \frac{de_p}{dt} + P_I(0)$$

Where, P = controller output in percent of full

scale ep = process error in percent of the maximum

Kp = Proportional gain

KI= Integral gain

KD= Derivative gain

PI(0) = initial controller integral output

The zero error term of the proportional mode is not necessary because the integral automatically accommodates for offset and nominal setting.

Implementation of three mode controller using op amps

Three mode controller can be implemented by a straight application of op amps as shown in Fig. 2.40a.

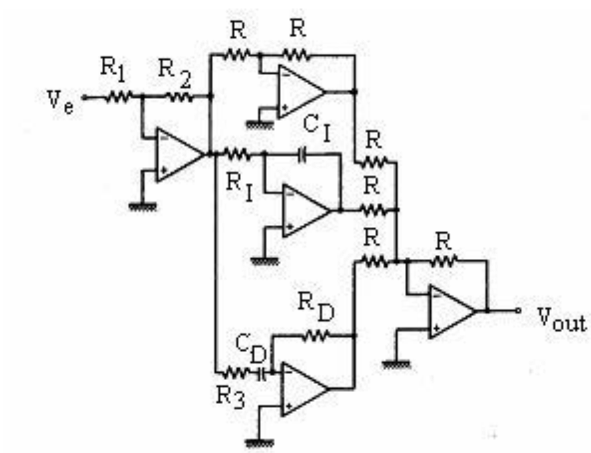


Fig. 2.40a. Implementation of a three mode (PID) controller with op amps

For the analysis, assume the voltages as indicated in Fig. 2.40b

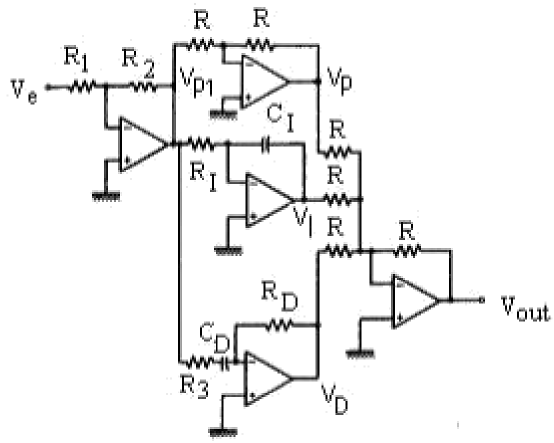


Fig. 2.40b. Implementation of a three mode (PID) controller with op amps

$$V_{p1} = -\frac{R_2}{R_1} V_e$$

$$V_p = \frac{R_2}{R_1} V_e$$

$$V_I = -\frac{1}{R_I C_I} \int_0^t V_{p1} dt \quad \text{or, } V_I = \frac{R_2}{R_1} \frac{1}{R_I C_I} \int_0^t V_e dt$$

$$V_D = -R_D C_D \frac{d}{dt} V_{p1} \quad \text{or, } V_D = \frac{R_2}{R_1} R_D C_D \frac{d}{dt} V_e$$

$- V_{out} = V_p + V_I + V_D$

$$-V_{out} = \left(\frac{R_2}{R_1}\right) V_e + \left(\frac{R_2}{R_1}\right) \frac{1}{R_I C_I} \int V_e dt + \left(\frac{R_2}{R_1}\right) R_D C_D \frac{dV_e}{dt} + V_{out}(0)$$

R_3 has been chosen from $2\pi f_{max} R_3 C_D = 0.1$ for stability. Comparing equations

$$G_p = \left(\frac{R_2}{R_1}\right), \; G_I = \left(\frac{1}{R_I C_I}\right) \text{ and } G_D = R_D C_D$$

$$-V_{out} = G_p V_e + G_p G_I \int V_e dt + G_p G_D \frac{dV_e}{dt} + V_{out}(0)$$

Adding an inverter at the output stage,

$$V_{out} = G_p V_e + G_p G_I \int V_e dt + G_p G_D \frac{dV_e}{dt} + V_{out}(0)$$

Suggested Readings and Websites:

1. Instrument Engineers Handbook: Volume 2-Process Control, by Bela J. Liptak, Chilton Book Company.
2. Computer based industrial control by Krishna Kant, PHI, 2002
3. Computer Control of Processes by M.Chidambaram, Narosa Pub., 2003
4. Computer Aided Process Control by S.K.Singh, PHI
5. www. controlmagazine.com
6. www. icsmagazine.com
7. www. xnet.com/~blatura/control.shtml
8. www.honeywell.com
9. www.controlguru.com/
10. www.processautomationcontrol.com

Glossary:

Control lag: It refers to the time for the process control loop to make necessary adjustments to the final control element.

Control parameter range : It is the range associated with the controller output

Control System: All the elements necessary to accomplish the control objective i.e. regulation of some parameters to have specific or desired values

Control: The methods/techniques to force parameters or variables in the environment / process to have specific values.

Controlled Variable : The process variable regulated by process control loop.

Controller: The element in a process control loop that evaluates error of the controlled variable and initiates corrective action by a signal to controlling variable.

Controlling Variable : The process variable changed by the final control element under the command of controller to effect regulation of controlled variable.

Cycling: It is defined as the oscillations of the error about zero value or nominal value.

Dead time : It is the elapsed time between the instant a deviation (error) occurs and the corrective action first occurs.

Direct Action: If the controller output increases with increase in controlled variable then it is called direct action.

Dynamic variable: The process variable that can change from moment to moment because of unknown sources.

Error: The algebraic difference between the measured value of variable and setpoint.

Process Control: It deals with the elements and methods of control system operations used in industry to control industrial processes.

Process Equation: It is a function which describes the process and provides the information about other process parameters which influence the controlled variable

Process Lag: It refers to the time consumed by the process itself to bring the controlled variable to setpoint value during load change

Process Load: It refers to set of all process parameters excluding the controlled variable in a process.

Process: In general, process constitutes a sequence of events in which a raw material will be converted into finished product. "Any system composed of dynamic variables, usually involved in manufacturing & production operations".

Regulation: It means to maintain a quantity or variable at some desired value regardless of external influences.

Reverse Action: If the controller output decreases with increase in controlled variable then it is called direct action

Self-regulation: Some processes adopt to stable value without being regulated via process control loop.

Setpoint: The desired value of a controlled variable in process control loop.

Variable range: The variable range can be expressed as the minimum and maximum value of the variable or the nominal value ± the deviation spread about the nominal value.

Question Bank

Part A

1. Why derivative mode of control is not recommended for a noisy process?

   Why is it necessary to choose controller settings that satisfy both gain margin and phase margin

2. What are the basic control actions in process control?
3. Define proportional band.
4. Define reset time
5. Define differential gap. Why is it introduced in a process?
6. Identify the two papameters of ON OFF controllers
7. What is meant by neutral zone in ON-OFF controller?
8. Define integral windup and Anti reset windup.
9. What are the advantages and disadvantages of PI control?
10. Derivative controls cannot be used alone. Justify your answer.
11. What is dead time?
12. Define 1⁄4 decay ratio.
13. What are the advantages and disadvantages of 2-position control?
14. What are the advantages, disadvantages and applications of PD controller?
15. What is meant by differential gap? What are its effects? Is it a desirable factor?
16. Design an electronic p-controller with a proportional gain 5.
17. What are the advantages and disadvantages of PID control actions?

PART B

1. With neat schematic diagram explain the single speed floating control.
2. With neat sketch explain the of P+I pneumatic controller.
3. Explain with neat diagram the working of electronic PID controller.
4. When an on-off controller is recommended? How its performance affected by process dead
5. A pi controller has 20% and integral time of 10sec.for a constant error of 5%.determine the controller output after 10sec.the controller offset is 25%.
6. Compare the features of ON & OFF,P,I,D control modes and draw their characteristics.
7. Explain the procedure for tuning pi controller using zielger Nicholas method.
8. What is cycling in the process output in which control mode it occurs.(may10)
9. A second order process with the transfer function of g(s)=5/(10s+1)(3s+1) is controlled by a proportional controller. Find the value of kp required offset due to unit step change in set point is 5% of steady state value of controlled variable.
10. Various steps involved in process reaction curve method of tuning off controllers
11. Explain the various time integral performance criteria with closed loop response

UNIT-III

CONTROLLER DESIGN

Controller Tuning

Controller tuning is the process by which a control engineer or technician selects values of user-adjustable controller parameters (for a PID controller these are the bias, gain, integral time, and derivative time) so that the closed loop dynamic response behaves as desired.

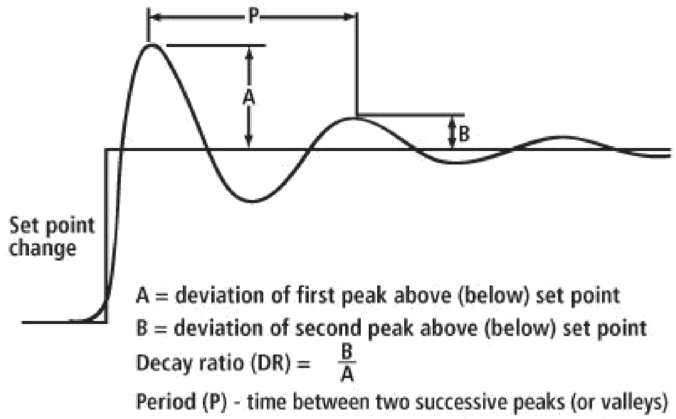
Loop tunings are the primary point of contact between an operations/manufacturing engineer and the plant control system. Controller settings determine the system response: a poorly tuned controller may be as bad as no controller at all.

Tuning is a exercise in compromise. Controller objectives, specifications, requirements, and performance always conflict to some degree or another. There are rarely absolute criteria for selecting tunings and so judgement is required.

As you prepare to tune a loop, you must consider a range of concerns and objectives.

1⁄4 DECAY RATIO

The ratio of the amounts above the ultimate value of two successive peaks is called as decay ratio. The decay ratio is expressed as, Decay ratio: $C/A = \exp(-2\zeta\pi / \sqrt{1 - \zeta^2})$. If C/A=1 then it is called ¼ decay ratio.



Time Integral performance criteria

The *error* in a control loop is usually defined as the deviation from setpoint. There are a variety of ways of quantifying the cumulative error:

- *Integral Squared Error*. Penalizes large errors more than small.

$$ISE = \int_0^{time} \left(e(t)\right)^2 dt$$

- *Integral Absolute Error*. The sum of areas above and below the setpoint, this penalizes all errors equally regardless of direction.

$$IAE = \int_0^{time} |e(t)| dt$$

- *Integral Time Weighted Absolute Error*. Penalizes persistent errors.

$$ITAE = \int_0^{time} t|e(t)| dt$$

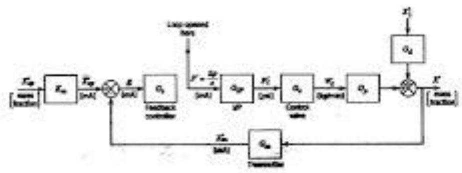
IAE - allows larger deviation than ISE (smaller overshoots)

ISE - longer settling time

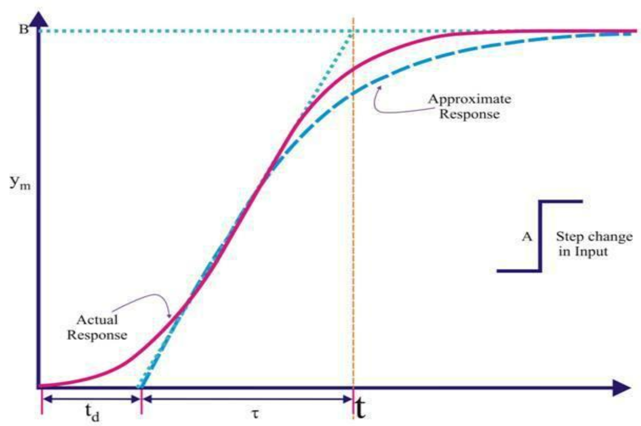
ITAE - weights errors occurring later more heavily

Tuning Of Controller By Process Reaction Curve Method:

Developed by Cohen-Coon which has been opened by disconnecting the controller from the final control element.



It is observed that the response of most of the processes under step change in input yields a sigmoidal shape



Process Reaction Curve for Cohen Coon Method

Such sigmoidal shape can be adequately approximated by the response of a first order process with dead time.

$$G_{PRC} = \frac{y_m(s)}{u(s)} \cong \frac{Ke^{-t_d s}}{\tau s + 1}; \qquad K = \frac{B}{A}$$

=

− s- slope of the sigmoidal response at the point of inflection

-time elapse until the system responded

K- static gain

-time constant

From the approximate response it is easy to estimate the parameters. The controllers are designed as given in Table

Controller settings using Cohen-Coon design method

	K_c	τ_I	τ_D
P	$\frac{1}{K}\frac{\tau}{t_d}\left(1+\frac{t_d}{3\tau}\right)$	–	–
PI	$\frac{1}{K}\frac{\tau}{t_d}\left(0.9+\frac{t_d}{12\tau}\right)$	$t_d\frac{30+3\,t_d/\tau}{9+20\,t_d/\tau}$	–
PID	$\frac{1}{K}\frac{\tau}{t_d}\left(\frac{4}{3}+\frac{t_d}{4\tau}\right)$	$t_d\frac{32+6\,t_d/\tau}{13+8\,t_d/\tau}$	$t_d\frac{4}{11+2\,t_d/\tau}$

Processes with very short time delay

- When is very small the process reaction curve reminds us of the response of a simple first order system
- The cohen coon settings dictate an extremely large value for the proportional gain

Closed loop tuning by Ziegler and Nichols:

Ziegler-Nichols Method: More than six decades ago, P-I controllers were more widely used than P-I-D controllers. Despite the fact that P-I-D controller is faster and has no oscillation, it tends to be unstable in the condition of even small changes in the input set point or any disturbances to the process than P-I controllers. Ziegler-Nichols Method is one of the most effective methods that increase the usage of P-I-D controllers.

The most often technique for closed loop technique was developed by the Ziegler and Nichols technique in 1942. The following steps illustrate the procedure to find the ultimate gain and ultimate period.

1. Set the integral and derivative constants to its maximum value i.e Ti is large and Td is „0‟ by leaving the proportional value. Maintain the controller in auto mode with closed loop.

2. Set the proportional constant to some arbitrary value and observe the response of the process by giving some upset to the process. The best way to provide upset to the process is to increase the set-point for small time and return back to the original value.
3. If the response curve is does not damp out and gives a increasing decay ratio (as in curve A) , illustrates that the gain (Kp) is too high (Proportional Band (PB) is too low). The gain should be reduced (PB should be increased) and repeat the same step-2.
4. If the response curve dams out (as shown in curve C) at steady state then the gain is too low (PB is too high), the gain should be increased (PB should be reduced) and repeat the step-2.
5. The step-2 should be repeated by changing the proportional gain until the response of the closed loop is oscillatory (similar to curve B). i.e without damping and without increasing decay ratio. The values of the ultimate gain and ultimate period are noted if the response is similar as shown in curve B. The ultimate gain which results the closed loop response with sustained oscillation is the ultimate sensitivity Su and

   ultimate period is Pu.

The ultimate gain and ultimate period are used to calculate the controller settings as per the tuning method.

Ziegler and Nichols correlated in the case of proportional control that the value of proportional setting should be half of the ultimate sensitivity Su. This setting often provides a closed response with one quarter decay ratio in case of proportional controller. Same way by following equations are found out as good rules of thumb for better combination of controller parameters.

Proportional controller

$$Kc = 0.5\ Su$$

Proportional- Integral (PI)controller

$$Kc = 0.45\ Su$$

$$Ti = Pu / 1.2$$

Proportional – Derivative (PD) controller

$$Kc = 0.6\ Su$$

$$Td = Pu / 8$$

Proportional – Integral – Derivative (PID) controller

$$Kc = 0.6\ Su$$

$$Ti = o.5\ Pu$$

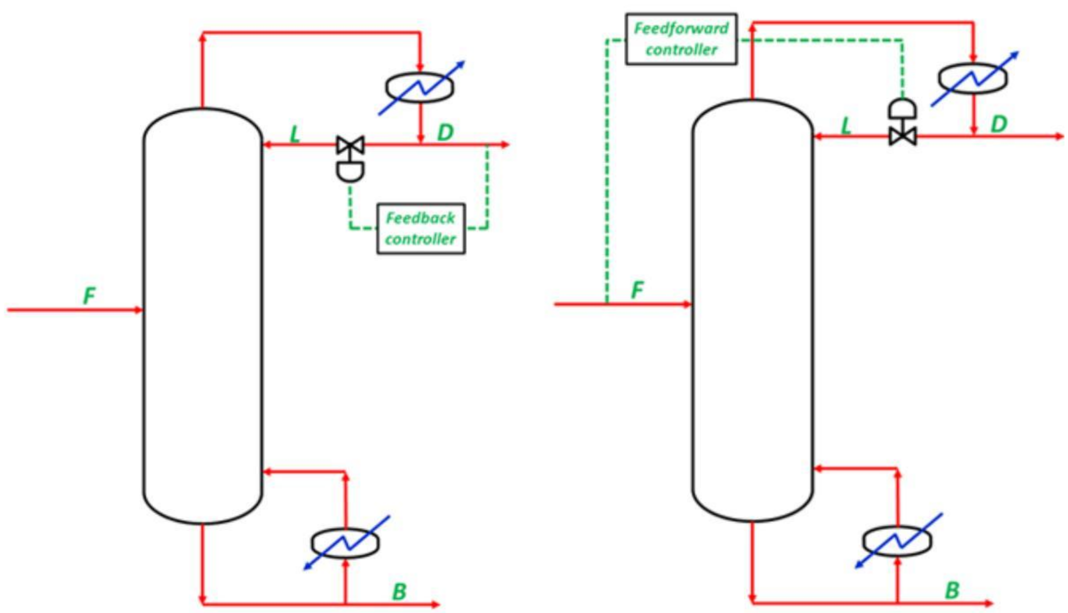
$$Td = Pu / 8$$

Again it should be that the above equations are empirical and inherent and chosen to achieve a decay ratio of one quarter.

Feed forward control

A feedback controller responds only after it detects a deviation in the value of the controlled output from its desired set point. On the other hand, a feedforward controller detects the disturbance directly and takes an appropriate control action in order to eliminate its effect on the process output.

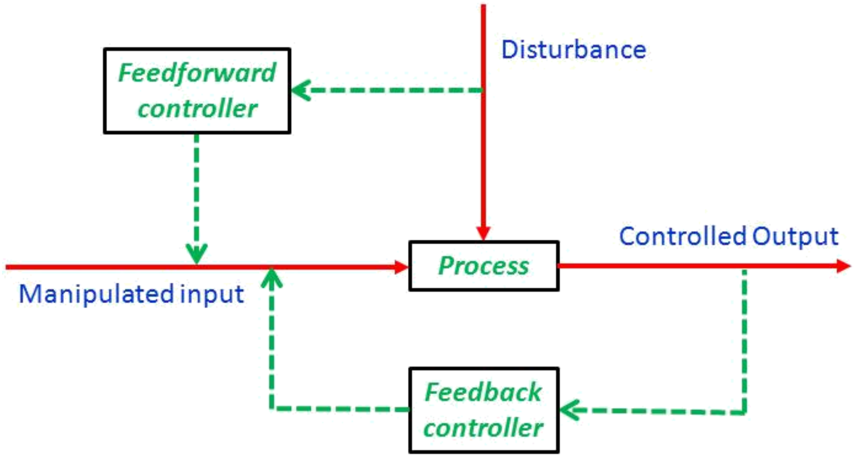
Consider the distillation column shown in Fig. The control objective is to keep the distillate concentration at a desired set point despite any changes in the inlet feed stream.



(a) Feedback control configuration (b) Feedforward control configuration

Feedback and Feedforward control configuration of a distillation column

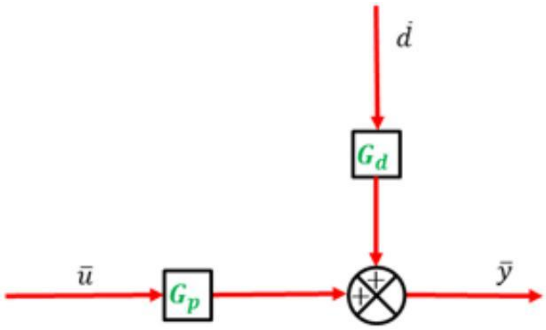
Fig. shows the conventional feedback loop, which measures the distillate concentration and after comparing it with the desired setpoint, increases or decreases the reflux ratio. A feedforward control system uses a different approach. It measures the changes in the inlet feed stream (disturbance) and adjusts the reflux ratio appropriately. Fig shows the feedforward control configuration.



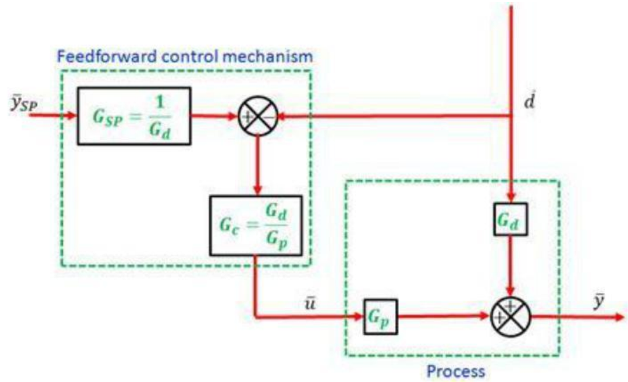
The comparative schematic of feedback and feedforward control structure

Fig shows the general form of a feedforward control system. It directly measures the disturbance to the process and anticipates its effect on the process output. Eventually it alters the manipulated input in such a way that the impact of the disturbance on the process output gets eliminated. In other words, where the feedback control action starts after the disturbance is “felt” through the changes in process output, the feedforward control action starts immediately after the disturbance is “measured” directly. Hence, feedback controller acts in a compensatory manner whereas the feedforward controller acts in an anticipatory manner.

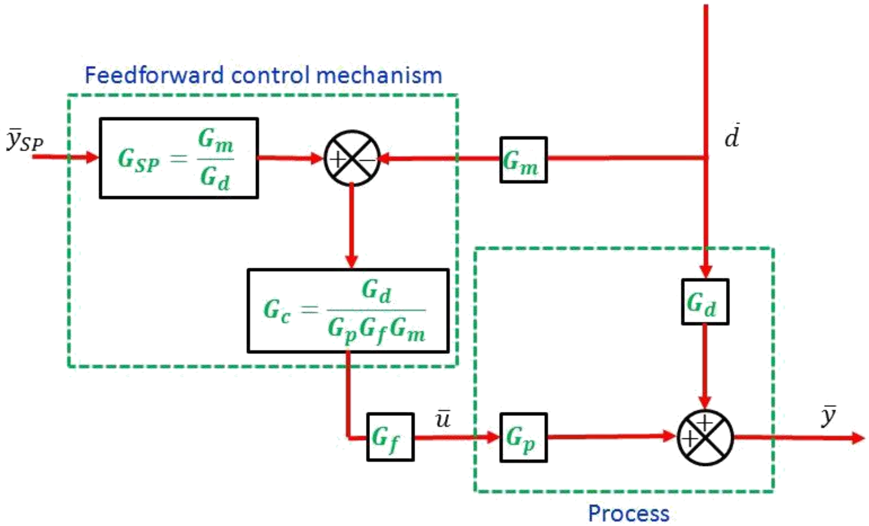
Design of feedforward controller::*Let us consider the block diagram of a process shown in Fig. The Fig.* presents the open-loop diagram of the process. The process and disturbance transfer functions are represented by G_p and G_d respectively. The controlled output, manipulated input and the disturbance variable are indicated as $\bar{y}, \bar{u}$ and $\bar{d}$ respectively.



(a) Open-loop process diagram



(b) Process diagram with feedforward controller



(c) Process diagram with feedforward controller, sensor and valve

The schematic of a feedforward controller mechanism

The process output is represented by

$$\bar{y} = G_p\bar{u} + G_d\bar{d} \tag{V.1}$$

The control objective is to maintain $\bar{y}$ at the desired setpoint $\bar{y}_{SP}$. Hence the eq (V.1) can be rewritten as

$$\bar{y}_{SP} = G_p\bar{u} + G_d\bar{d} \tag{V.2}$$

The eq. (V.2) can be rearranged in the following manner:

$$\bar{y}_{SP} - G_d\bar{d} = G_p\bar{u}$$

or

$$\bar{u} = \frac{1}{G_p}\left(\bar{y}_{SP} - G_d\bar{d}\right) = \frac{G_d}{G_p}\left(\frac{1}{G_d}\bar{y}_{SP} - \bar{d}\right) = G_c\left(G_{SP}\bar{y}_{SP} - \bar{d}\right) \tag{V.3}$$

The eq. (V.3) can be schematically represented by Fig V.3(b).

For the sake of simplicity, measuring element and final control element were not considered as parts of the feedforward control configuration as shown in Fig V.3(b). In a more generalized case, when such elements are added in the controller configuration, the resulting control structure takes the form of Fig V.3(c). A generalized form of controller equation can be written as

$$\bar{u} = G_c G_f \left(G_{SP} \bar{y}_{SP} - G_m \bar{d} \right) \quad \text{(V.4)}$$

And

$$\bar{y} = G_p \bar{u} + G_d \bar{d} = G_p \{ G_c G_f \left(G_{SP} \bar{y}_{SP} - G_m \bar{d} \right) \} + G_d \bar{d} = \{ G_p G_c G_f G_{SP} \} \bar{y}_{SP} + \{ G_d - G_p G_c G_f G_m \} \bar{d} \quad \text{(V.5)}$$

In case of regulatory problem (disturbance rejection) i.e. when $\bar{y}_{SP} = 0$, the controller should be able to reject the effect of disturbance and ensure no deviation in the output, i.e. $\bar{y} = 0$. In other words,

$$G_d - G_p G_c G_f G_m = 0 \quad \text{(V.6)}$$

or

$$G_c = \frac{G_d}{G_m G_f G_p} \quad \text{(V.7)}$$

In case of servo problem (setpoint tracking), i.e. when $\bar{d} = 0$, the controller should be able to ensure that output tracks the setpoint, i.e. $\bar{y} = \bar{y}_{SP}$. In other words,

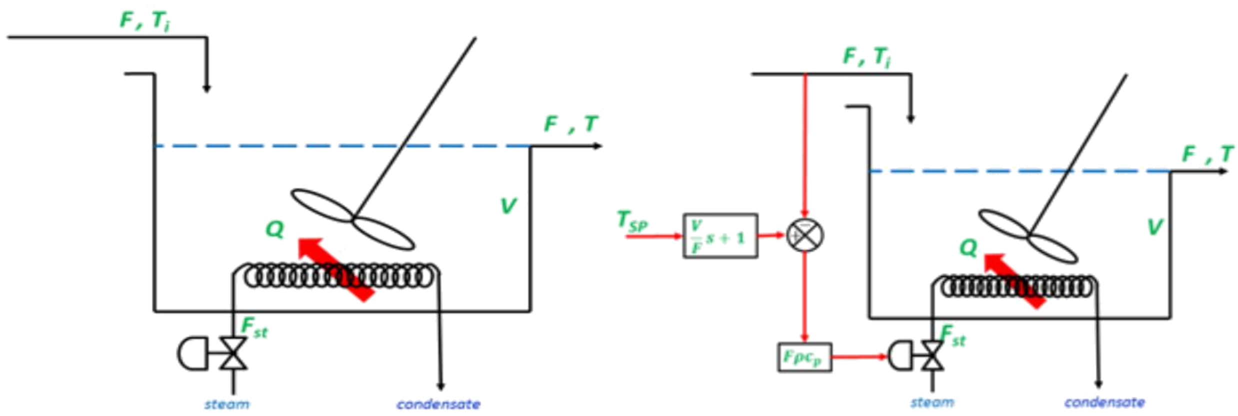
$$G_p G_c G_f G_{SP} = 1 \quad \text{(V.8)}$$

or

$$G_{SP} = \frac{1}{G_p G_c G_f} = \frac{1}{G_p \left(\frac{G_d}{G_m G_f G_p} \right) G_f} = \frac{G_m}{G_d}$$

Example of design of feedforward controller:

Consider an overflow type continuous stirred tank heater shown in Fig V.4. The fluid inside the tank is heated with steam whose flow rate is $\boldsymbol{F_{st}}$ and supplying heat at a rate of $\boldsymbol{Q}$ to the fluid. Temperatures of the inlet and outlet streams are $\boldsymbol{T_i}$ and T respectively. V is the volume of liquid which is practically constant in an overflow type reactor. A control valve in the steam line indicates that the steam flow rate can be manipulated in order to keep the liquid temperature at a desired setpoint. Temperature of the inlet stream flow is the source of disturbance (change in $\boldsymbol{T_i}$) to the process.



(a) Process without a controller (b) Process with feedforward controller

Feedforward control configuration of an overflow type continuous stirred tank heater

A simple energy balance exercise will yield the model equation of the above process as:

$$V\frac{dT}{dt} = F(T_i - T) + \frac{Q}{\rho c_p} \tag{V.10}$$

All the variables are assumed to be in the deviation form. Hence, taking Laplace transform on both sides we obtain:

$$VsT(s) = F\{T_i(s) - T(s)\} + \frac{Q(s)}{\rho c_p} \tag{V.11}$$

or,

$$VsT(s) + FT(s) = FT_i(s) + \frac{Q(s)}{\rho c_p} \tag{V.12}$$

$$\frac{V}{F}sT(s) + T(s) = T_i(s) + \frac{Q(s)}{F\rho c_p} \tag{V.13}$$

or,

$$T(s) = \left\{\frac{1}{\left(\frac{V}{F}s + 1\right)}\right\}T_i(s) + \left[\left\{\frac{1}{F\rho c_p}\right\}\left\{\frac{1}{\left(\frac{V}{F}s + 1\right)}\right\}\right]Q(s) \tag{V.14}$$

The feedforward controller is meant for ensuring $T = T_{SP}$. Hence,

$$\left(\frac{V}{F}s + 1\right)T_{sp}(s) = T_i(s) + \left[\left\{\frac{1}{F\rho c_p}\right\}\right]Q(s) \tag{V.15}$$

or

$$Q(s) = F\rho c_p\left\{\left(\frac{V}{F}s + 1\right)T_{sp}(s) - T_i(s)\right\} \tag{V.16}$$

Hence, one needs to set F_{st} in such a way that Q amount of heat as given in eq.(V.16) is transferred to the process. Fig (b) represents the feedforward structure of the controller.

Remarks:

• The feedforward controller ideally does not get any feedback from the process output. Hence, it solely works on the merit of the model(s). The better a model represents the behavior of a process,the better would be the performance of a feedforward controller designed on the basis of that model. Perfect control necessitates perfect knowledge of process and disturbance models and this is practically impossible. This inturn is the main drawback of a feedforward controller.

• The feedforward control configuration can be developed for more than one disturbance in multi-controller configuration. Any controller in that configuration would act according to the disturbance for which it is designed.

• External characteristics of a feedforward loop are same as that of a feedback loop. The primary measurement (disturbance in case of feedforward control and process output in case of feedback control) is compared to a setpoint and the result of the comparison is used as the actuating signal for the controller. Except the controller, all other hardware elements of the feedforward control configuration such as sensor, transducer, transmitter, valves are same as that of an equivalent feedback control configuration.

• Feedforward controller cannot be expressed in the feedback form such as P, PI and PID controllers. It is regarded as a special purpose computing machine .

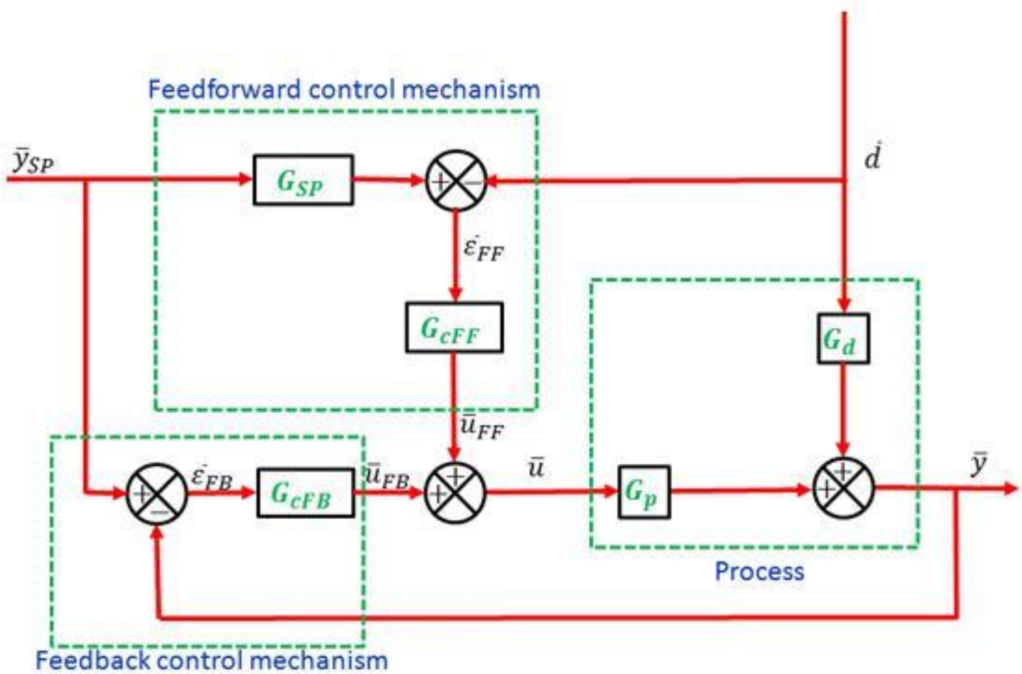
Combination of Feedforward-Feedback Controller:

Table: Merits and demerits of feedforward and feedback controllers

Merits	**Demerits**
Feedforward controllers	
Takes corrective action before the process “feels” the disturbance	Requires measurement of all disturbances affecting the system
Good for sluggish systems and/or system with large deadtime	Sensitive to variation in process parameters
Does not affect the stability of the process	Requires a “near perfect” model of the process

Feedback controllers	
Does not require disturbance measurement	Acts to take corrective action after the process "feels" the disturbance
Insensitive to mild errors in modeling	Bad for sluggish systems and/or system with large deadtime
Insensitive to mild changes in process parameters	May affect the stability of the process

Let us now explore how a combination of feedforward and feedback controller would perform when they are designed to act simultaneously. The schematic of a feedforward-feedback controller is shown in Fig



The schematic of a feedforward-feedback controller

Without losing the generality we shall ignore the transfer functions of the measuring element and the final control element.

Now the closed loop transfer function of feedforward-feedback controller can be derived in the following manner:

$$\begin{aligned}\bar{y} = G_p\bar{u} + G_d\bar{d} &= G_p(\bar{u}_{FF} + \bar{u}_{FB}) + G_d\bar{d} = G_p(G_{cFF}\bar{\varepsilon}_{FF} + G_{cFB}\bar{\varepsilon}_{FB}) + G_d\bar{d} \\ &= G_p\left(G_{cFF}\{G_{SP}\bar{y}_{SP} - \bar{d}\} + G_{cFB}\{\bar{y}_{SP} - \bar{y}\}\right) + G_d\bar{d}\end{aligned} \qquad (V.17)$$

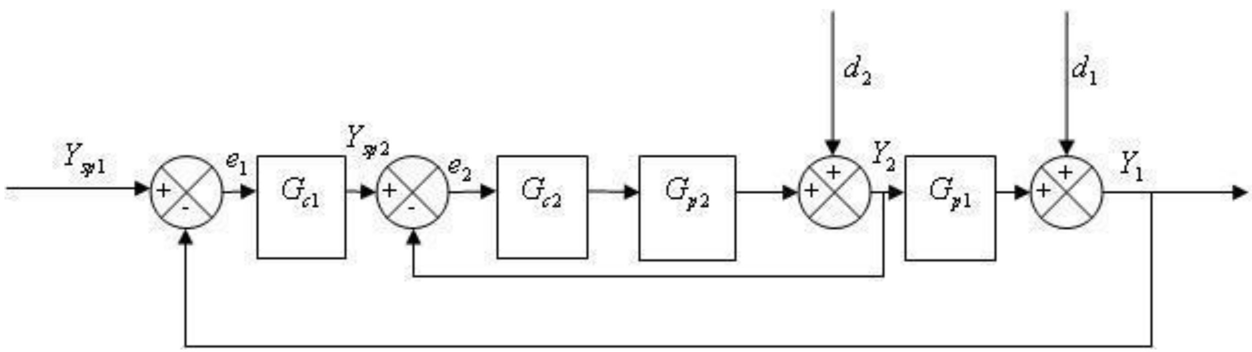
Rearranging the above we get,

$$\bar{y} = \frac{G_p(G_{cFB} + G_{SP}G_{cFF})}{1 + G_pG_{cFB}}\bar{y}_{SP} + \frac{G_d - G_pG_{cFF}}{1 + G_pG_{cFB}}\bar{d} \quad \text{(V.18)}$$

It is observed that the stability of the closed loop response is determined by the roots of the characteristic equation: $1 + G_pG_{cFB} = 0$. Hence, the stability characteristics of a process does not change with the addition of a feedforward loop.

Cascade Control

The primary disadvantage of conventional feedback control is that the corrective action for disturbances does not begin until after the controlled variable deviates from the setpoint. In other words, the disturbance must be "felt" by the process before the control system responds. Feedforward control offers large improvements over feedback control for processes that have large time constant and/or delay. However, feedforward control requires that the disturbances be measured explicitly and that a model be available to calculate the controller output. Cascade control is an alternative approach that can significantly improve the dynamic response to disturbances by employing a secondary measurement and a secondary feedback controller. The secondary measurement point is located so that it recognizes the upset condition sooner than the controlled variable, but the disturbance is not necessarily measured.



Cascade Control Structure

Let us consider the following block diagram of cascade control structure.The outer loop and its controller are called master loop and master controller whereas the inner loop and its controller are called slave loop and slave controller respectively.

$$Y_2 = G_{p2}G_{c2}e_2 + d_2 = G_{p2}G_{c2}\left(Y_{sp2} - Y_2\right) + d_2 \quad \text{V.20}$$

Simplifying

$$Y_2 = \frac{G_{p2}G_{c2}}{(1+G_{p2}G_{c2})}Y_{sp2} + \frac{1}{(1+G_{p2}G_{c2})}d_2 \qquad \text{V.21}$$

Similarly

$$\begin{aligned} Y_1 &= G_{p1}Y_2 + d_1 \\ &= G_{p1}\left[\frac{G_{p2}G_{c2}}{(1+G_{p2}G_{c2})}Y_{sp2} + \frac{1}{(1+G_{p2}G_{c2})}d_2\right] + d_1 \\ &= G_{p1}\left[\frac{G_{p2}G_{c2}}{(1+G_{p2}G_{c2})}G_{c1}e_1 + \frac{1}{(1+G_{p2}G_{c2})}d_2\right] + d_1 \\ &= G_{p1}\left[\frac{G_{p2}G_{c2}}{(1+G_{p2}G_{c2})}G_{c1}(Y_{sp1}-Y_1) + \frac{1}{(1+G_{p2}G_{c2})}d_2\right] + d_1 \end{aligned} \qquad \text{V.22}$$

Again simplifying the above eqn:

$$Y_1 = \frac{G_{p1}G_{p2}G_{c2}G_{c1}}{1+G_{p2}G_{c2}+G_{p1}G_{p2}G_{c2}G_{c1}}Y_{sp1} + \frac{G_{p1}}{1+G_{p2}G_{c2}+G_{p1}G_{p2}G_{c2}G_{c1}}d_2 + \frac{1+G_{p2}G_{c2}}{1+G_{p2}G_{c2}+G_{p1}G_{p2}G_{c2}G_{c1}}d_1 \qquad \text{V.23}$$

Now see what happens if the secondary loop is absent. In that case:

$$Y_2 = G_{p2}Y_{sp2} \qquad \text{V.24}$$

and

$$\begin{aligned} Y_1 &= G_{p1}Y_2 + d_1 \\ &= G_{p1}\left[G_{p2}Y_{sp2} + d_2\right] + d_1 \\ &= G_{p1}\left[G_{p2}G_{c1}e_1 + d_2\right] + d_1 \\ &= G_{p1}\left[G_{p2}G_{c1}(Y_{sp1}-Y_1) + d_2\right] + d_1 \end{aligned} \qquad \text{V.25}$$

Simplifying

$$Y_1 = \frac{G_{p1}G_{p2}G_{c1}}{(1+G_{p1}G_{p2}G_{c1})}Y_{sp1} + \frac{G_{p1}}{(1+G_{p1}G_{p2}G_{c1})}d_2 + \frac{1}{(1+G_{p1}G_{p2}G_{c1})}d_1 \qquad \text{V.26}$$

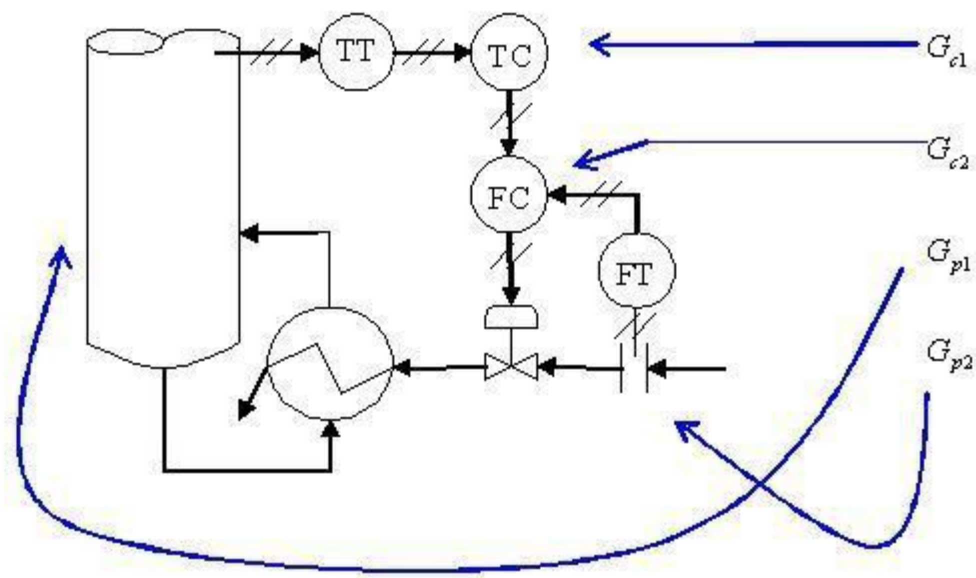
Few points to remember on Cascade controller:

• The slave loop should be tuned before the master loop. After the slave loop is tuned and closed, the master loop should be designed based on the dynamics of inner loop.

• There is little or no advantage to using cascade control if the secondary process is not significantly faster than the primary process dynamics. In particular, if there is long dead time in the secondary process, it is unlikely that the cascade controller will be better than the standard feedback control.

• The most common cascade control loop involves flow controller (eg. TC/FC example in distillation column) as the inner loop. This loop easily rejects the disturbances in fluid steam pressure, either upstream or downstream of the valve.

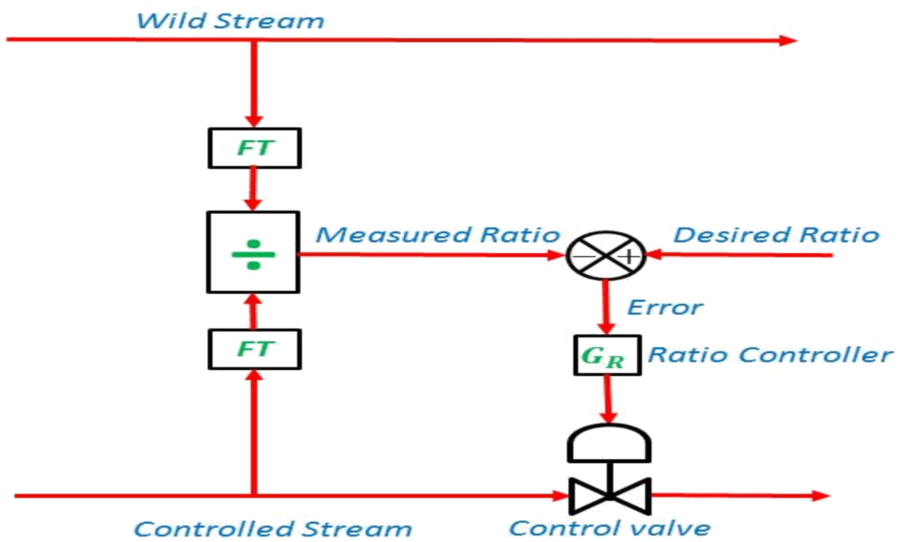
The Fig presents the process and Instrumentation diagram of a distillation column on which a Cascade controller structure has been employed. Only the bottom part of the column has been shown in the figure.



Partial P & ID of a distillation column with cascade structure of temperature a flow control.

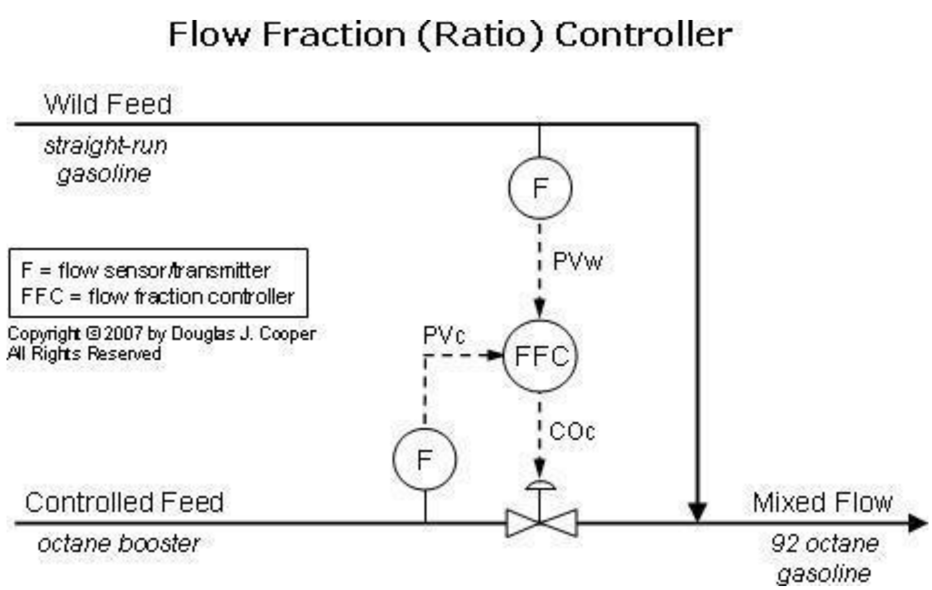
Ratio Control

A ratio controller is a special type of feedforward controller where disturbances are measured and their ratio is held at a desired set point by controlling one of the streams. The other uncontrolled stream is called wild stream. Fig shows the schematic of a ratio controller. The ratio of flow rates of two streams are being held at a desired ratio by controlling the flow rate of one stream. The flow rates are measured through flow transmitters (FTs).



The schematic of a ratio control structure

In this configuration we measure both flow rates and take their ratio. This ratio is compared to the desired ratio and the deviationbetween the measured and desired ratios constitutes the actuating signal for the ratio controller



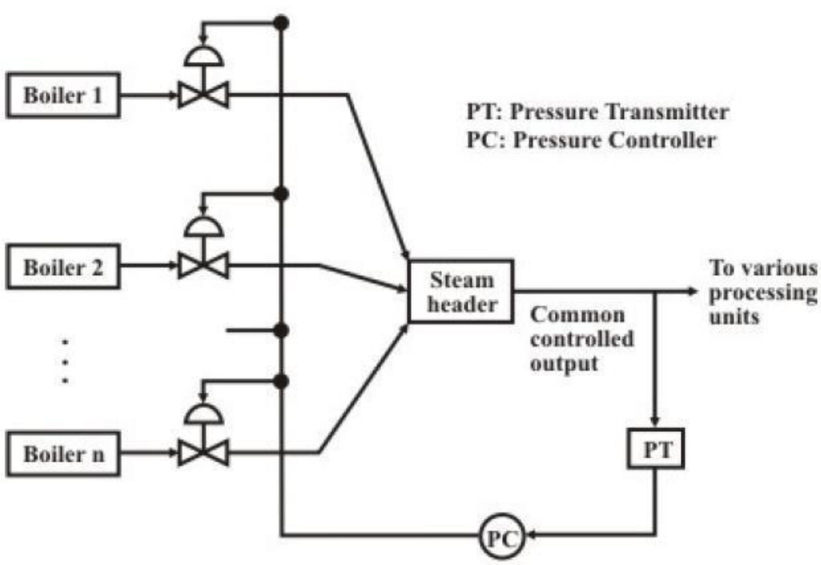
In this configuration we measure the flow rate of the wild feed and multiply it with desired ratio. The result is the flow rate that the stream should have and constitutes the setpoint value which is compared to the measured flow rateof the controlled feed. The deviation constitutes the actuating signalfor the controllerwhich adjusts appropriately thr flow of controlled feed.

The chemical process industries have various applications for ratio controllers. Following are a few such examples:

- Reflux ratio and reboiler feed ratio in a distillation column
- Maintaining the stoichiometric ratio of reactants in a reactor
- Keeping air/fuel ratio in a combustion process

Split Range Control

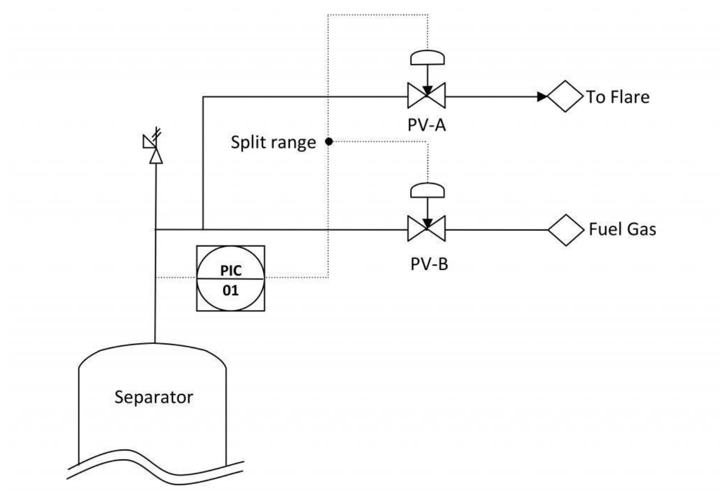
This type of control is used, where there are several manipulated variables, but a single output variable. The coordination among different manipulated variables is carried out by using Split Range Control. In a split range control loop, output of the controller is split and sent to two or more control valves. The splitter defines how each valve is sequenced as the controller output changes from 0 to 100%. In most split range applications, the controller adjusts the opening of one of the valves when its output is in the range of 0 to 50% and the other valve when its output is in the range of 50% to 100%.



Steam header with split-range control

Fig shows an example of a typical split range control scheme. The steam discharges from several boilers are combine at a steam header. Overall steam pressure at the header is to be maintained constant through a pressure control loop. The command from the pressure controller is used for controlling simultaneously the steam flow rates from the boilers in parallel. Clearly, there is a single output variable (steam header pressure) while there are a number of manipulating variables (discharge from different boilers).

The principle of a split range control is illustrated in the following example:

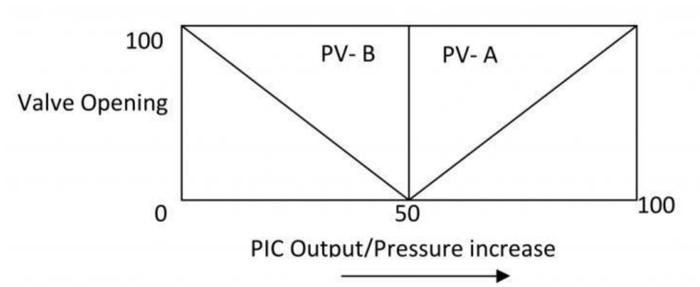


In figure1, PIC-01 controls the pressure of the separator for liquid-vapor hydrocarbons, by mean of a split range controller with the output signal split and sent to two pressure control valves PV-A and PV-B. When pressure increases, the fluid shall be discharged to flare. When the pressure decreases, Fuel gas is introduced to compensate the pressure of the separator

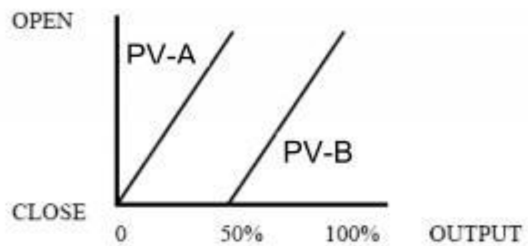
The fuel gas valve (PV-B) needs to close in response to increasing of pressure of the separator, while the flare valve (PV-A) will need to open when the pressure increases beyond setpoint.

When the pressure increases beyond setpoint in range of with 0-50% controller output, PV-B shall close from fully open to fully close.

When the pressure increases beyond setpoint in range of with 50-100% controller output, PV-A shall open from fully close to fully open.



In this case, the service of both control valves is different, with respect to use of fuel gas and flaring for pressure control. Another case of use of split range control loop is when one control valve cannot be suitably designed to cover the complete operating range of the controller. In that case, valve with a smaller Cv operated between 0-50% range and the other operates for 50-100% range. For example, a pressure controller for accumulator drum in overhead of a stabilizer column splits range to open 2 control valves. In the low range (0-50 % range in response to high pressure of the stabilizer), the off gas is routed to a gas plant downstream, in the high range (50-100% range in response to high high pressure of the stabilizer) the off gas goes to flare by opening of valve B. The control valve actions is as follow:



Multivariable control

When more than one input and output exist in a process, several control configurations are possible depending on which output is controlled by manipulating which input. In order to arrive at the best possible configuration, one needs to answer the following crucial design questions:

• What are the control objectives, *i.e* . how many and which process variables are needed to be controlled?

• Which outputs should be measured and whether they are primary measurements (outputs to be controlled are measured directly, *eg* . temperature, pressure *etc*.) or secondary measurements (outputs to be controlled are estimated from auxiliary measurements, *eg* . concentrations *etc* .)?

• Which inputs should be manipulated? Manipulations should be direct, fast and should have strong effects on controlled outputs. Some inputs are quite reliable in manipulation (*e.g.* liquid flow) and some are not (*e.g.* solid/slurry flow).

• What are the possible control configurations (input/output mapping) and what is the best among them? How to measure its superiority quantitatively?

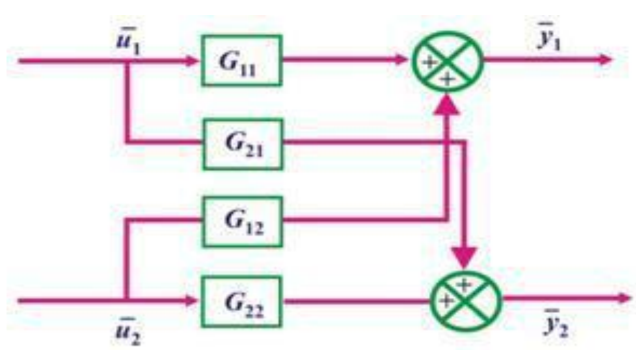
Recall the concept of degrees of freedom (DoF) discussed in Module II. It is clear that in order to determine a system completely, its DoF should be equal to zero. There are two sources of additional specifications/equations that would reduce the DoF to zero, *viz* . control system and its surroundings. The surrounding refers to everything outside the domain of closed loop system which has the capability of influencing the operating conditions. The process disturbances and few externally specified inputs fall under this category. Remaining equations are provided by the control system that imposes certain relationships between outputs to be controlled and the inputs to be manipulated. These relationships are nothing but the controller equations. It is evident that number of controlled outputs should be equal to number of controller equations. Let us assume that there are *N* number of controlled outputs and *M* number of inputs are available for their manipulation $(M \geq N)$. To have a unique solution to the N number of controller equations one has to have exactly N number of freely adjustable manipulations. In other words, for a perfect design of control system one needs to identify the best N number of manipulated variables among the available M number of inputs for possible manipulation. This is termed as *square system* where number of manipulated inputs and number of controlled outputs are same. It is however possible to design a*non-square control system* where more number of inputs are manipulated to control less number of outputs. This concept is however out of the scope of this course.

Interaction between control loops

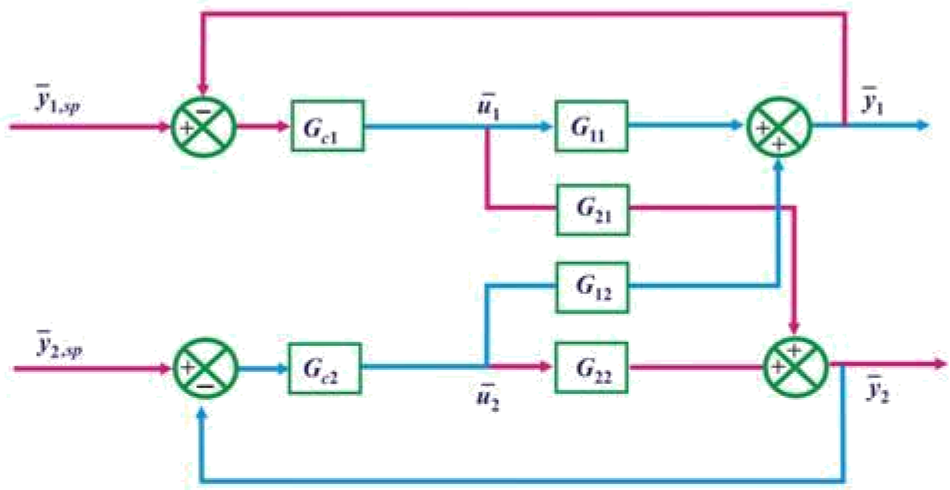
Consider a process with two inputs and two outputs:

$$\bar{y}_1 = G_{11}\bar{u}_1 + G_{12}\bar{u}_2 \tag{VI.6}$$
$$\bar{y}_2 = G_{21}\bar{u}_1 + G_{22}\bar{u}_2 \tag{VI.7}$$

The schematic of such process is given by the Fig



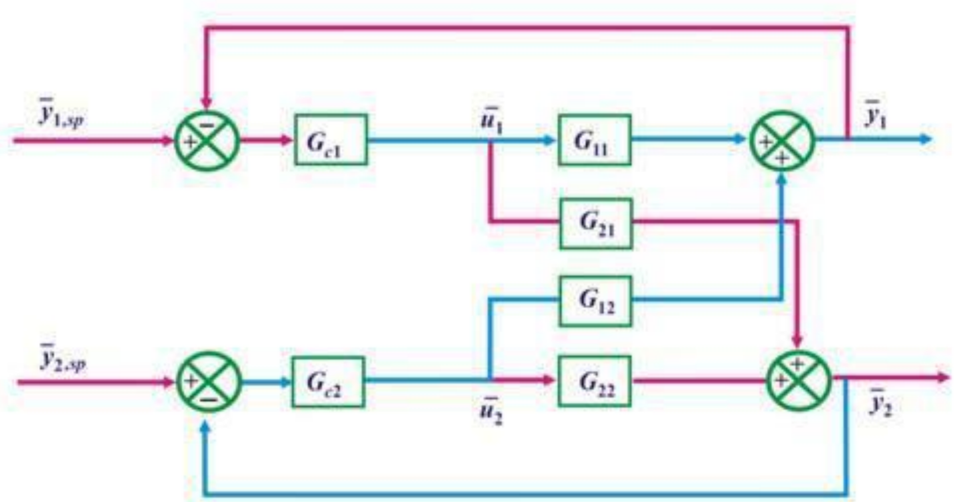
OPEN LOOP



CLOSED LOOP

Schematic of a two input and two output system

Any change in either of the inputs will lead to change in the values of both the outputs. This phenomenon persists even when two control loops are formed as in Fig. This is termed as control loop interaction. There are two types of effects of an input on an output, *viz* . direct and indirect. The effects are best explained by the Fig.VI.4 where blue line indicates direct effect of input $\bar{u}_1$ on output $\bar{y}_1$ within its own loop, *i.e.* , loop 1 whereas green line indicates indirect effect of input $\bar{u}_1$ on output that $\bar{y}_1$ yields through loop 2.



Direct and indirect effects of control loop interaction

The concept of this loop interaction will be used for understanding Relative Gain Array (RGA).

Relative Gain Array and selection of control loops

The RGA provides a quantitative criterion for selection of control loops that would lead to minimum interaction among the loops. Consider the following two experimentations (or simulations):

- Let us open the loops and detach the controllers from the process. Let us keep $\bar{u}_2$ constant and introduce a step input in $\bar{u}_1$. That would yield a static gain $K = \left.\frac{\Delta \bar{y}_1}{\Delta \bar{u}_1}\right|_{\bar{u}_2 = constant}$ that would indicate the direct effect of input on output .
- Let us now close only the loop 2 and attach the corresponding controller with the process. Let us now introduce a step input in $\bar{u}_1$ while maintaining $\bar{y}_2$ at its desired setpoint through the loop 2 controller. That would yield another open loop gain $K' = \left.\frac{\Delta \bar{y}_1'}{\Delta \bar{u}_1}\right|_{\bar{y}_2 = at\ setpoint}$ that would indicate the direct as well as indirect effect of $\bar{u}_1$ input on output $\bar{y}_1$.

The ratio of above two open loop gains is defined as the relative gain

$$\lambda_{11} = \frac{K}{K'} = \frac{\left.\frac{\Delta \bar{y}_1}{\Delta \bar{u}_1}\right|_{\bar{u}_2 = constant}}{\left.\frac{\Delta \bar{y}_1'}{\Delta \bar{u}_1}\right|_{\bar{y}_2 = at\ setpoint}} \qquad \text{(VI.8)}$$

In the similar manner, relative gains between other input-output combinations can be calculated as $\lambda_{12}, \lambda_{21}, \lambda_{22}$. The relative gain array is expressed in the matrix form as

(VI.9)

It should be noted that sum of relative gains along any row or column should be 1.

The relative gain provides useful information on interaction

- If $\lambda_{11} = 0$, input $\bar{u}_1$ does not have any effect on output and thus they should not be paired.

- If $\lambda_{11} = 1$, input $\bar{u}_2$ does not have any effect on output and hence the system is *completely decoupled* . Thus pairing of input and output is ideal.

- If $0 < \lambda_{11} < 1$, interaction exists. Smaller the value of relative gain, larger will be the interaction.

- If $\lambda_{11} < 0$, input $\bar{u}_2$ has strong effect on output $\bar{y}_1$ and that also in the opposite direction of that of input $\bar{u}_1$. Here the interaction effect is dangerous an detrimental for the system.

Let us now analyze the following conditions for RGA:

- When $\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the system is completely decoupled. Pairing of $\{u_1 vs\ y_1, u_2 vs\ y_2\}$ is ideal.

- When $\Lambda = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ the system is completely decoupled in the reverse manner. Pairing of $\{u_1 vs\ y_2, u_2 vs\ y_1\}$. is ideal.

- When $\Lambda = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$, strong interaction exists in the system and it does not matter which ever pairing is resorted to.

- When $\Lambda = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$, mild interaction exists in the system. However, pairing of $\{u_1 vs\ y_1, u_2 vs\ y_2\}$ is favourable in this case.

Quantitative analysis with RGA can be extended to systems with more number of inputs and outputs in the similar manner, however, the situation might not be straightforward in all such cases. Needless to mention, RGA is very much sensitive to model uncertainty if it is produced through simulation. Reassessment of modeling equations is required if RGA analysis is too poor to be conclusive.

Decoupling of control loops

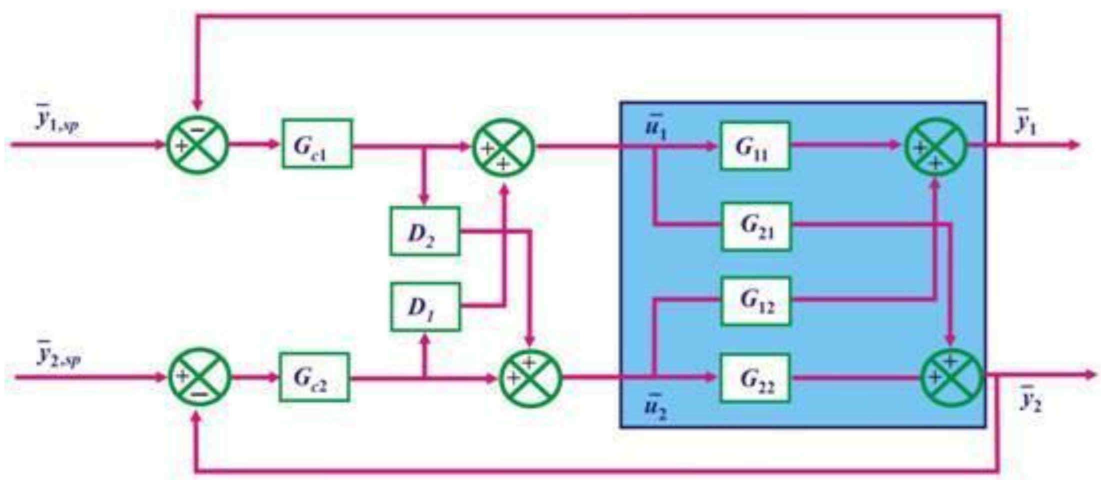
The relative gain array indicates how the inputs should be paired with the outputs. However, if the interaction between the loops is beyond acceptable limit, then a control designer ought to seek a solution whereby he/she can implement some technique that would decouple the loops from one another and make several non-interacting loops in result. For the present 2 x 2 system, it is evident that $\bar{u}_2$ does affect the $\bar{y}_1$ however it is possible to negate the effect by appropriately manipulating $\bar{u}_1$ in addition to whatever guidance it obtains from the controller G_{c1}. From eq. (VI.6), it can be concluded that if output is maintained static at its nominal point then deviation $\bar{y}_1 = 0.$ Hence, the steady state relationship between two inputs is

$$\bar{u}_1 = \left(-\frac{G_{12}}{G_{11}}\right)\bar{u}_2 = D_1\bar{u}_2 \qquad \text{(VI.10)}$$

Similarly, from eq. (VI.7), it can be concluded that if output is maintained static at its nominal point then deviation $\bar{y}_2 = 0.$ Hence, the steady state relationship between two inputs would be

$$\bar{u}_2 = \left(-\frac{G_{21}}{G_{22}}\right)\bar{u}_1 = D_2\bar{u}_1 \qquad \text{(VI.11)}$$

Now let us introduce two transfer functions, *viz* ., decouplers, D_1 and D_2 in the following fashion



Decouplers

Now the manipulated inputs of the renewed loops are:

$$\bar{u}_1 = G_{c1}\left(\bar{y}_{1,sp} - \bar{y}_1\right) + D_1 G_{c2}\left(\bar{y}_{2,sp} - \bar{y}_2\right) \qquad \text{(VI.12)}$$

$$\bar{u}_2 = G_{c2}\left(\bar{y}_{2,sp} - \bar{y}_2\right) + D_2 G_{c1}\left(\bar{y}_{1,sp} - \bar{y}_1\right) \qquad \text{(VI.13)}$$

Using eqs. (VI.12) and (VI.13), in eq. (VI.6) one obtains,

$$
\begin{aligned}
\bar{y}_1 &= G_{11}G_{c1}\left(\bar{y}_{1,sp}-\bar{y}_1\right)+G_{11}D_1G_{c2}\left(\bar{y}_{2,sp}-\bar{y}_2\right)+G_{12}G_{c2}\left(\bar{y}_{2,sp}-\bar{y}_2\right)\\
&\quad +G_{12}D_2G_{c1}\left(\bar{y}_{1,sp}-\bar{y}_1\right)\\
&= \left(G_{11}G_{c1}+G_{12}D_2G_{c1}\right)\left(\bar{y}_{1,sp}-\bar{y}_1\right)+\left(G_{11}D_1G_{c2}+G_{12}G_{c2}\right)\left(\bar{y}_{2,sp}-\bar{y}_2\right)\\
&= \left(G_{11}G_{c1}-G_{12}\frac{G_{21}}{G_{22}}G_{c1}\right)\left(\bar{y}_{1,sp}-\bar{y}_1\right)+\left(-G_{11}\frac{G_{12}}{G_{11}}G_{c2}+G_{12}G_{c2}\right)\left(\bar{y}_{2,sp}\ -\right.\\
&= G_{c1}\left(G_{11}-G_{12}\frac{G_{21}}{G_{22}}\right)\left(\bar{y}_{1,sp}-\bar{y}_1\right)
\end{aligned}
\tag{VI.14}
$$

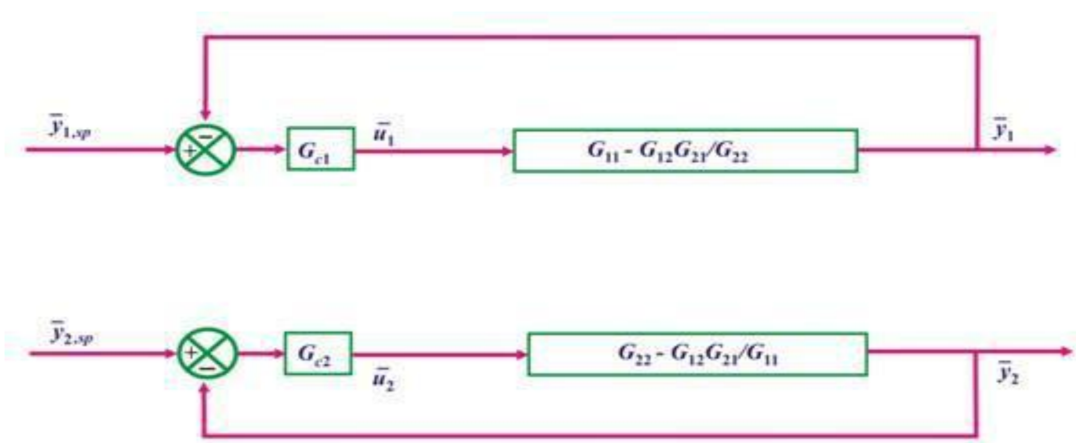
or rearranging the above equation we obtain,

$$
\bar{y}_1=\left\{\frac{G_{c1}\left(G_{11}-G_{12}\frac{G_{21}}{G_{22}}\right)}{1+G_{c1}\left(G_{11}-G_{12}\frac{G_{21}}{G_{22}}\right)}\right\}\bar{y}_{1,sp}
\tag{VI.15}
$$

Similarly using eqs. (VI.12) and (VI.13), in eq. (VI.7) one obtains,

$$
\bar{y}_2=\left\{\frac{G_{c2}\left(G_{22}-G_{12}\frac{G_{21}}{G_{11}}\right)}{1+G_{c2}\left(G_{22}-G_{12}\frac{G_{21}}{G_{11}}\right)}\right\}\bar{y}_{2,sp}
\tag{VI.16}
$$

The eqs. (VI.15) and (VI.16) are the results of the following system

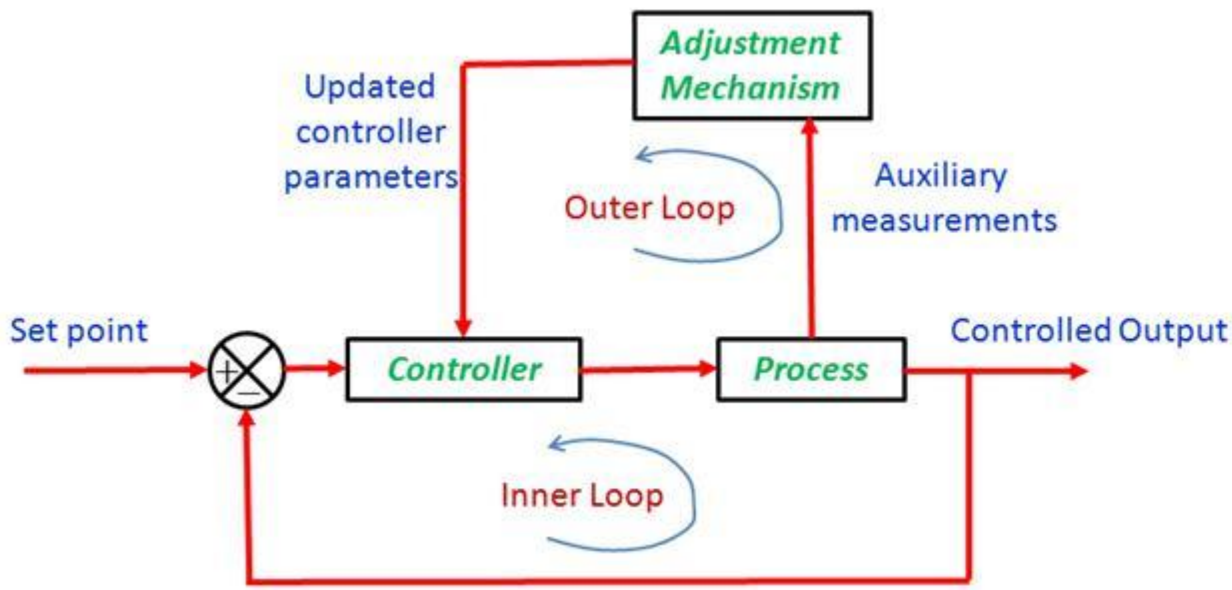


Decoupled process

Where the 2 x 2 process loops are completely decoupled from each other. It should be noted that decouplers are essentially feedforward control elements. These elements are very much sensitive to operating conditions of the process. As the transfer functions are developed at the operating conditions, any substantial shift in the nominal operating region would deteriorate decoupling effect. Adaptive decouplers are useful for this purpose, however, they are beyond the scope of this course. Again process dead time plays an important role in assigning the decoupler transfer functions. In case the resultant decoupler transfer function obtains a time lead (rather than a time delay), it yields a physically unrealizable decoupler.

Adaptive Control

It is understood in the previous chapters that task of controller design for a process is very much domain specific. First the model of the process is linearized around a certain nominal point and the controller is designed on the basis of that linearized model and finally implemented on the process. Hence, the controller is applicable for certain domain around the nominal operating point around which the model has been linearized. However, if the process deviates from the nominal point of operation, controller will lose its efficiency. In such cases, the parameters of the controller need to be re-tuned in order to retain the efficiency of the controller. When such retuning of controller is done through some "automatic updating scheme", the controller is termed as *adaptive controller*. The technique can be illustrated with the following figure.



Adaptive control

One of the most popular adaptive control techniques is *gain scheduling* technique. The overall gain of an open loop process is usually given as

$$K_{overall} = K_c K_p K_m K_f \tag{V.33}$$

It is customary to keep the overall gain constant. In case of changes in the process (or valve characteristics or measuring element), K_c should be tuned in such a manner that overall gain remains constant.

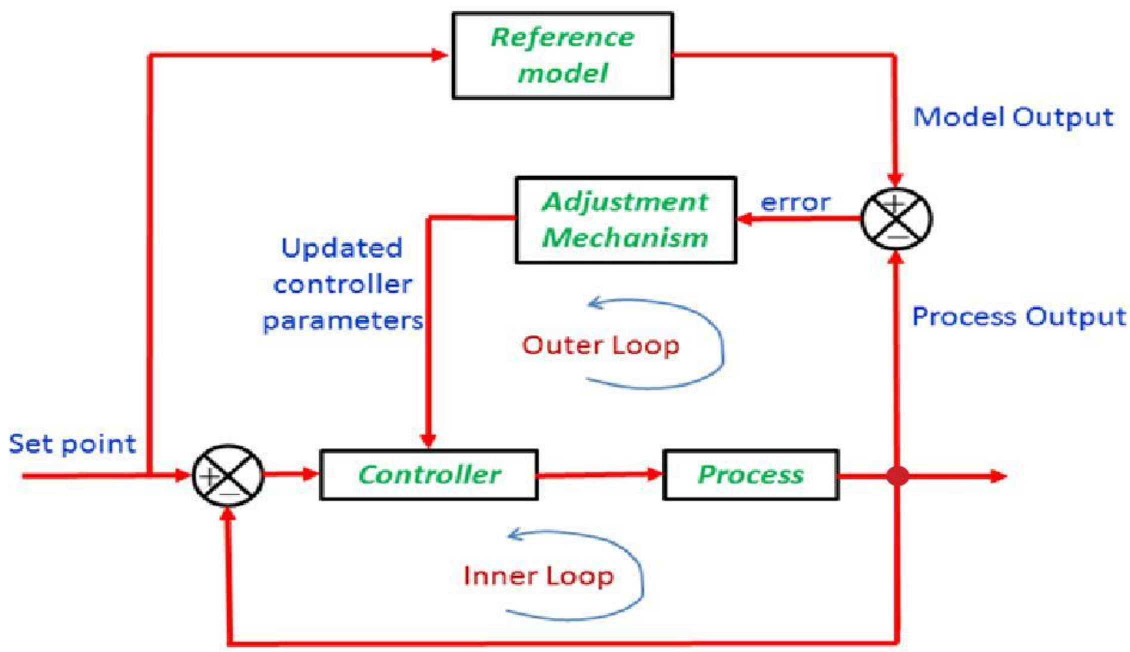
$$K_c = \frac{K_{overall}}{K_p K_m K_f} \qquad \text{(V.34)}$$

The above is called the *gain scheduling* control law.

When the process is poorly known, one cannot rely much on the correctness of the value of K_P In such cases, the *self-adaptive control* may be helpful. A self adaptive controller optimizes the value of certain objective function (criterion) in order to obtain updated controller parameters. Two examples of self adaptive controllers are Model Reference Adaptive Control (MRAC) and Self-Tuning Regulator (STR)

Model Reference Adaptive Control

The following figure shows the schematic of a Model Reference Adaptive Controller.

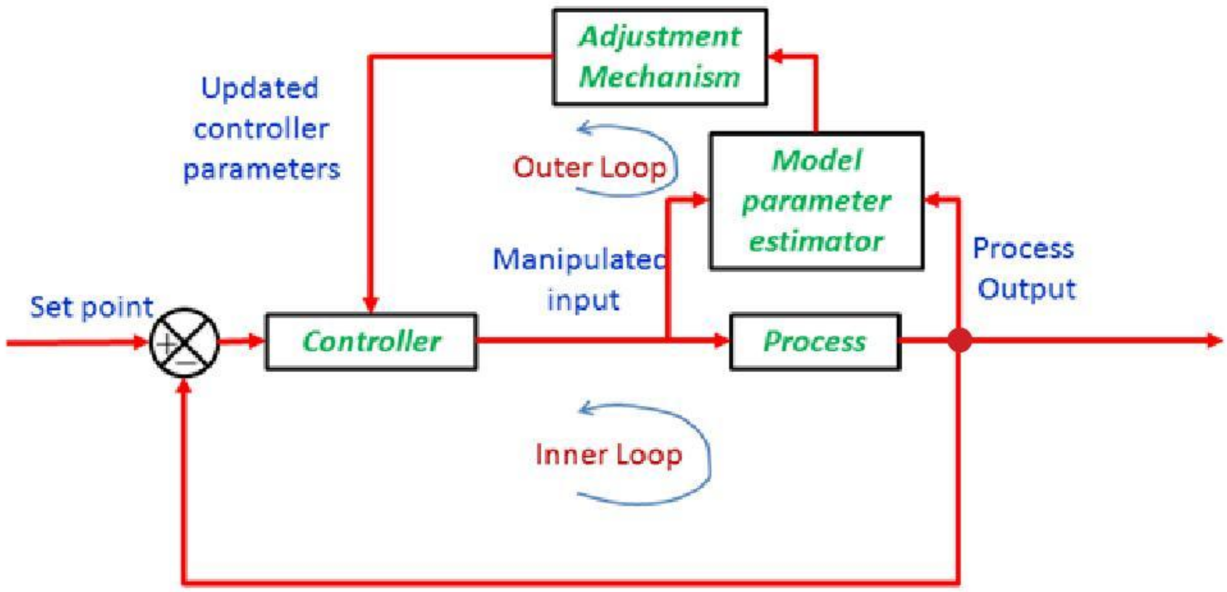


Model Reference Adaptive control

It contains of two loops. The inner loop contains the regular feedback mechanism whereas the outer loop contains an ideal reference model which the process needs to follow. The process and model outputs are compared and the error function is minimized through a suitable optimization routine in order to arrive at the re-tuned controller parameters.

Self Tuning Regulator

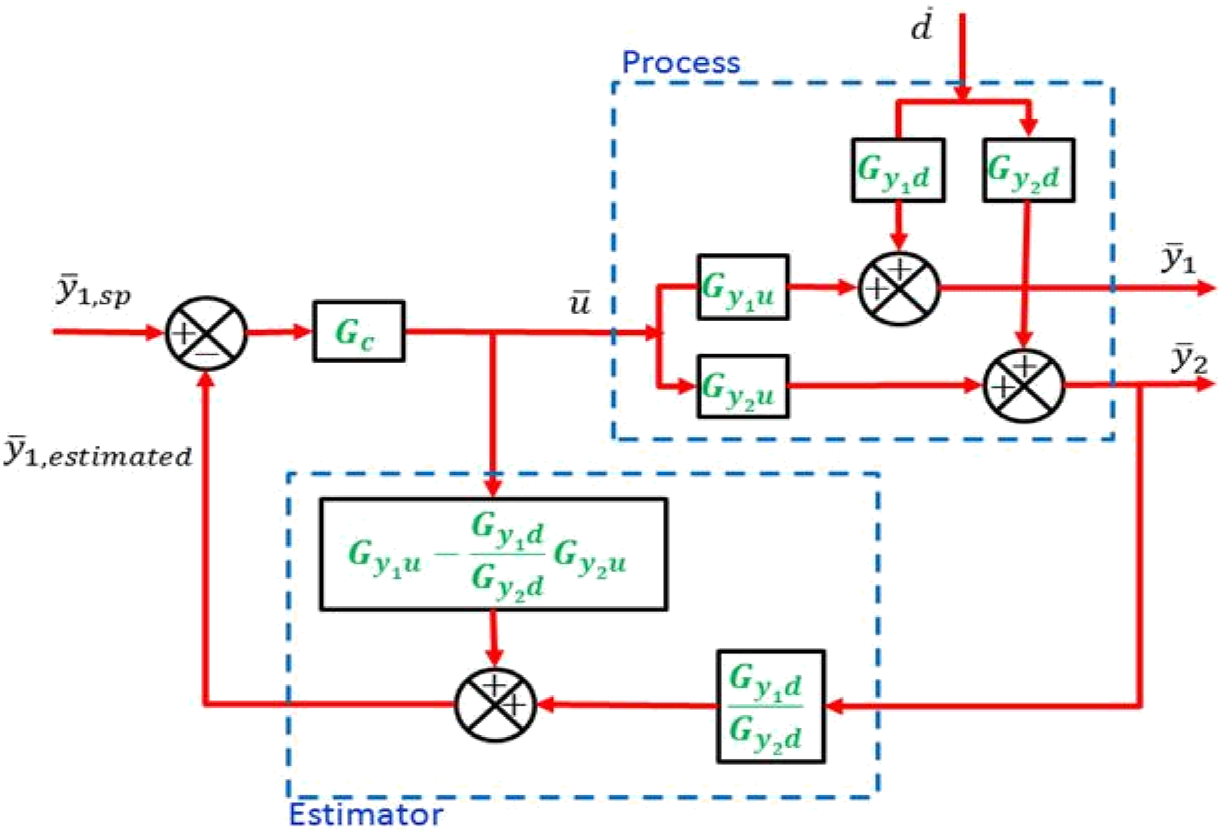
Self-Tuning Regulator on the other hand estimates the model parameters by measuring process inputs and outputs. The re-tuned model eventually guides the controller parameter adjustment mechanism. Figure V.17 shows the schematic of Self Tuning Regulator.



Self Tuning Regulator

Inferential Control

Often the process plant has certain variables that cannot be measured on-line, however, needs to be controlled on-line. In such cases, the unmeasured variables to be controlled can be estimated by using other measurements available from the process. Consider the following example:



Inferential Control

The process has two outputs y_1 (unmeasured) and y_2 (measured). The disturbance d affects the process adversely that needs to be nullified by manipulating input u. The open loop model equations can be written as

$$y_1 = G_{y_1u}u + G_{y_1d}d \tag{V.35}$$

$$y_2 = G_{y_2u}u + G_{y_2d}d \tag{V.36}$$

In this case disturbance, which is usually not measured can be expressed in terms of two measurable quantities

$$d = \left(\frac{1}{G_{y_2d}}\right)y_2 - \left(\frac{G_{y_2u}}{G_{y_2d}}\right)u \tag{V.37}$$

And hence,

$$y_1 = G_{y_1 u} u + G_{y_1 d}\left\{\left(\frac{1}{G_{y_2 d}}\right) y_2 - \left(\frac{G_{y_2 u}}{G_{y_2 d}}\right) u\right\} = \left(\frac{G_{y_1 d}}{G_{y_2 d}}\right) y_2 + \left\{G_{y_1 u} - \left(\frac{G_{y_1 d}}{G_{y_2 d}}\right) G_{y_2 u}\right\} u \quad \text{(V.38)}$$

In other words, the variable y_1 is estimated through two measurable quantities y_2 and u . The rest is similar to regular feedback control. This control mechanism is termed as inferential control because here the controlled variable y_1 is never measured, rather it has been estimated through the inference drawn from measurement of other variables (y_2 and u in this case).

Inferential Control Uses easily measure process variables (T, P, F)ν to infer more difficult to measure quantities such as compositions and molecular weight. Can substantially reduce analyzer delay.ν Can be much less expensive in terms of capitalν and operating costs. Can provide measurements that are notν available any other way.

Summary of Inferential Control Summary of Inferential Control Advantages: Enable the use of a desired control loop despite the lack ofν measurement devices. Free from dependence on delayed data (off-line analysis),ν leading to better control. Disadvantageν Knowledge on the process must be known.ν Wrong estimation leads to wrong control action and henceν detrimental to process operation. How to Improve?ν Provide measured data period.

Part A Questions

1. What is the decay ratio.
2. What is the advantage of Ziegler-Nichols tuning
3. What is the advantage of ratio control over cascade control
4. What are ITAE and when to go for it.
5. Define frequency response analysis
6. Differentiate between feedback and feedforward control
7. What is IAE
8. What is ITAE
9. What is meant by continuous cycling method
10. What is meant by split range control
11. What is inferential control
12. Give the different types of adaptive control
13. What is meant by relative gain aray
14. What is meant by decoupling
15. What is split range control?

Part B Questions

1. Deduce the optimum settings of mathematically described using time response

2. Explain the tuning of controllers by Ziegler- Nichols method
3. What is Cascade control? Explain it with an example? When would you recommend such a control system?
4. Explain process reaction method of tuning of controllers.
5. Describe the frequency response method of tuning proportional controller. What do you mean by Optimum Controller setting?
6. Explain in detail about logic of feed forward control with a suitable example.
7. Explain in detail about multivariable control.
8. What is meant by adaptive control? Explain in detail

UNIT-IV

FINAL CONTROL ELEMENTS

I/ P CONVERTER

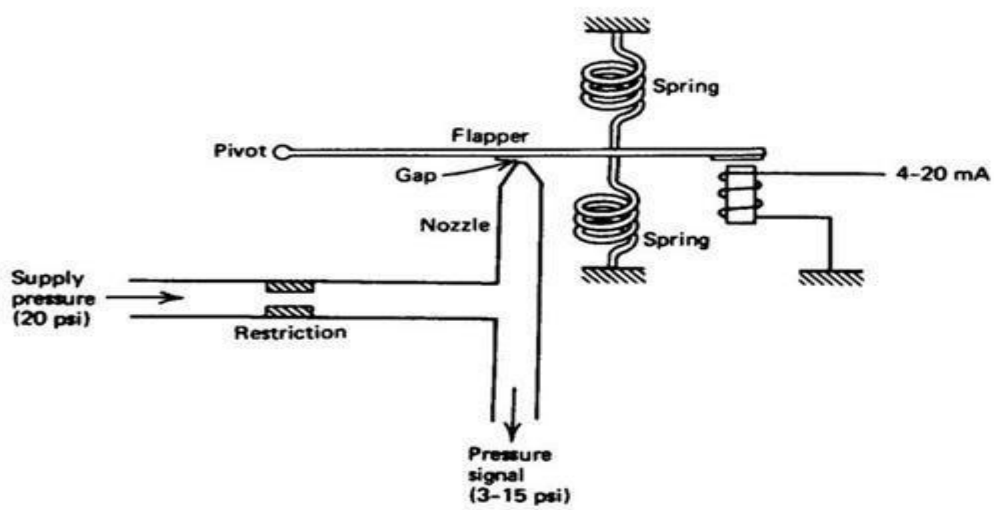
An I/P converter is a very simple device that converts a 4-20mA signal to a 3-15psi air signal. It gives a linear conversion between the current and pressure signal.

The basic design of I/P converter involves use of Flapper Nozzle system.

Working:

The current through the coil produces a force that tends to pull down the flapper, thus closing the nozzle. This increases the back pressure in the nozzle. Thus high current produces high pressure. Calibration is done (by adjustment of spring, position relative to the pivot) for a current of 4mA to be converted to 3 psi and 20mA to 15 psi.

Diagram:



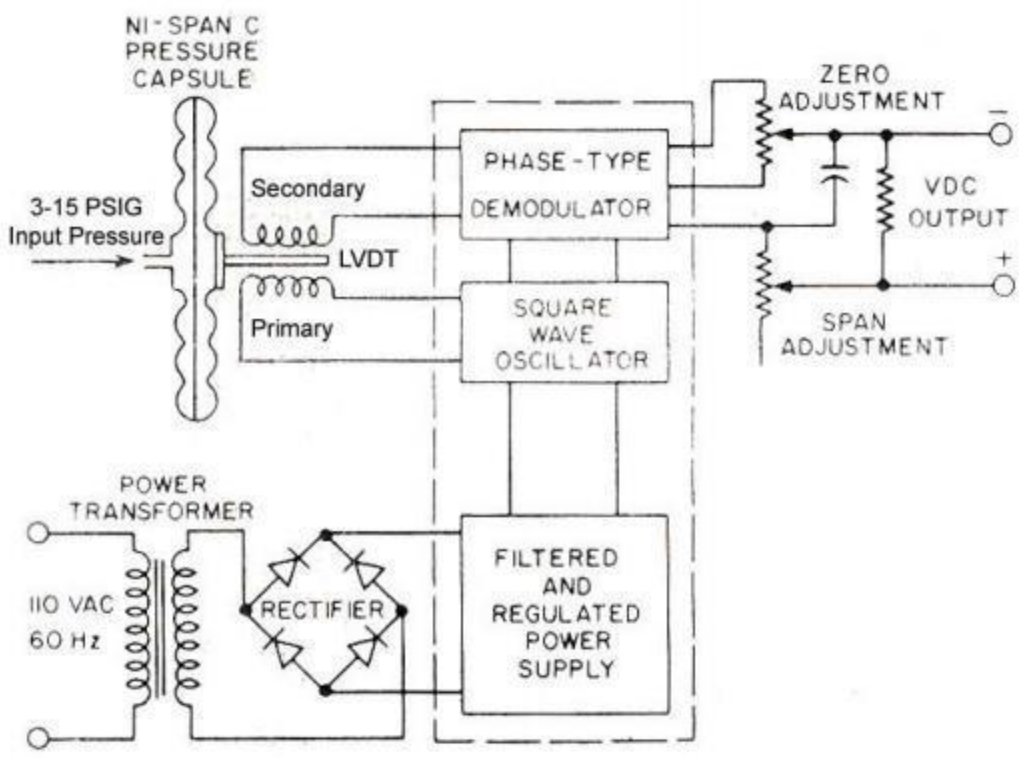
P / I CONVERTER

It is a device which converts 3-15 psi air signal to 4-20 mA current signal.

Reason for conversion of air signal to current signal:

1. For transmission over long distances.
2. To input the data to computers.
3. To input the data to a telemetering equipment.

The diagram shown below converts a pressure signal to a voltage signal and then using a V /I converter, the voltage signal is further converter to current signal.



Pneumatic to Voltage Converter

Working:

When 3-15 PSIG input pressure is applied to pressure capsule or bellow, it leads to movement of the LVDT core between the primary and secondary coils. The primary coil has input excitation from a square wave oscillator. The movement of the core leads to generation of differential potential across two secondary which is demodulated to obtain dc voltage output, which will be directly proportional to applied input pressure.

ACTUATORS:

An actuator is a type of motor that is responsible for moving or controlling a mechanism or system. It is operated by a source of energy, typically electric current, hydraulic fluid pressure, or pneumatic pressure, and converts that energy into motion.

A valve can open or close. The actuator is a device which is responsible for movement of the valve stem, thereby opening or closing the valve.

Types of Actuators:

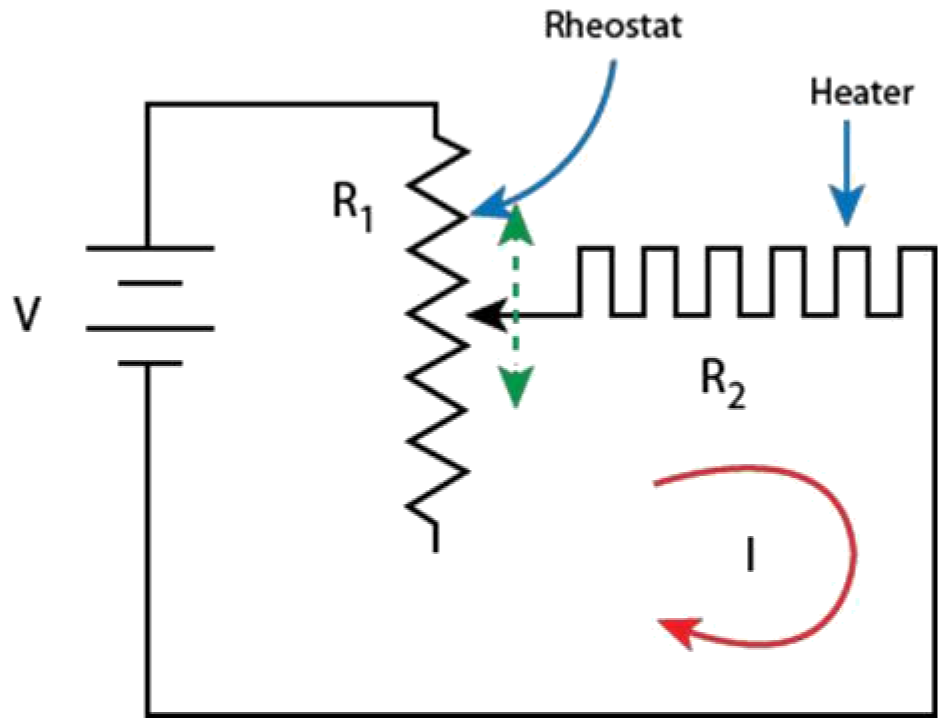
1. Electric
2. Pneumatic
3. Hydraulic
4. Combined

Electrical Actuators:

Electric actuators can be of the following forms: Electric current/voltage, Solenoid, Stepping Motor and DC/AC Motor. Current or voltage can easily be regulated to adjust the flow of energy into the process, *e.g.* heater, fan speed regulator *etc*. Energy supplied by the heater element is

$$W = I^2Rt$$

where I =current, R=resistance, t=time of heating. The current/voltage can be regulated using potentiometer (or rheostat), amplifier or a switch.



Rheostat

Rheostat

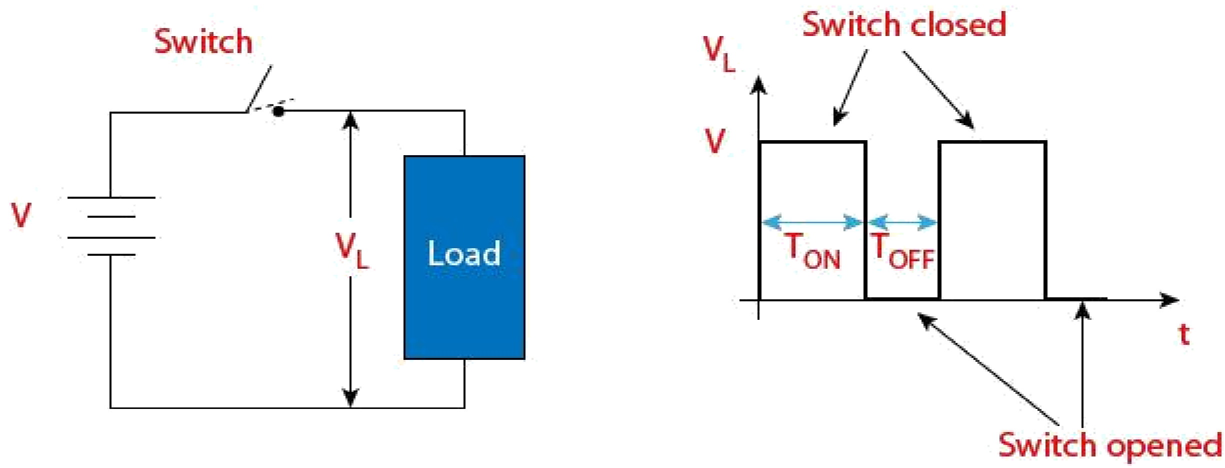
A rheostat is a device that has variable resistance to current flow (Fig. VII.13). The current flowing through the circuit is

$$I = V/(R_1 + R_2),$$

where R_1 can be varied by suitably sliding the pointer. The power transmitted to the heater would be

$$P = I^2 R_2.$$

As the pointer slides towards a lesser value of R_1, heater receives more power for heating.



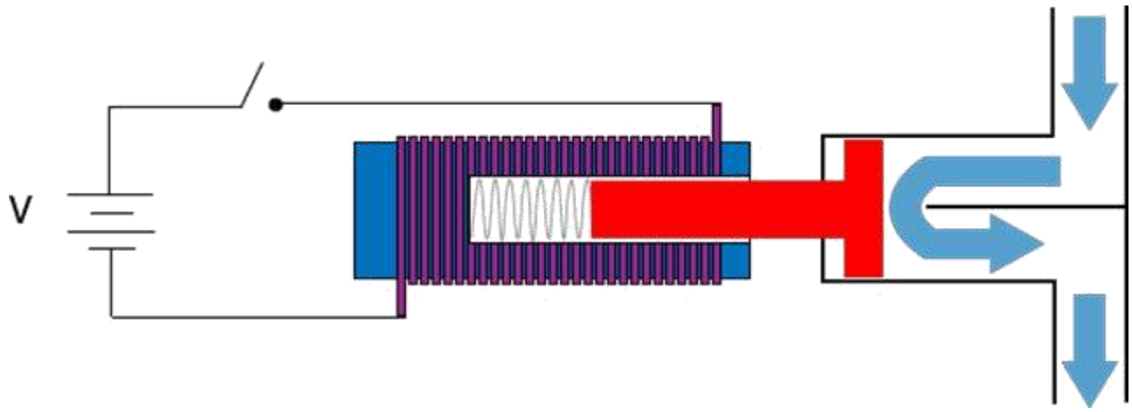
Switch

Switch

A switch is a device which has two states of operation *viz* ., ON and OFF. The duration of the switch to remain in either state can be modulated as per requirement. If T_{ON} and T_{OFF} are the switch on and switch off time respectively, then duty cycle of the switch is defined as $T_{ON}/(T_{ON}+T_{OFF}) \times 100\%$ where $(T_{ON}+T_{OFF})$ is kept constant for any operation. This methods is often called as Pulse Width Modulation (PWM). Transistor and Thyristor are the examples of switches.

Solenoid

A solenoid is a coil wound into a tightly packed helix which produces a controlled magnetic field when an electric current is passed through it. Solenoids can be used as electromagnets which convert electromagnetic energy into linear motion of some mechanical part. It is often used as a valve which actuates a piston to restrict a flow in a pneumatic/hydraulic pipeline.

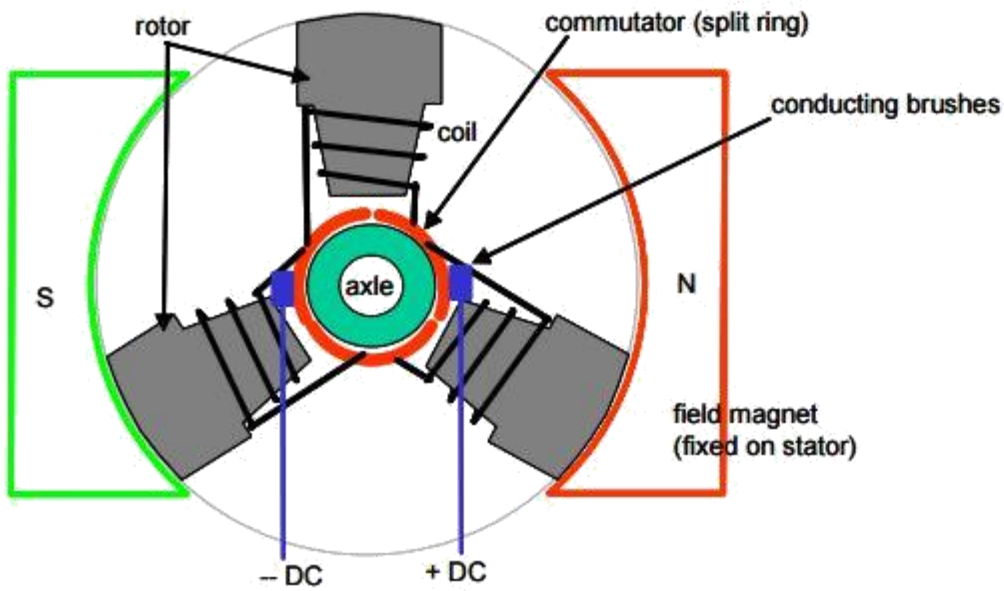


Solenoid

The above figure shows a schematic of a solenoid valve. The blue bar indicates an electromagnet that gets energized on flowing current through the helix. The red colored piston is a metallic object whose movement is controlled by the duration of energization of electromagnet. In normal situation the spring forces the piston to move far and block the fluid flow in the pipeline. As the electromagnet gets energized, the piston is pulled back yielding the fluid to flow unrestricted.

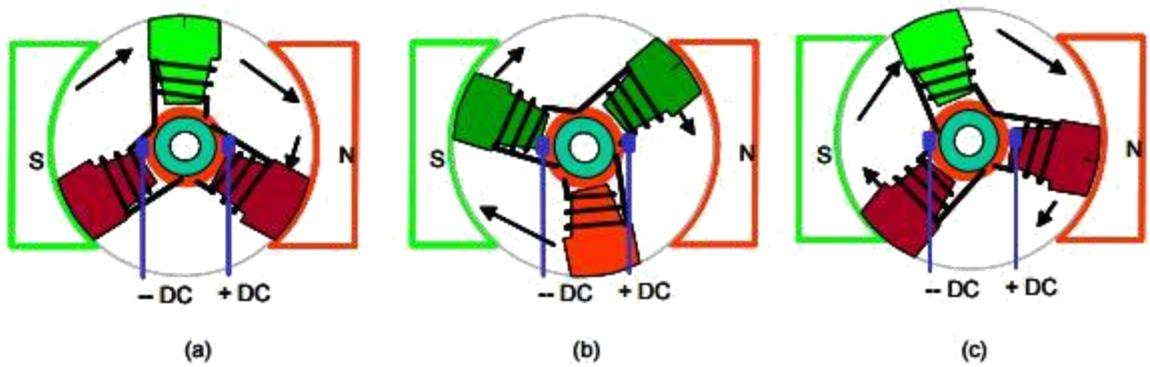
Electric Motors:

All electric motors use electromagnetic induction to generate a force on a rotational element called the rotor. The torque required to rotate the rotor is created due to the interaction of magnetic fields generated by the rotor, and the part surrounding it, which is fixed, and called the stator. The figure below shows a typical DC motor. A high-strength permanent magnet (field magnet) creates a magnetic field in the space occupied by the rotor. In this DC motor, the rotor is made of an axle with three radial arms fixed at equal angles around it. The axle is supported to the stator by a bearing, so it can rotate freely with respect to the stator. On each of the three arms, a conducting wire is wound in a coil, as shown; the direction of this winding is important. The coils are connected to three separate electrical contacts as shown. These contacts are a short tube of copper that has been cut to separate it into three pieces; the split ring made by these contacts is called the commutator. S N rotor coil commutator (split ring) -- DC + DC field magnet (fixed on stator) axle conducting brushes S N rotor coil commutator (split ring) -- DC + DC field magnet (fixed on stator) axle conducting brushes



Structure of a 3-pole DC motor

A current flows through the coil on the top, inducing a magnetic field, and this arm behaves like a magnetic South pole. At the same time, the two coils in the bottom also experience a voltage drop across them. In this position, the two coils are connected in series; by the left-hand rule, both behave as magnetic North poles, but since they are in series, they are of less strength than the coil on top. North poles in red color, and south poles in green. Also, half strength magnetization is shown by hashed line patterns. [Fleming's left hand rule: when a current-carrying coil of wire is grasped in the left hand, the fingers curled around the coil in the direction of electron flow, from negative (-) to positive (+), the thumb will point toward the North pole of the electromagnet.] It is clear that in the position shown, due to the magnetic forces between the field magnet and the three electromagnets in the stator, there is a torque forcing the rotor to rotate Clockwise. But as soon as the rotor turns a little, the connections between the input voltage and the coils change due to the commutator, changing the magnetic poles in the rotor arms.



In the above figure, we can see that as the rotor rotates, the commutator keeps changing the direction of the electromagnets in the rotor to maintain a continuous torque in the Clockwise direction – thus making motor to do work.

Almost all small DC motors are a permanent magnet type; sometimes they have more than three poles, but the basic principle of operation is the same. In industrial DC motors, the electromagnetic fields in both, the rotor as well as the stator, are generated by passing the DC current through the electric wire coils in them. There are several types of large DC motors, depending on how the DC source is connected to the stator and rotor coils (namely, in series, in parallel, etc.) The table below summarizes the basic characteristics and uses of some of these.

Motor type	Speed regulation	Starting torque	Applications
DC series	very high	very high	Hoists, Gates, Trams, Trains, Bridges
DC shunt	low	medium	Printing press, Machine tools, Conveyors
DC compound	medium	high	Punch press, Shears, Crushers

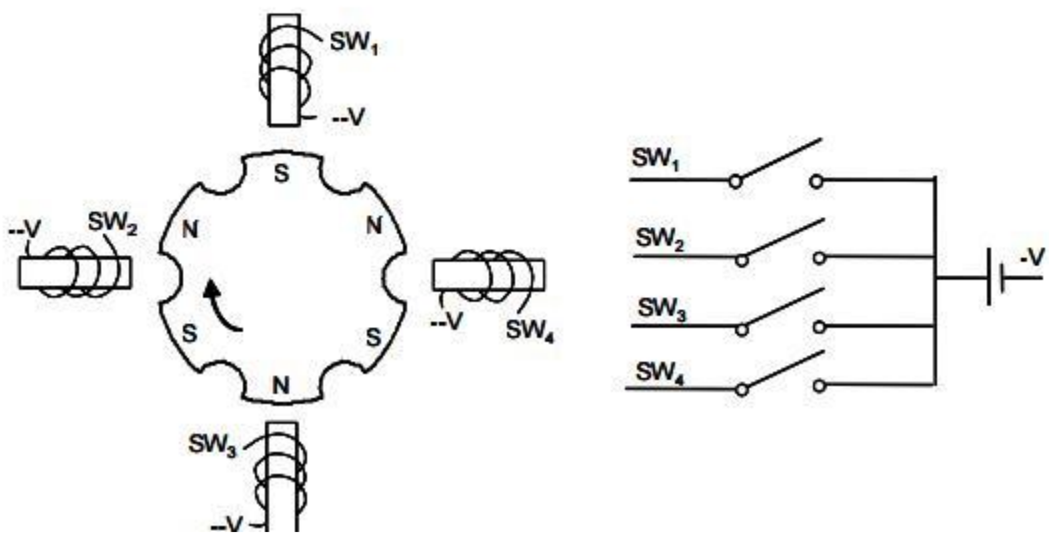
Note: ***Speed regulation*** = (no load speed – full load speed)/no load speed

In AC motors, only the stator has coils through which current passes. This current creates an alternating electromagnetic field, which induces an alternating current in the rotor. The interaction of the two electromagnetic fields is used to create the driving torque. The study of AC motors in more detail is out of the scope of this course. The table below summarizes the main types of AC motors, their characteristics and typical applications. AC motors are very commonly seen in machine tools in labs and workshops, since AC power sources are available everywhere.

Motor type	Speed	Power	Applications
1-phase AC induction	Nearly constant in operational range, slightly less than the to synchronous speed	~ 3 KW or less	Almost all medium power pumps, fans, blowers, machines
3-phase AC induction	Nearly constant over large range of loads	High, > 1.5 KW	High power pumps, machinery

Stepper Motors

Stepper motors rotate in discrete steps (e.g. 2° for each step); they have many uses, especially in motion for robots and locating or indexing tables. Their working principle is similar to DC motors, but they are controlled by digital electronics: an electronic circuit turns a series of switches ON and OFF at each electrical pulse input to the stepper motor control.



Full Steps (CW)

Degs	SW_1	SW_2	SW_3	SW_4
0	ON			
30		ON		
60			ON	
90				ON
120	ON			
150		ON		

Half Steps (CW)

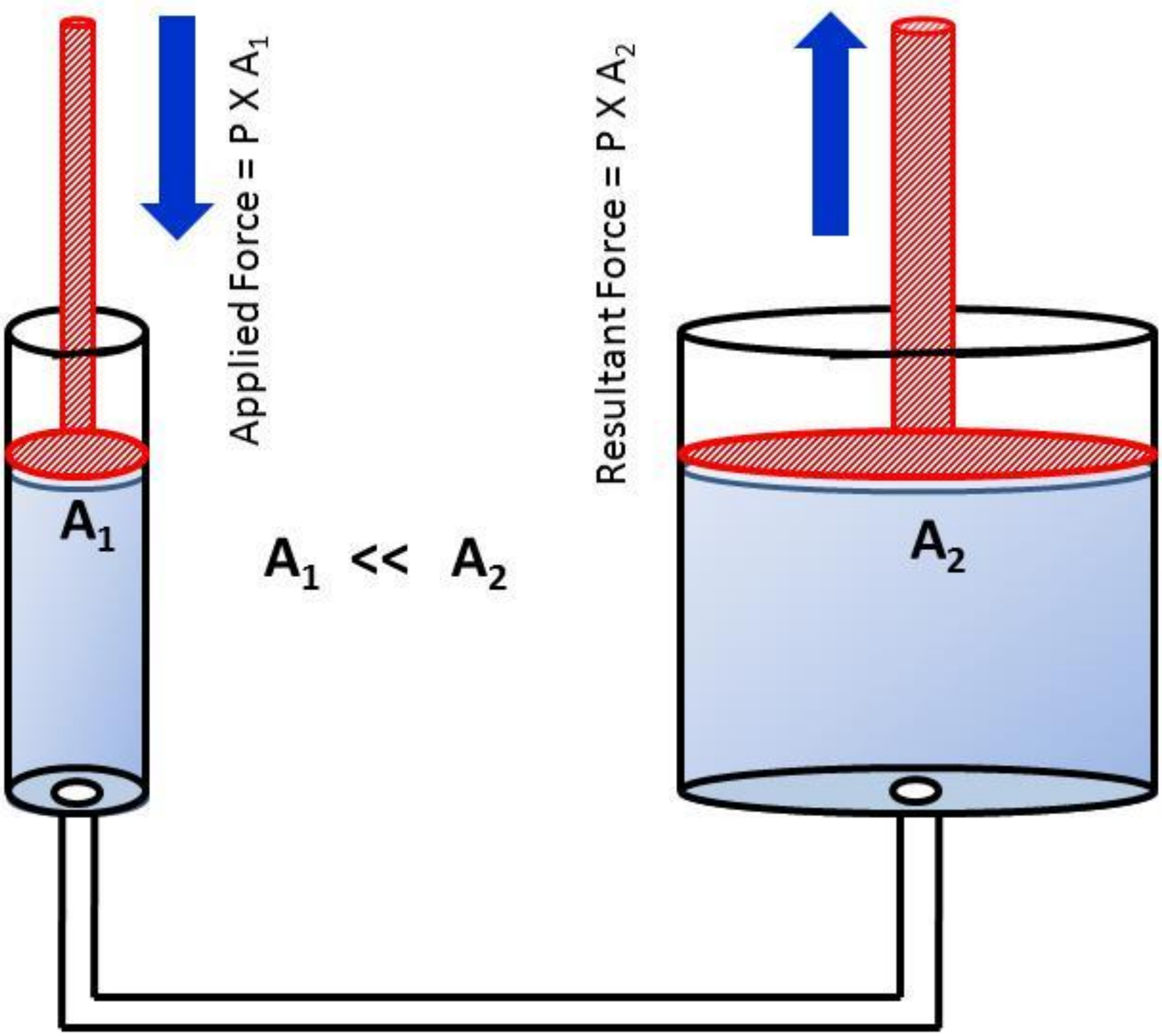
Degs	SW_1	SW_2	SW_3	SW_4
0	ON			
15	ON	ON		
30		ON		
45		ON	ON	
60			ON	
75			ON	ON
90				ON

The principle of the Stepper Motor

The stepper motor is driven by feeding it a stream of electric pulses. Each pulse makes the motor rotate by a fixed angle. The rotor is a permanent magnet, configured so as to have a series of equally spaced (angularly) sets of poles along the circumference. The stator has a corresponding number of coils. The stepper motor can be operated in three different methods: full steps, half steps, and micro-steps. Depending on the method used, the step size (i.e. degrees rotated per pulse) is different. In full-step mode, only one of the stator coils is activated at a time, causing the rotor to align its poles in opposition to the electromagnetic poles of the energized coil. In half-step mode, a combination of coils are activated in a way so as to increase the resolution of the motor (i.e. step sizes are smaller). The third method used by stepper motor drive units is application of pulses of different voltages to different coils. By proper selection of the voltage levels applied, smaller steps can be produced; this method is called micro-stepping.

Hydraulic actuator

Hydraulic actuators employ hydraulic pressure to move a target device. These are used where high speed and large forces are required. Pressure applied to a confined incompressible fluid at any point is transmitted throughout the fluid in all directions and acts upon every part of the confining vessel at its interior surfaces.

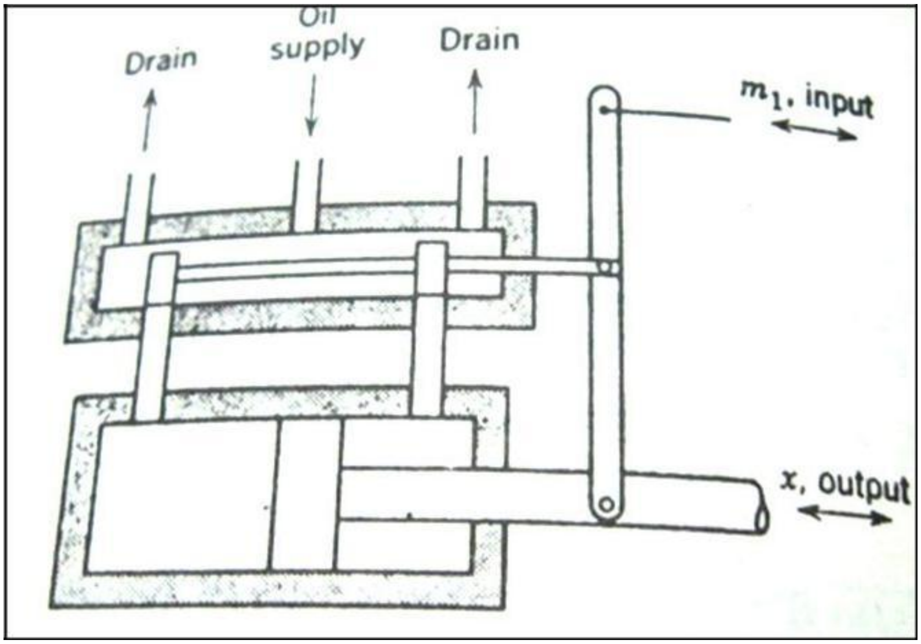


Schematic of hydraulic Actuator

According to Pascal's Law, since pressure P applied on an area A yields a force F, given as,

$F = P \times A$,

if a force is applied over a small area to cause a pressure P in a confined fluid, the force generated on a larger area can be made many times larger than the applied force that created the pressure.



The above diagram shows a hydraulic piston actuator.

It requires a continuously running electric motor and pump to provide a source of high pressure oil and a drain or sump to collect the return.

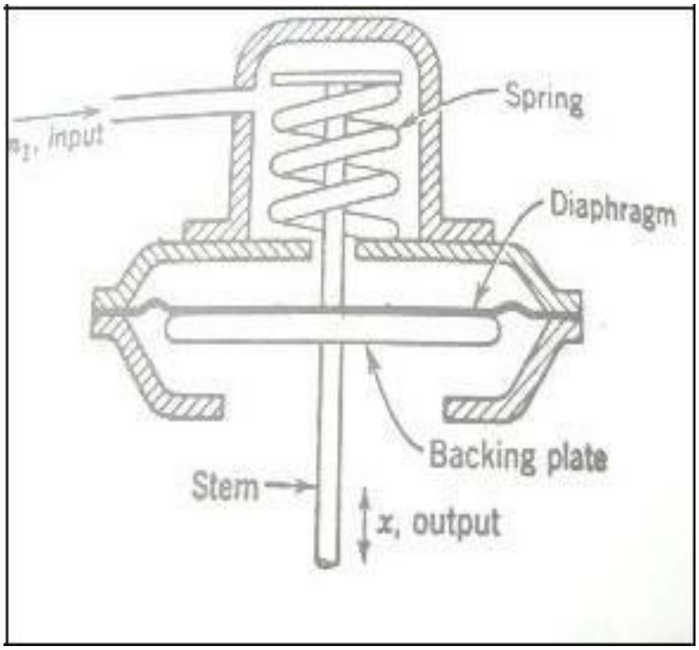
Operation: When input increases, it pushes the pilot piston to the left. This causes the left end of the piston open to the oil supply, increasing the pressure there. The right end of piston is open to drain. The large power piston moves right. Thus output x is proportional to input m1.

Pneumatic Actuators

Following types:

1. Spring Actuator
2. Spring Actuator With Positioner
3. Springless Actuator
4. Piston Actuator
5. Motor Actuator

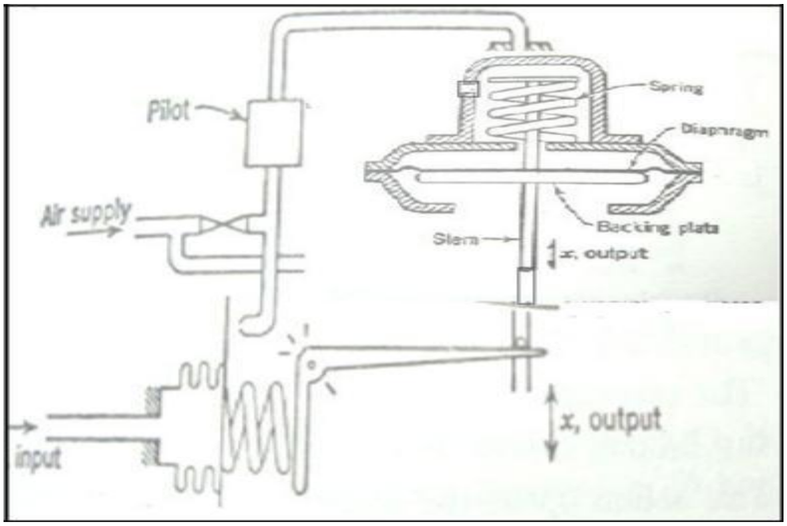
Spring Actuator:



The input air pressure acts against the diaphragm, causing a downward force, which compresses the spring. The diaphragm is usually made of fabric-base rubber, supported by a backing plate. The displacement produced in the stem is proportional to the input air pressure. For a pressure of 3 –15 psi, the displacement produced is 0.25- 3 inches. Limitations: Small strokes of the diaphragm. For longer strokes, piston-spring combination is used.

Spring Actuator With Positioner

When static forces are large or when the response of the motor is too slow, then the spring actuator requires a positioner for it to perform. The positioner consists of an input bellows, a nozzle and amplifying pilot, a feedback lever and spring.



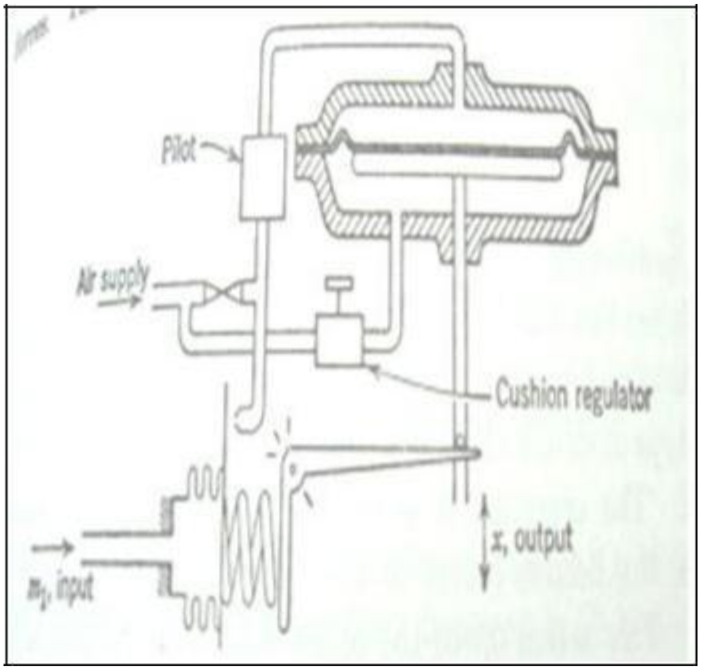
The air supply required for operation is 20-100 psig.

Operation: When input air pressure increases, bellows of positioner moves forward to cover the nozzle. Back pressure in nozzle increases, amplified by the pilot and transmitted to diaphragm. The diaphragm moves down, causing the feedback lever to compress the spring to return the baffle to its normal position. Advantages of positioner:

1. Reduces hysterisis, improves linearity
2. Actuator can handle much higher static friction due to amplifying pilot
3. Variable thrust forces on the motor does not disturb the stem position
4. Speed of response improved because pneumatic controller must supply sufficient air to fill the small input bellows rather than the large actuating chamber.

Disadvantage: positioner requires maintenance

Springless Actuator

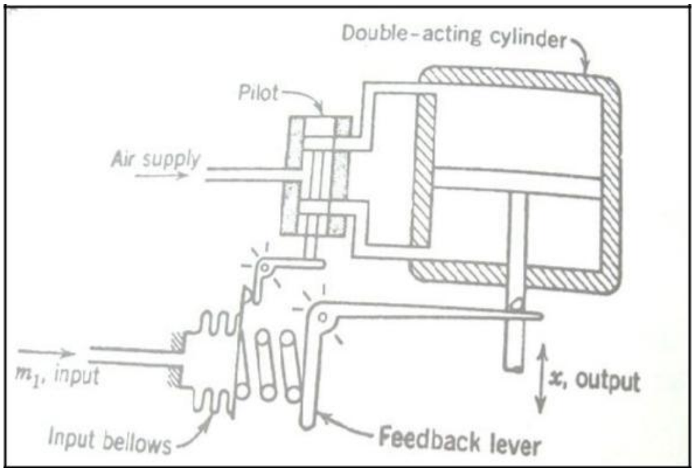


Spring of actuator is replaced by pressure regulator, which maintains a constant pressure on the underside of the diaphragm. Requires air supply of 20-100 psig for operation.

Working principle: Let us assume that pressure at the underside of the diaphragm is 9 psi. When the input pressure increases, the nozzle back pressure increases. Therefore the pressure at the upper side of the diaphragm is higher than the underside. This causes the diaphragm to move downwards, causing the actuator stem to move down. The upper side pressure also reaches a value of 9 psi, and static balance is attained.

Advantage: It can handle 3-10 times the thrust force handled by the spring actuator with or without positioner.

Piston Actuator



The above figure shows a double acting piston actuator.

Used for larger thrust forces and for long strokes.

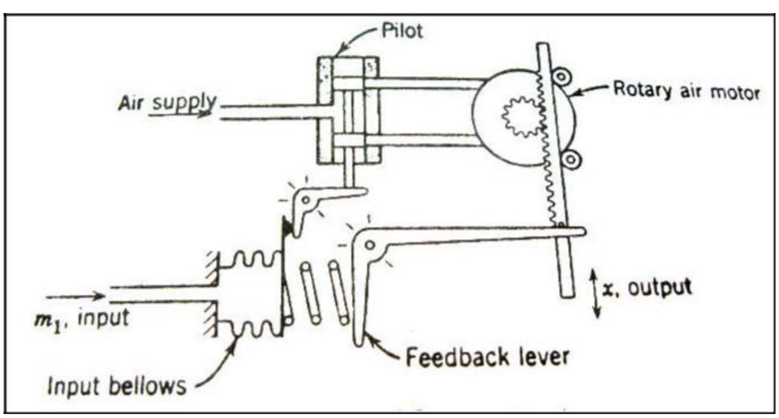
Requires an air supply of 30-100 psig for operation.

Pilot is a spool type diverting valve.

Operation: When input pressure increases, the bellows moves right, causing the pilot spool to move upwards. This causes the air to be supplied on the upper side of the piston cylinder, and lower side is open to the atmosphere. Thus position of piston is proportional to the input pressure.

Advantage: it can handle very large thrust forces and long strokes.

Motor Actuator



The above figure shows a rotary actuator.

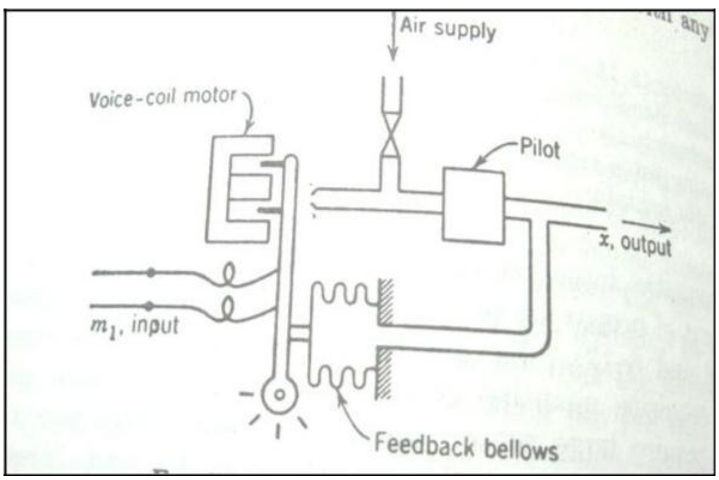
The air motor is a reversible vane type or positive displacement type motor. Operating range is 80- 100psig air pressure.

Operation: When input pressure increases, bellows cause the pilot piston to move upwards, thus applying pressure at the top side of the air motor, whereas the downside is open to atmosphere. This causes the motor to drive the rack downwards. When the stem moves down, it compresses the spring, bringing back the pilot piston to its normal position.

Range of application of Pneumatic Actuators

- SPRING ACTUATOR : TO 3 In, Thrust Force Upto 800 Lb
- SPRING ACTUATOR WITH POSITIONER : TO 3 In, Thrust Force Upto 800 Lb
- SPRINGLESS ACTUATOR: TO 3 In, Thrust Force Upto 2000 Lb
- PISTON ACTUATOR: TO 36 In, Thrust Force Upto 5000 Lb
- MOTOR ACTUATOR: TO 60 In,, Thrust Force Upto 100,000 Lb

Electro-Pneumatic Actuators



The above figure shows an electro-pneumatic pilot.

It converts an input electric signal to a proportional air pressure output. Its primary component is a voice coil motor. The voice-coil motor is the transducer which converts direct current to mechanical force. The motor in the voice-coil motor is constructed by winding a coil of fine wire in a cylindrical form. The coil is maintained in an air gap between pole pieces connected to a permanent magnet. A current through the coil causes a force to push the coil up or down through the gap. In this, the coils are attached to the flapper. A movement of the coil brings a movement of the flapper thus closing the nozzle.

VALVE POSITIONER

The purpose of having a valve positioner is to guarantee that the valve moves to the position where the controller wants it to be. It is the job of the positioner to protect the controlled variable from being upset by any of the variations. The valve positioner is a high gain plain proportional controller which measures the valve stem position, compares it o the set point, and based on the error, correction is done.

CONTROL VALVES

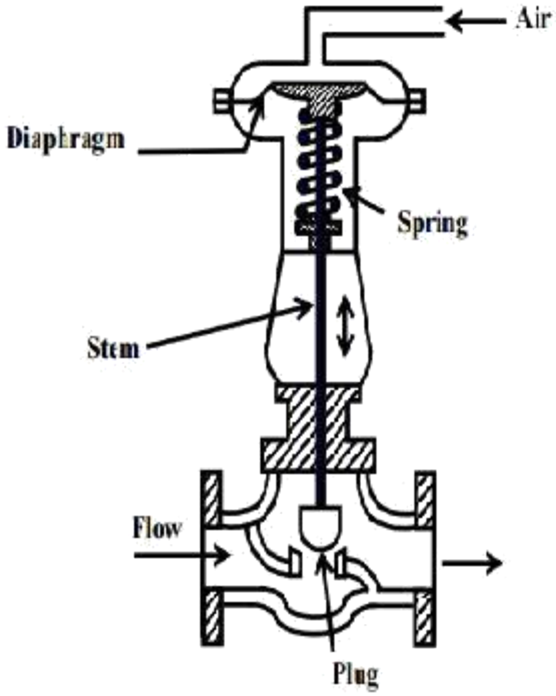
A control valve is used to regulate the flow rate in a fluid delivery system. It changes the flow rate by changing the pressure in a flow system. The pressure is changed by placing a constriction in the pipe, through which the fluid flows.

Characteristics of control valve:

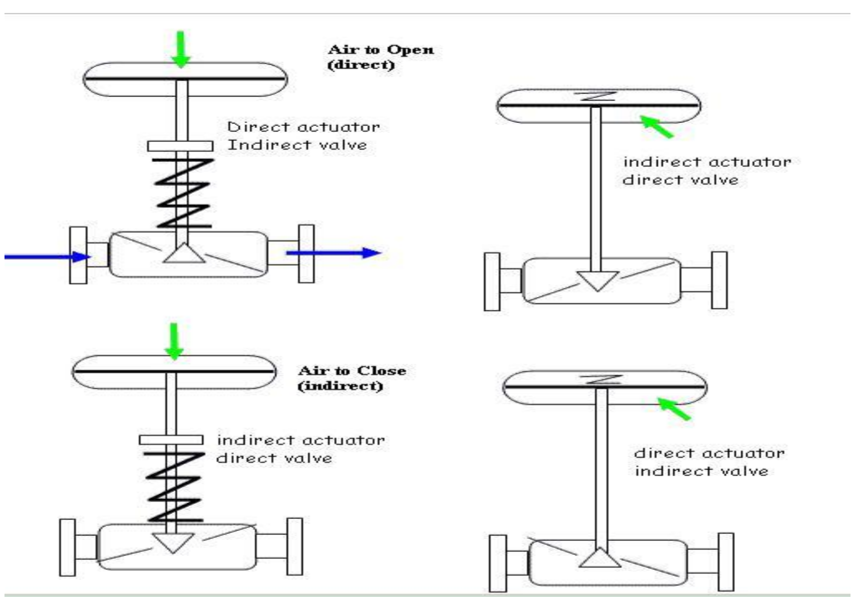
Based on the relation between the valve stem position and the flow rate through the valve, control valves are classified into different types. The control valves possess two types of characteristics

1. *Inherent Characteristics*: It is based on the assumptions that the stem position indicates the extent of valve opening and that the pressure difference is due to the valve alone.
2. *Installed Or Effective Characteristics:* the control valve when installed in a process with pipe lines, downstream and upstream equipment will exhibit a different flow rate- stem position relation.

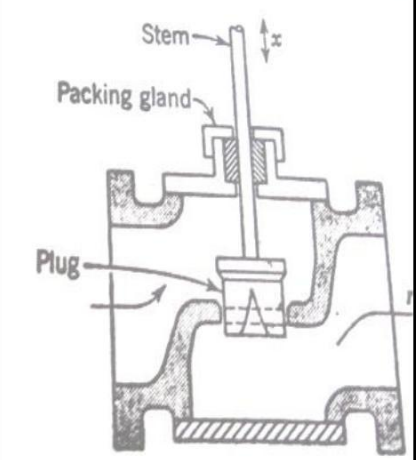
Control valves are available in different types and shapes. They can be classified in different ways; based on: (a) action, (b) number of plugs, and (c) flow characteristics.



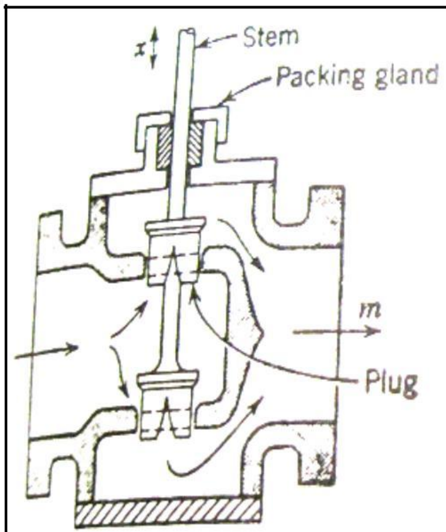
(a) Action: Control valves operated through pneumatic actuators can be either (i) air to open, or (ii) air to close. They are designed such that if the air supply fails, the control valve will be either fully open, or fully closed, depending upon the safety requirement of the process. For example, if the valve is used to control steam or fuel flow, the valve should be shut off completely in case of air failure.



(b) Number of plugs: Control valves can also be characterized in terms of the number of plugs present, as single-seated valve and double-seated valve.



Single-seat control valve



Double-seat control valve

Referring the first figure above, only one plug is present in the control valve, so it is single seated valve. The advantage of this type of valve is that, it can be fully closed and flow variation from 0 to 100% can be achieved. But looking at its construction, due to the pressure drop across the orifice a large upward force is present in the orifice area, and as a result, the force required to move the valve against this upward thrust is also large. Thus this type of valves is more suitable for small flow rates. On the other hand, in the second figure shown above, there are two plugs in a double-seated valve; flow moves upward in one orifice area, and downward in the other orifice. The resultant upward or downward thrust is almost zero. As a result, the force required to move a double-seated valve is comparatively much less. But the double-seated valve suffers from one disadvantage. The flow cannot be shut off completely, because of the differential temperature expansion of the stem and the valve seat. If one plug is tightly closed, there is usually a small gap between the other plug and its seat. Thus, single-seated valves are recommended for when the valves are required to be shut off completely. But there are many processes, where the valve used is not expected to operate near shut off position. For this condition, double-seated valves are recommended.

(c) Flow Characteristics: It describes how the flow rate changes with the movement or lift of the stem. The shape of the plug primarily decides the flow characteristics. However, the design of the shape of a control valve and its shape requires further discussions. The flow characteristic of a valve is normally defined in terms of (a) inherent characteristics and (b) effective characteristics. An inherent characteristic is the ideal flow characteristics of a control valve and is decided by the shape and size of the plug. On the other hand, when the valve is connected to a pipeline, its overall performance is decided by its effective characteristic.

Ideal Characteristics

The control valve acts like an orifice and the position of the plug decides the area of opening of the orifice. Recall that the flow rate through an orifice can be expressed in terms of the upstream and downstream static pressure heads as:

$= {}_1 \qquad (h_1 - h_2)$

where q = flow rate in m^3 /sec.

K1 = flow coefficient

a = area of the control valve opening in m^2

h1 = upstream static head of the fluid in m

h2 = downstream static head of the fluid in m

g = acceleration due to gravity in m/sec^2 .

Now the area of the control valve opening (a) is again dependent on the stem position, or the lift. So if the upstream and downstream static pressure heads are somehow maintained constant, then the flow rate is a function of the lift (z), i.e. q= f(z)

(2) The shape of the plug decides, how the flow rate changes with the stem movement, or lift; and the characteristics of q vs. z is known as the inherent characteristics of the valve.

Let us define

$= \underline{\qquad}$ and $= \underline{\qquad}$

where, q_{max} is the maximum flow rate, when the valve is fully open

and z_{max} is the corresponding maximum lift.

So the above equation can be rewritten in terms of m and x as: m= f(x)

and the valve sensitivity is defined as dm/ dx , or the slope of the curve m vs. x. In this way, the control valves can be classified in terms of their m vs. x characteristics, and three types of control valves are normally in use.

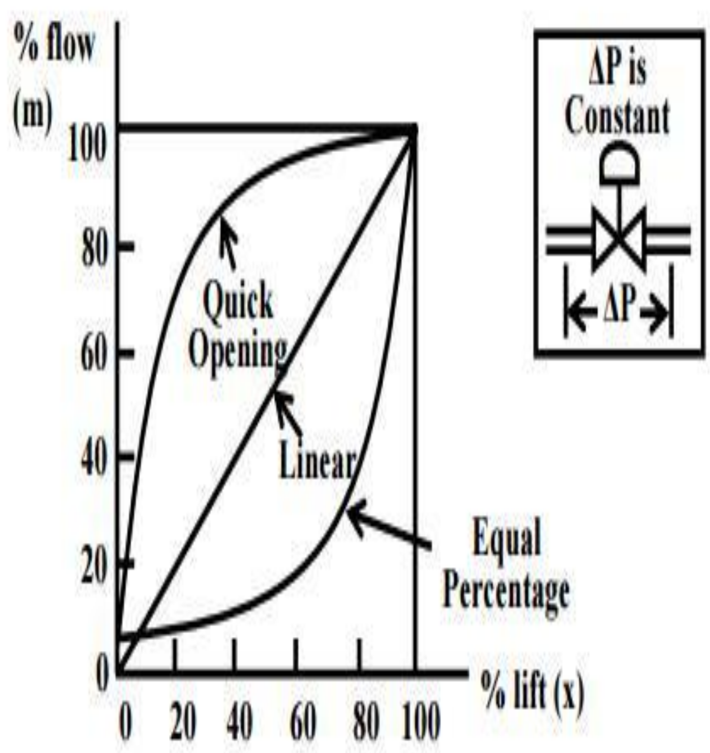
They are:

(a) Quick opening

(b) Linear

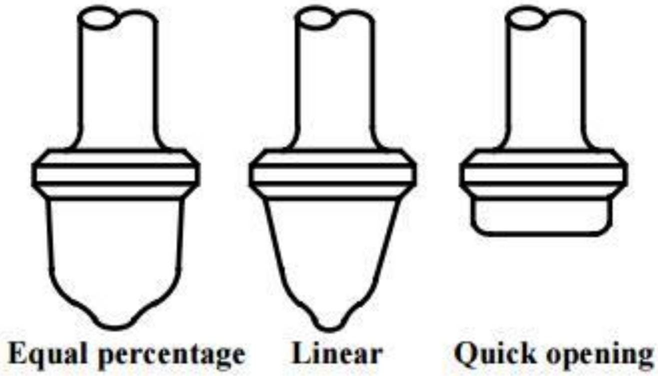
(c) Equal Percentage.

The characteristics of these control valves are shown in Fig. 4. It has to be kept in mind that all the characteristics are to be determined after maintaining constant pressure difference across the valve as shown in Fig.4.



Flow characteristics of control valve

Different flow characteristics can be obtained by properly shaping the plugs. Typical shapes of the three types of valves are shown in figure below



For a linear valve, dm /dx =1 , and the flow characteristics is linear throughout the operating range.

On the other hand, for an equal percentage valve, the flow characteristics is mathematically expressed as: a= βm where β is a constant.

The above expression indicates, that the slope of the flow characteristics is proportional to the present flow rate, justifying the term equal percentage.

This flow characteristics is linear on a semilog graph paper. The minimum flow rate m_0 (flow rate at x=0) is never zero for an equal percentage valve and m can be expressed as

$m = m_0\, e^{\beta x}$(5)

Rangeability of a control valve is defined as the ratio of the maximum controllable flow and the minimum controllable flow.

Thus Rangeability = maximum controllable flow / minimum controllable flow

Rangeability of a control valve is normally in between 20 and 70.

Sensitivity is defined as ratio between change in flow rate to change in stem position, dm/dx.

Quick opening valve is also called as decreasing sensitivity type. The valve sensitivity at any flow decreases with increasing flow.

Linear valve has constant sensitivity.

Equal percentage valve is also called as increasing sensitivity type, as the valve sensitivity increases with increasing flow rate. The valve sensitivity at any given flow rate is a constant percentage of the given flow rate.

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Special characteristic valves:

Square root

$m = \sqrt{x}$

Hyperbolic

$m = 1/\alpha - (\alpha - 1)x$

Effective Characteristics

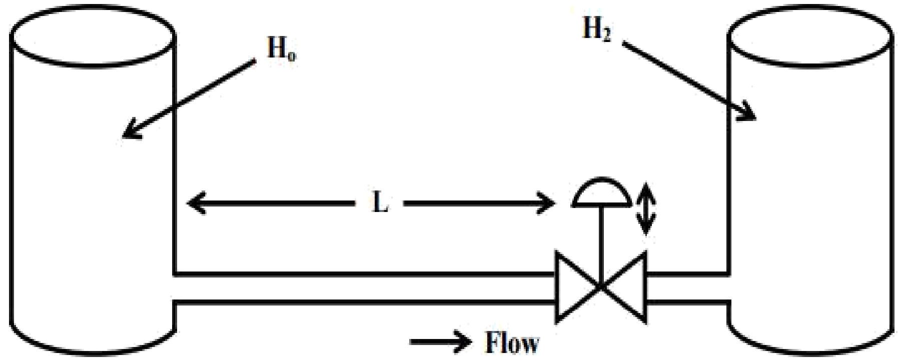
So far we have discussed about the ideal characteristics of a control valve. It is decided by the shape of the plug, and the pressure drop across the valve is assumed to be held constant. But in practice, the control valve is installed in conjunction with other equipment, such as heat exchanger, pipeline, orifice, pump etc. The elements will have their own flow vs. pressure characteristics and cause additional frictional loss in the system and the effective characteristics of the valve will be different from the ideal characteristics. In order to explain the deviation, let us consider a control valve connected with a pipeline of length L in between two tanks, as shown in figure below. We consider the tanks are large enough so that the heads of the two tanks H0 and H2 can be assumed to be constant. We also assume that the ideal characteristic of the control valve is linear.

For a linear valve

$K_1 a = Kz$

where K is a constant and z is the stem position or lift.

Now the pipeline will experience some head loss that is again dependent on the velocity of the fluid.



Effect of friction loss in pipeline for a control valve.

The head loss Δh_L will affect the overall flow rate q

$$q = \left[K\sqrt{2g(H_0 - H_2 - \Delta h_L} \right] z$$

The head loss (in m) can be calculated from the relationship:

$$\Delta h_L = F \frac{L}{D} \frac{v^2}{2g}$$

where F = Friction coefficient
L = Length of the pipeline in m
D = inside diameter of the pipeline in m
v = velocity of the flow in m.

Further, the velocity of the fluid can be related to the fluid flow q (in m^3/sec) as:

$$v = \frac{q}{\frac{\pi}{4} D^2}$$

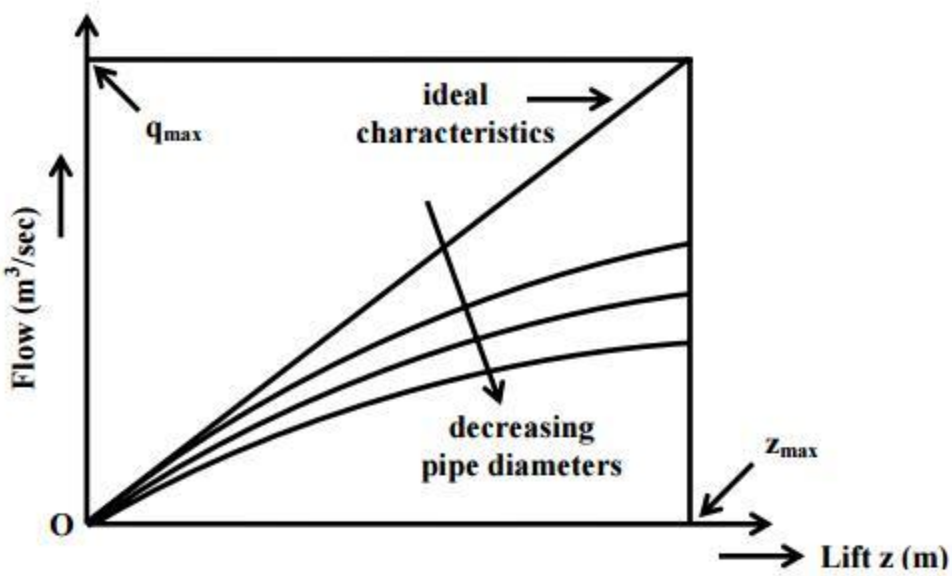
$$\Delta h_L = \frac{8}{\pi^2} \frac{FL}{gD^5} q^2$$

Substituting (9) in (6) and further simplifying, one can obtain:

$$q = \left[K\sqrt{\frac{2g(H_0 - H_2)}{1 + \alpha z^2}} \right] z$$

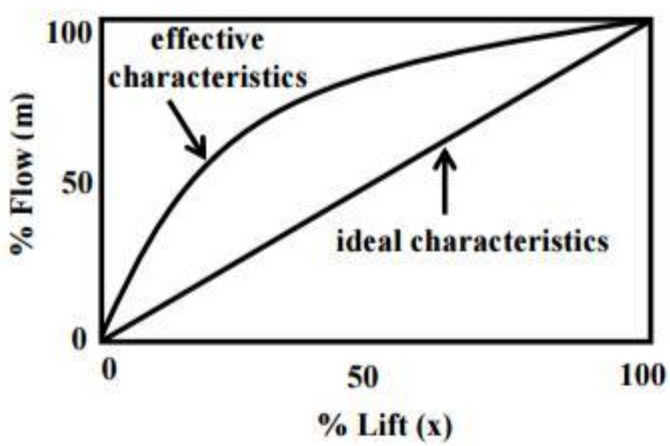
where $\alpha = \dfrac{16FLK^2}{\pi^2 D^5}$

it can be concluded that q is no longer linearly proportional to stem lift z, though the ideal characteristics of the valve is linear. This nonlinearity of the characteristics is dependent on the diameter of the pipeline D; i.e. smaller the pipe diameter, larger is the value of α and more is the nonlinearity. The nonlinearity of the effective valve characteristics can be plotted as shown in the figure below



Effect of pipeline diameter on the effective flow characteristics of the control valve

The nonlinearity introduced in the effective characteristics can be reduced by mainly (i) increasing the line diameter, thus reducing the head loss, (ii) increasing the pressure of the source H0,(iii) decreasing the pressure at the termination H2.



Comparison of ideal and effective characteristics for a linear valve

Thus linear valves are recommended when pressure drop across the control valve is expected to be fairly constant. On the other hand, equal percentage valves are recommended when the pressure drop across the control valve would not be constant due to the presence of series resistance in the line. As the line loss increases, the effective characteristics of the equal percentage valve will move closer to the linear relationship in m vs. x characteristics.

There are basically three types of valves employed in hydraulic systems:

- Directional control valves
- Flow control valves
- Pressure control valves

1. Direction control valve

Directional control valves are used to control the distribution of energy in a fluid power system. They provide the direction to the fluid and allow the flow in a particular direction. These valves are used to control the start, stop and change in direction of the fluid flow. These valves regulate the flow direction in the hydraulic circuit. These control valves contain ports that are external openings for the fluid to enter and leave. The number of ports is usually identified by the term ‘way'. For example, a valve with four ports is named as four-way valve. The fluid flow rate is responsible for the speed of actuator (motion of the output) and should controlled in a hydraulic system. This operation can be performed by using flow control valves. The pressure may increase gradually when the system is under operation. The pressure control valves protect the system by maintaining the system pressure within the desired range. Also, the output force is directly proportional to the pressure and hence, the pressure control valves ensure the desired force output at the actuator.

Directional control valves can be classified in the following manner:

1. Type of construction:

- Poppet valves
- Spool valves

2. Number of ports:

- Two- way valves
- Three – way valves

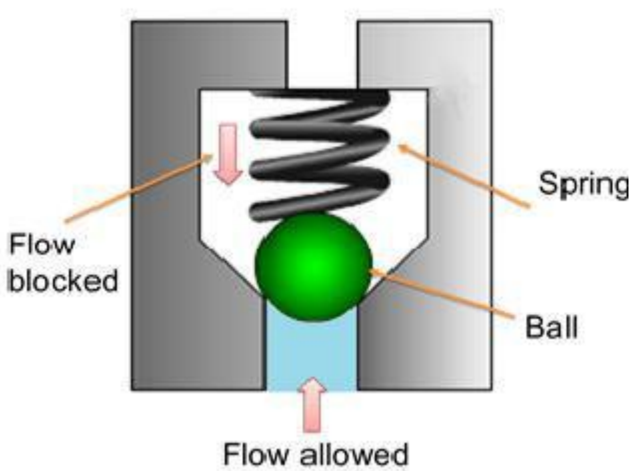
- Four- way valves.

3. Number of switching position:

- Two – position
- Three - position

4. Actuating mechanism:

- Manual actuation
- Mechanical actuation
- Solenoid actuation
- Hydraulic actuation
- Pneumatic actuation
- Indirect actuation

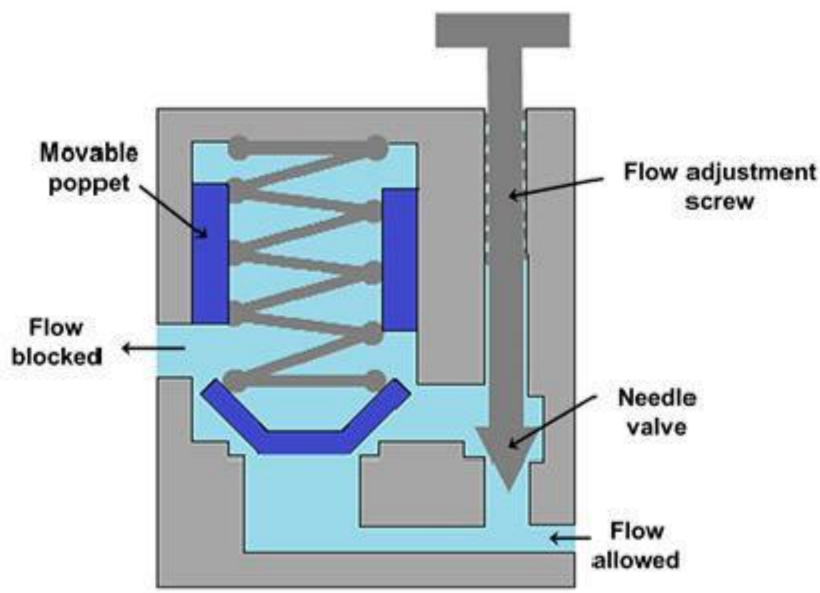


Check valve

These are unidirectional valves and permit the free flow in one direction only. These valves have two ports: one for the entry of fluid and the other for the discharge. They consist of a housing bore in which ball or poppet is held by a small spring force. The valve having ball as a closing member is known as ball check valve. The various types of check valves are available for a range of applications. These valves are generally small sized, simple in construction and inexpensive. Generally, the check valves are automatically operated. Human intervention or any external control system is not required. These valves can wear out or can generate the cracks after prolonged usage and therefore they are mostly made of plastics for easy repair and replacements.

The ball is held against the valve seat by a spring force. It can be observed from the figure that the fluid flow is not possible from the spring side but the fluid from opposite side can pass by lifting the ball against. However, there is some pressure drop across the valve due to restriction by the spring force. Therefore these valves are not suitable for the application of high flow rate. When the operating pressure increases the valve becomes more tightly seated in this design.

The advantages of the poppet valves include no leakage, long life and suitability with high pressure applications. These valves are commonly used in liquid or gel mini-pump dispenser spigots, spray devices, some rubber bulbs for pumping air, manual air pumps, and refillable dispensing syringes.

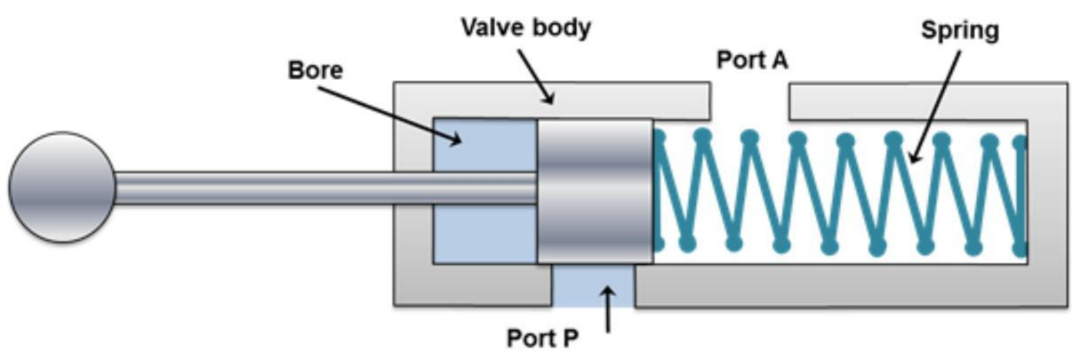


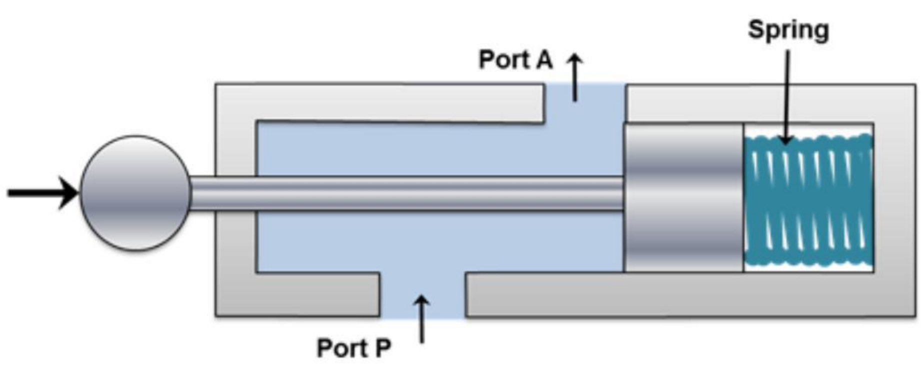
Poppet valve or restriction check valve

When the closing member is not a ball but a poppet energized by a spring is known as poppet valve. Some valves are meant for an application where free flow is required in one direction and restricted flow required in another direction. These types of valves are called as restriction check valve. These valves are used when a direction sensitive flow rate is required.

Spool valve

The spool valves derive their name from their appearance. It consists of a shaft sliding in a bore which has large groove around the circumference. This type of construction makes it look like a spool. The spool is sealed along the clearance between moving spool and housing (valve body). The quality of seal or the amount of leakage depends on the amount of clearance, viscosity of fluid and the level of the pressure. The grooves guide the fluid flow by interconnecting or blocking the holes (ports). The spool valves are categorized according to the number of operating positions and the way hydraulic lines interconnections. One of the simplest two way spool valve is shown in Figure below. The standard terms are referred as Port ‘P' is pressure port, Port ‘T' is tank port and Port ‘A' and Port ‘B' are the actuator (or working) ports. The actuators can move in forward or backward direction depending on the connectivity of the pressure and tank port with the actuators port.





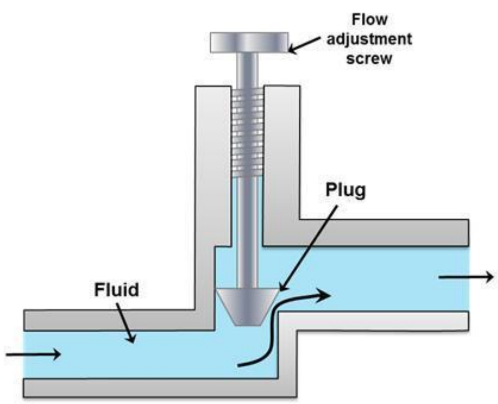
The first figure above shows the valve closed and the above figure shows the open condition of the valve.

Types of Flow Control Valves

The flow control valves work on applying a variable restriction in the flow path. Based on the construction; there are mainly four types viz. plug valve, butterfly valve, ball valve and balanced valve.

Plug or glove valve

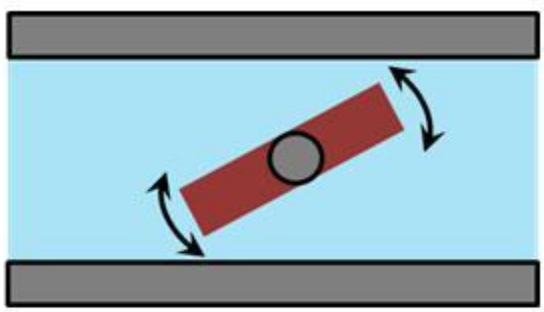
The plug valve is quite commonly used valve. It is also termed as glove valve. Schematic of plug or glove valve is shown in Figure below: This valve has a plug which can be adjusted in vertical direction by setting flow adjustment screw. The adjustment of plug alters the orifice size between plug and valve seat. Thus the adjustment of plug controls the fluid flow in the pipeline. The characteristics of these valves can be accurately predetermined by machining the taper of the plug. The typical example of plug valve is stopcock that is used in laboratory glassware. The valve body is made of glass or teflon. The plug can be made of plastic or glass. Special glass stopcocks are made for vacuum applications. Stopcock grease is used in high vacuum applications to make the stopcock air-tight.



Plug or Glove Valve

Butterfly valve

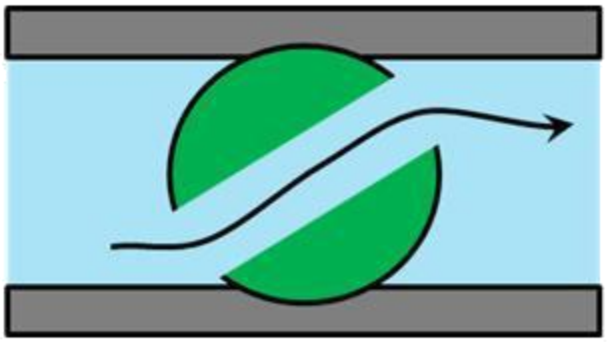
A butterfly valve is shown in Figure below. It consists of a disc which can rotate inside the pipe. The angle of disc determines the restriction. Butterfly valve can be made to any size and is widely used to control the flow of gas. These valves have many types which have for different pressure ranges and applications. The resilient butterfly valve uses the flexibility of rubber and has the lowest pressure rating. The high performance butterfly valves have a slight offset in the way the disc is positioned. It increases its sealing ability and decreases the wear. For high-pressure systems, the triple offset butterfly valve is suitable which makes use of a metal seat and is therefore able to withstand high pressure. It has higher risk of leakage on the shut-off position and suffer from the dynamic torque effect. Butterfly valves are favored because of their lower cost and lighter weight. The disc is always present in the flow therefore a pressure drop is induced regardless of the valve position.



Butterfly valve

Ball Valve

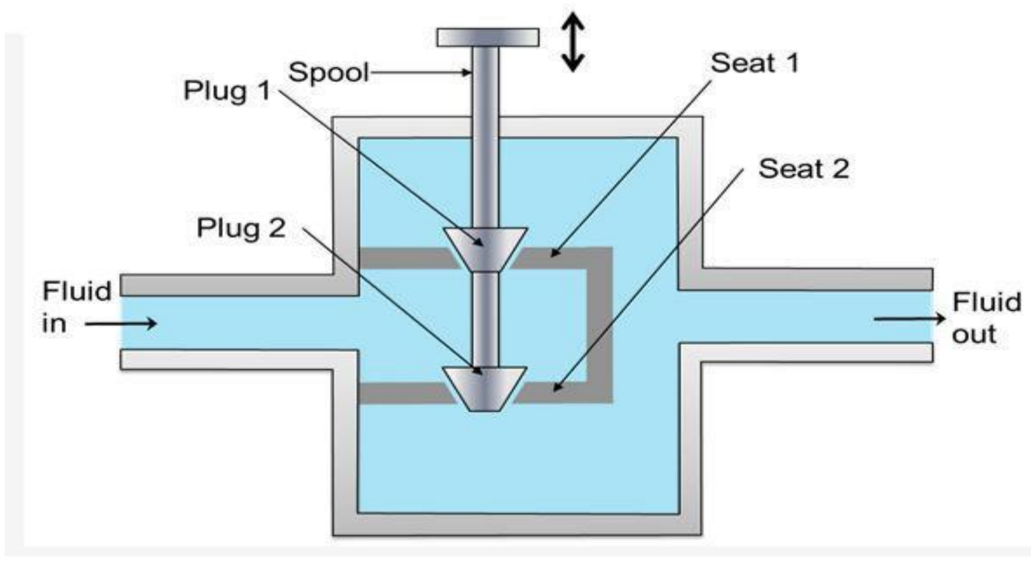
The ball valve is shown in Figure below. This type of flow control valve uses a ball rotated inside a machined seat. The ball has a through hole as shown in Figure below. It has very less leakage in its shut-off condition. These valves are durable and usually work perfectly for many years. They are excellent choice for shutoff applications. They do not offer fine control which may be necessary in throttling applications. These valves are widely used in industries because of their versatility, high supporting pressures (up to 1000 bar) and temperatures (up to 250°C). They are easy to repair and operate.



Ball valve

Balanced valve

Schematic of a balanced valve is shown in figure below. It comprises of two plugs and two seats. The opposite flow gives little dynamic reaction onto the actuator shaft. It results in the negligible dynamic torque effect. However, the leakage is more in these kind of valves because the manufacturing tolerance can cause one plug to seat before the other. The pressure-balanced valves are used in the houses. They provide water at nearly constant temperature to a shower or bathtub despite of pressure fluctuations in either the hot or cold supply lines.



Balanced valve

CONTROL VALVE SIZING

The proper sizing of control valve is important. If the valve is oversized, thevalve will operate at low lift and minimum controllable flow is too large. If the valve is undersized, the maximum flow desired for operation of a process may not be provided.

Factors that influence the size of control valve:

- Pressure drop across the control valve
- Flow rate through the valve
- Specific gravity

Other factors that influence the size are:

Type of fluid: gas or liquid

Critical flow conditions for gases and vapours

Viscosity of liquids

Flow Coefficient: It indicates the amount of flow of control valve can handle under a given pressure drop across the control valve.

C_V Flow coefficient :It is defined as the flow rate of water in gallons per minute at 60 ºF through a valve at maximum opening with a pressure drop of 1 psi measured in the inlet and outlet pipes directly adjacent to the valve body.

K_V Flow coefficient :It is defined as the flow rate of water in m^3 /hr at 30 ºC through a valve at maximum opening with a pressure drop of 1 kg/cm^2 measured in the inlet and outlet pipes directly adjacent to the valve body.

$$C_V = 1.17\ K_V \quad \text{or}$$
$$K_V = 0.86\ C_V$$

Flow rate Vs Flow Coefficient

$Q = C_V \left(\sqrt{(\Delta P / SG)} \right)$

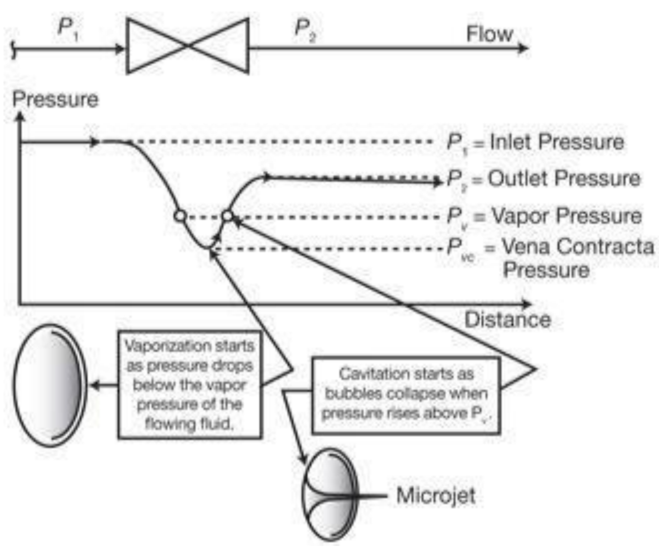
ΔP = pressure drop across control valve
SG= Specific gravity of the liquid

Guidelines for Sizing of Control Valves:

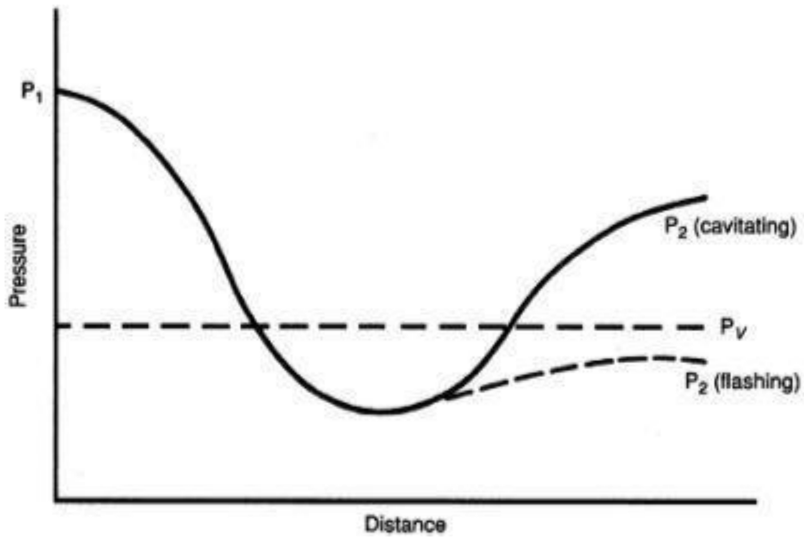
1. The valve shall be sized for the actual flow condition and not for the ultimate design capacity of the system. Normal maximum flow rate is normally about 70% of the ultimate design capacity.
2. Most of the pressure drop of the system should be across the control valve. (about 70%)
3. When the pipe line is dimensioned with normal allowable velocities the control valve will be a few sizes smaller than the pipe line. Only in extreme cases where very high velocities have been used in the pipe line, the size of the control valve will be the same as that of the pipe line.
4. The selection must be done such that the calculated C_V is attained at about 75 to 80% of the full wave travel.
5. Regardless of the application such as flow control or pressure control the valve sizing is done on the basis of flow coefficient C_V.

CAVITATION AND FLASHING

To avoid dangerous and costly problems in their process systems industrial valve users should understand the basics of the ways in which cavitation and flashing affect control valves. Cavitation occurs in liquid systems when local pressure fluctuations near the liquid's vapor pressure result in the sudden growth and collapse of vapor bubbles (cavities) within the liquid. The cavity collapse produces localized shock waves and liquid microjets. If these impact on the adjacent surfaces of pumps, valves, or pipe, severe pitting and erosion damage can occur, which can reduce critical wall thickness. Cavitation often produces high levels of noise and vibration across a broad range of frequencies. Excessive vibration can loosen flange bolting, damage piping support structures, and destroy process equipment. These hazards and excessive noise create dangerous conditions for people and their environment. The pressure-reducing characteristics that make control valves useful also make them susceptible to cavitation. However, an effective combination of system design, valve selection and design, and material selection can minimize or eliminate the unwanted effects of cavitation.



Flashing is a vaporizing process similar to cavitation. However, flashing differs from cavitation in that the vapor phase persists and continues downstream because the downstream pressure remains at or below the vapor pressure of the liquid. High velocities and mixed-phase flow are generated by the expansion of the liquid * into vapor, which can cause erosion and thinning of pressure boundary walls. Although with flashing noise and vibration are usually much less than with severe cavitation, flashing can generate excessive vibration associated with high-velocity flow. Reducing velocity and using erosion-resistant materials are effective design strategies that minimize the damage from flashing.



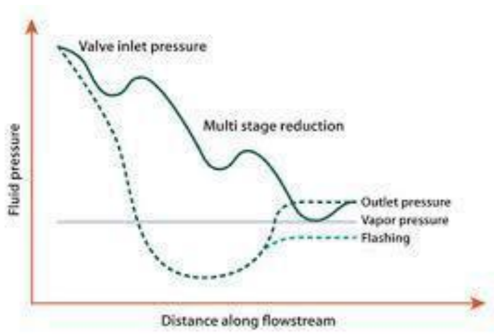
Elimination of Cavitation:

Revised process conditions:

In order to eliminate cavitation, one can change the process conditions. A reduction of operating temperature can lower the vapor pressure sufficiently to eliminate it. Similarly, increased upstream and downstream pressures, with Δp unaffected, or a reduction in the Δp can both relieve cavitation. Therefore, control valves that are likely to cavitate should be installed at the lowest possible elevation in the piping system and operated at minimum Δp. Moving the valve closer to the pump will also serve to elevate both the up and downstream pressures. If cavitating conditions are unavoidable, one can increase operating temperature or decrease outlet pressure to cause flashing which eliminates cavitation by converting the incompressible liquid into a compressible mixture.

Revised valve:

Where the operating conditions cannot be changed, one should use valves with a treacherous flow path with low recovery and therefore high F_c and F_L coefficients and avoid high recovery valves (ball, butterfly, gate), having low F_L and F_c coefficients. For example, in the " Swiss cheese” type design, small holes in the skirt or cage are arranged in pairs on opposite sides of the centerline of the valve. Labyrinth-type valves avoid cavitation by having a series of right-angle turns with negligible pressure recovery at each turn. Multistep or multiple valves in series can also avoid cavitation by replacing a single and deep vena contracta with several small vena contracta points as the pressure drop is distributed between several ports working in series. This solution is likely to work only if the vapor pressure of the process fluid is below the outlet pressure of the valve(s).



Gas Injection:

One can alleviate cavitation by introducing air or nitrogen into the region where cavitation is anticipated. The gas may be admitted through the valve shaft or through downstream taps on either side of the pipe in line with the shaft, and as close to the valve as possible. It also absorbs some of the pressure drop in restriction orifices, chokes, or in partially open block valves upstream or downstream of the valve.

IMPORTANT QUESTIONS

PART A

Define cavitation.

What is flashing in control valve?

What is the purpose of final control element?

Define the flow coefficient of control valve.

How are the valves classified?

State the purpose of valve positioned.

What do you mean by cavitations?

What are the advantages of actuator with positioner?

Explain the principle of Flapper Nozzle method?

Why control valve to be sized?

Explain the need for valve positioned in control valves.

What is the function of valve positioner?

Define cavitation in control valve.

Define valve flow coefficient.

Why is an equal percentage valve is called so?

Define sizing.

How to avoid cavitations in control valves?

What is an equal percentage valve characteristic?

Define cavitation?

PART B

1.Explain the principle of operation of I/P and P/I converters.

2.Write notes on the following.

a) Hydraulic actuators.

b) Characteristics of control valve.

3. aExplain with neat diagram the working of I/P converters

b.Whether control valve should be oversized or undersized? Why?

4.Explain Cavitations and Flashing in control valves.

5.a What are the characteristics of control valves?

(b) What are the applications of a valve positioner?

6.(a) What are the different types of pneumatic actuators? Explain in detail.

(b) Why is the size of a control valve important? Explain.

7.(a) Explain the construction and working of I/P converter.

(b) What is an equal percentage valve? What is meant by 'rangeability' of a control valve? 8.(a) Describe the working of a pneumatic actuator with positioner.

(b) Briefly write about 'Valve sizing' of control valves 9.Explain I/P and P/I converter.

10.Explain the harmful effect of cavitations and flashing in control valves.

11.Write short notes on the following.

(a) I/P converters (b) Hydraulic actuators

12.With neat diagram explain the functioning and applications of any two types of valves

13a) Define cavitation and flashing. How to prevent them?

(b) Describe the characteristics of control valve.

14.(a) What are the factors required for selection of a control valve?

(b) Explain the operation and benefits of valve positioner.

15 Explain the characteristics of control valve with the valve positioner in various modes.

16.Write short notes on control valve sizing & flashing in control valve.

17. Explain the following

  (a) Electronic Actuators

  (b) Hydraulic actuators

18.(a) Explain the types control valve characteristics with neat diagram.

(b) Explain Butter fly valve and Globe valves.

UNIT V

MULTILOOP CONTROL

Piping and instrumentation diagram, also called P&ID, illustrates the piping processes and interactions with other installed equipment and instrumentation. P&IDs are often used in the process industry to show the process flow and other installed equipment and instrument.

Piping and instrumentation diagram plays a crucial role in the design and engineering of process plants and piping systems, by showing the interaction of process used to control the process. P&ID is a schematic illustration of functional relationship of piping, instrumentation and system equipment components

Difference between PFD and P&ID

Instrumentation detail varies with the degree of design complexity. For example, simplified or conceptual designs, often called process flow diagrams (PFDs), provide less detail than fully developed piping and instrumentation diagrams (P&IDs). Since a PFD shows less detail than a P&ID, it is used only to understand how the process works. Figure 5.1 Shows the difference between PFD & PID diagram.

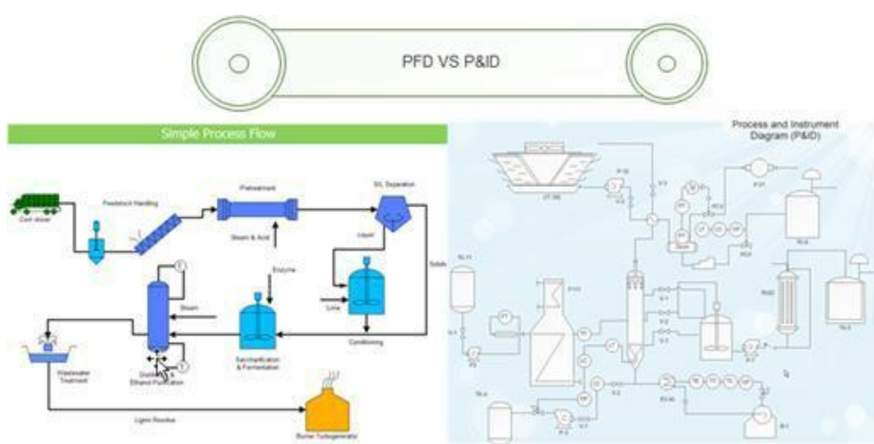


Fig 5.1 Difference between PFD & PID diagram

Building the P&ID

The P&ID will use symbols and circles to represent each instrument and how they are inter-connected in the process.

Process and Instrumentation Symbols

Valves

Gate valve is a device used to control the flow of liquids and gases.

Check valve, also known as one-way valve, is to prevent the line of medium back.

Globe valve is a mechanism used to control or stop the flow of liquid or gas through a pipe.

Ball valve is a valve with a spherical disc, the part of the valve which controls the flow through it.

Butterfly valve is installed between two flanges using a separate set of bolts for each flange.

Angle valve is oriented at an angle of 90 deg of gate valve.

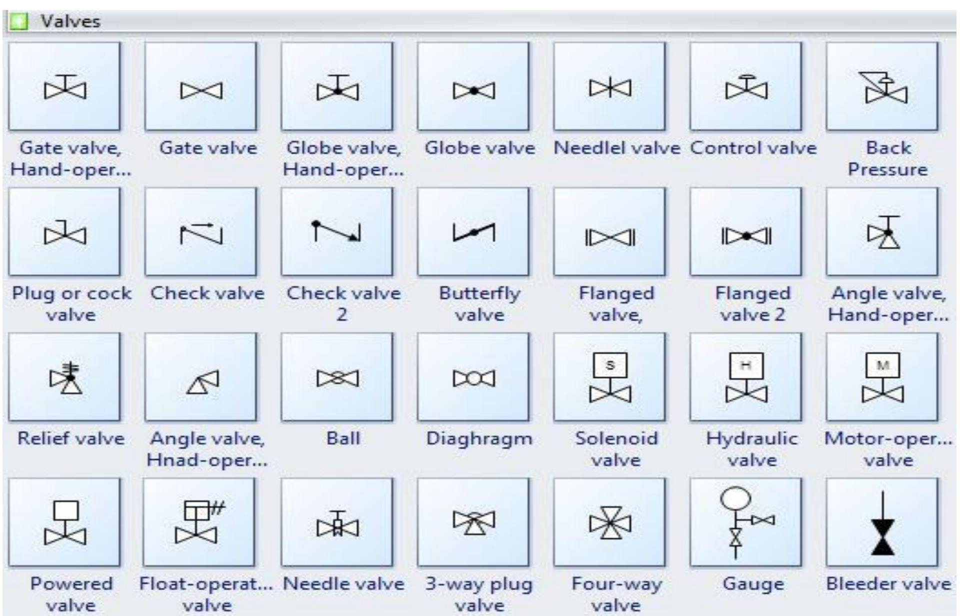


Fig 5.2 P&I Symbols of valves

Piping Lines

Process flow diagrams use special piping lines to represent how signals are transmitted between equipments. These symbols are used to identify how the instruments in the process connect to each other. And what type of signal is being used. (electrical, pneumatic, data, etc)

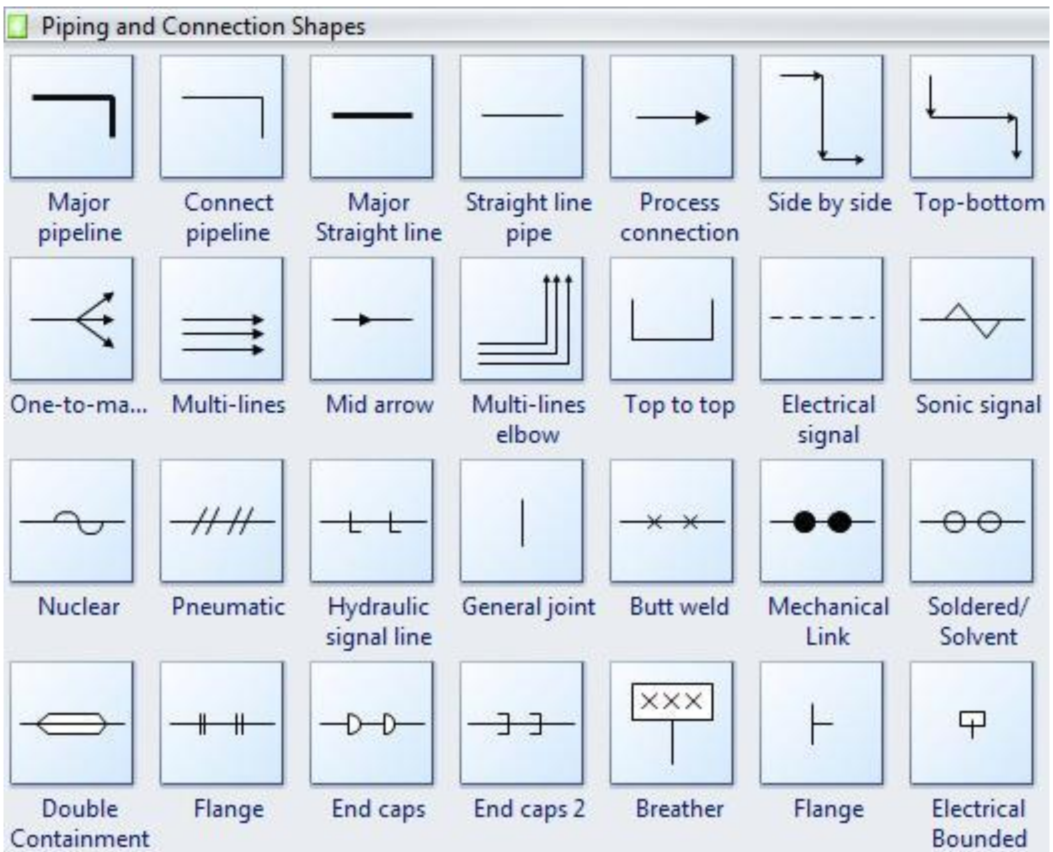


Fig 5.3 P&I Symbol - Piping and Connection Shapes

All lines to be fine in relation to process piping lines.

Major pipeline is used to connect the equipment in any position.

Major straight line is used to connect the equipment in the same horizontal or vertical position.

Process connection help to create the process flow between equipments. Double click process connection to edit description.

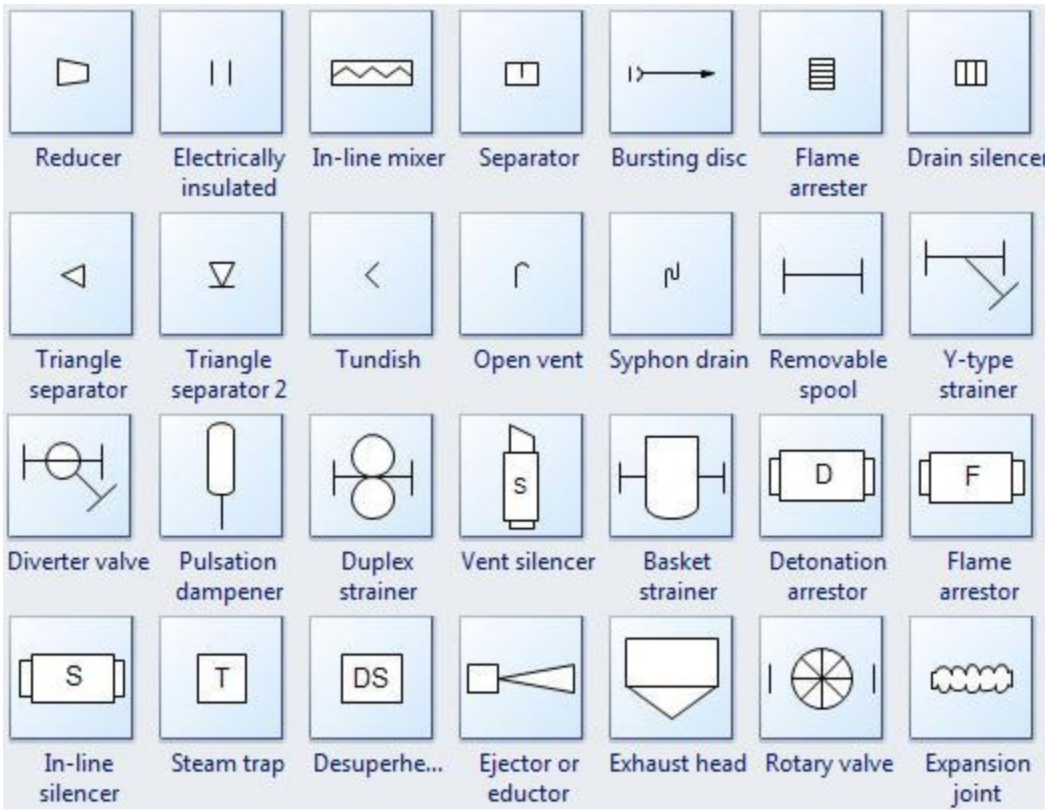


Fig 5.4 P&I Symbol – Piping Joints

Process and Instrumentation Symbols - Instruments

Process Flow Diagram use symbols and circles to represent each instrument and how they are inter-connected in the process.

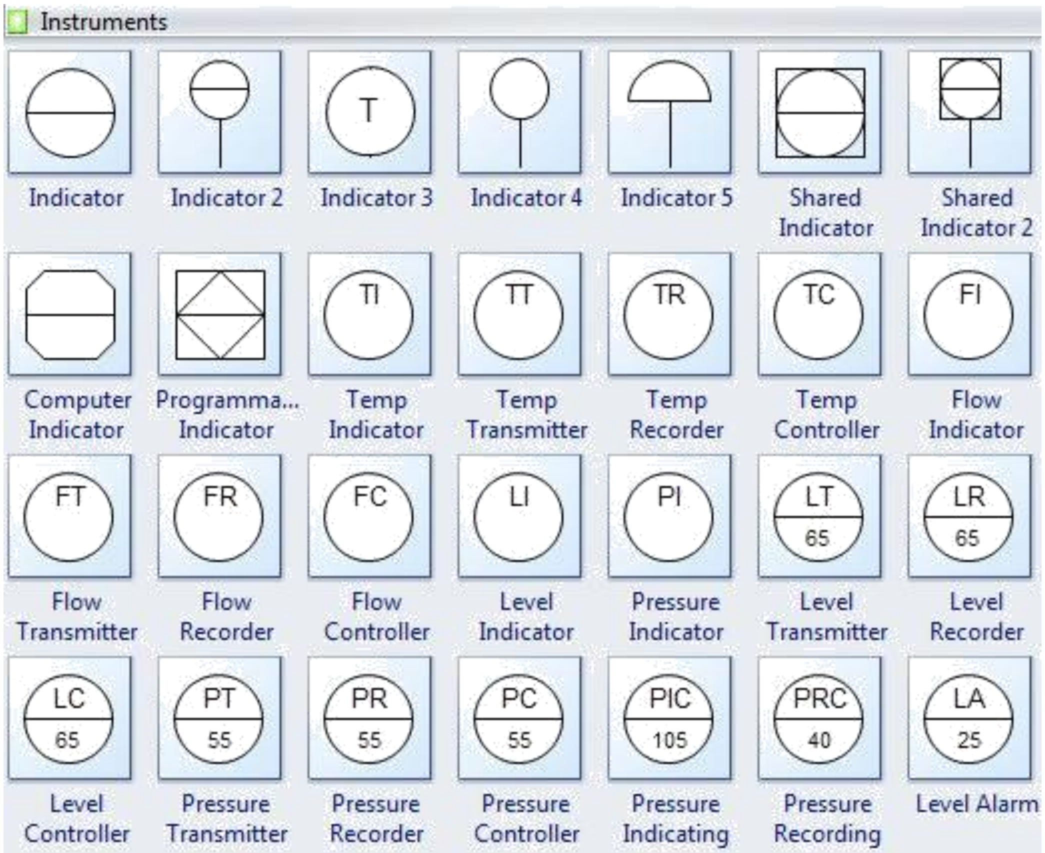


Fig 5.5 P&I Symbol - Instruments

Tag Numbers

Tag ―numbers‖ are letters and numbers placed within or near the instrument to identify the type and function of the device.

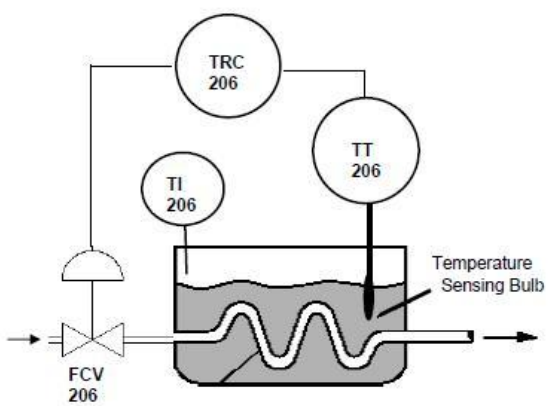
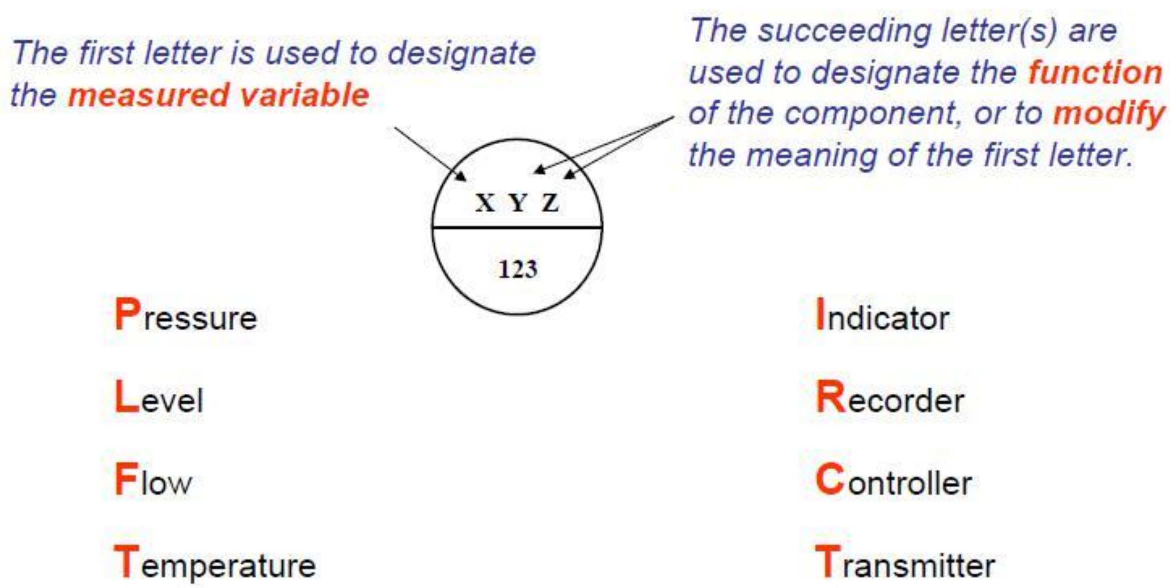


Fig 5.6 Example of P&I with Tag Numbers

Tag Descriptors



Tag —numbers‖ areletters and numbers placed within or near the instrument to identify the type and function of the device.

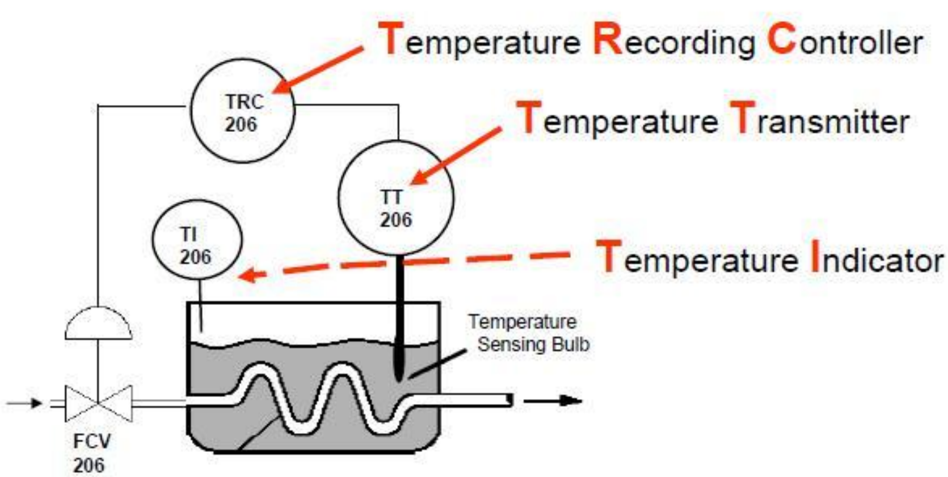


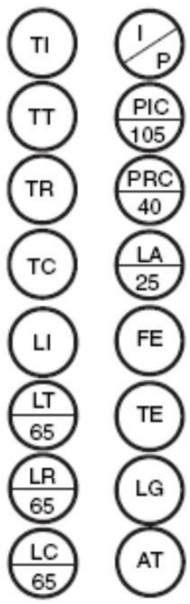
Fig 5.7 Detailed Example for P&I with tag numbers

ISA Identification Letters

Table 5.1 ISA Identification Letters

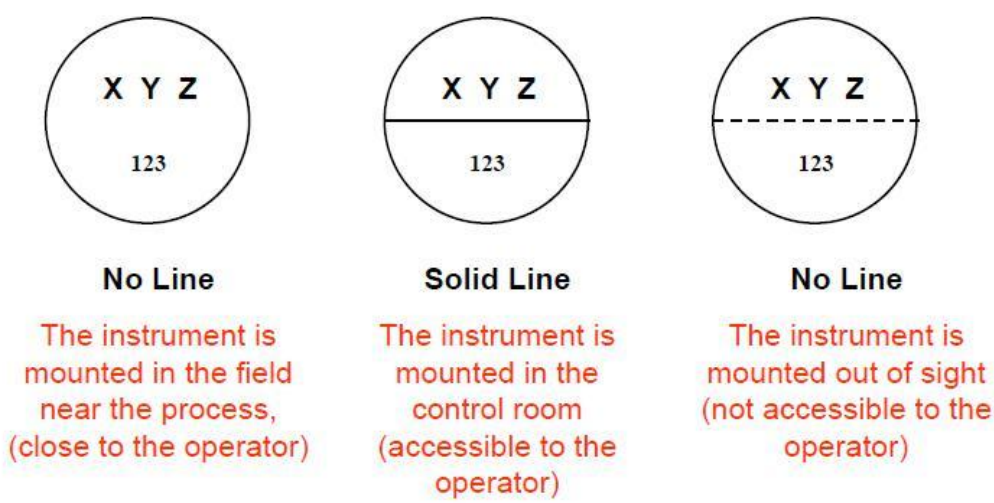
Letters	First Letter		Succeeding Letters		
	Measured or Initiating Variables	**Modifier**	**Readout Function**	**Output Function**	**Modifier**
A	Analysis				
C				Control	
D		Differential			
F	Flow Rate	Ratio			
H	Hand				High
I	Current		Indicate		
L	Level				Low
P	Pressure, Vacuum				
Q	Quantity	Totalizer			
S		Safety		Switch	
T	Temperature			Transmit	
V	Vibration			Valve, Damper	
Z	Position			Actuator	

Examples



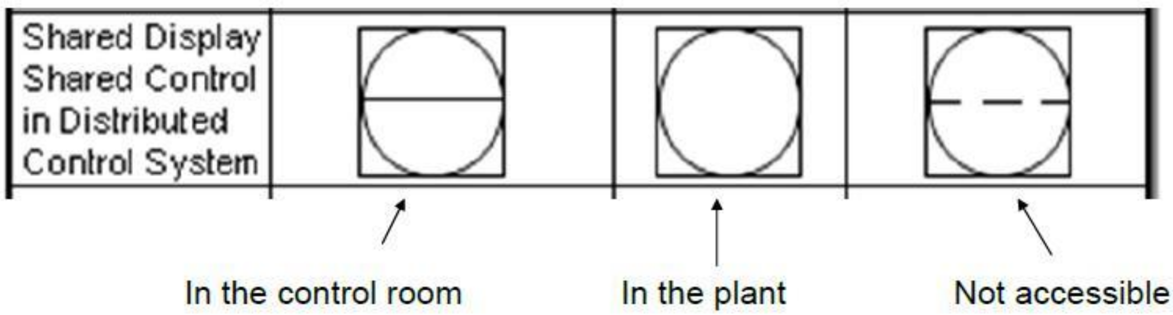
Instrument Location

The presence or absence of a line determines the location of the physical device. For example no line means the instrument is installed in the field near the process.

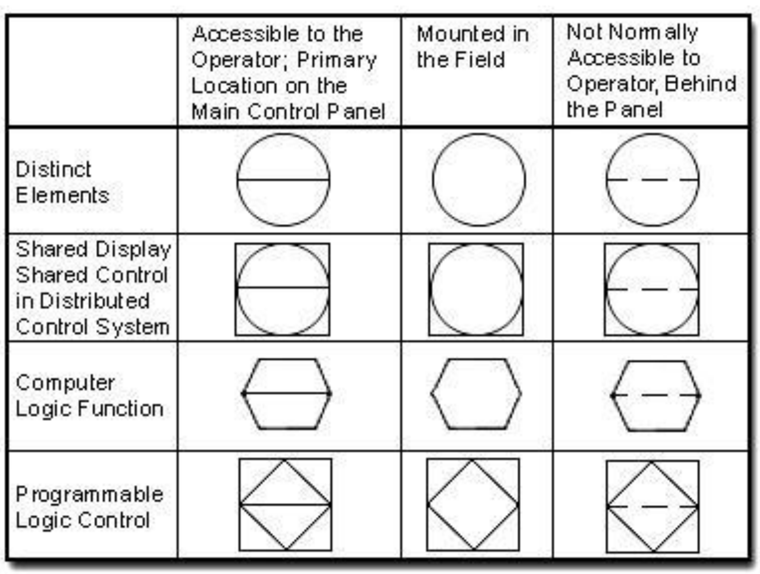


Shared Displays/Shared Control

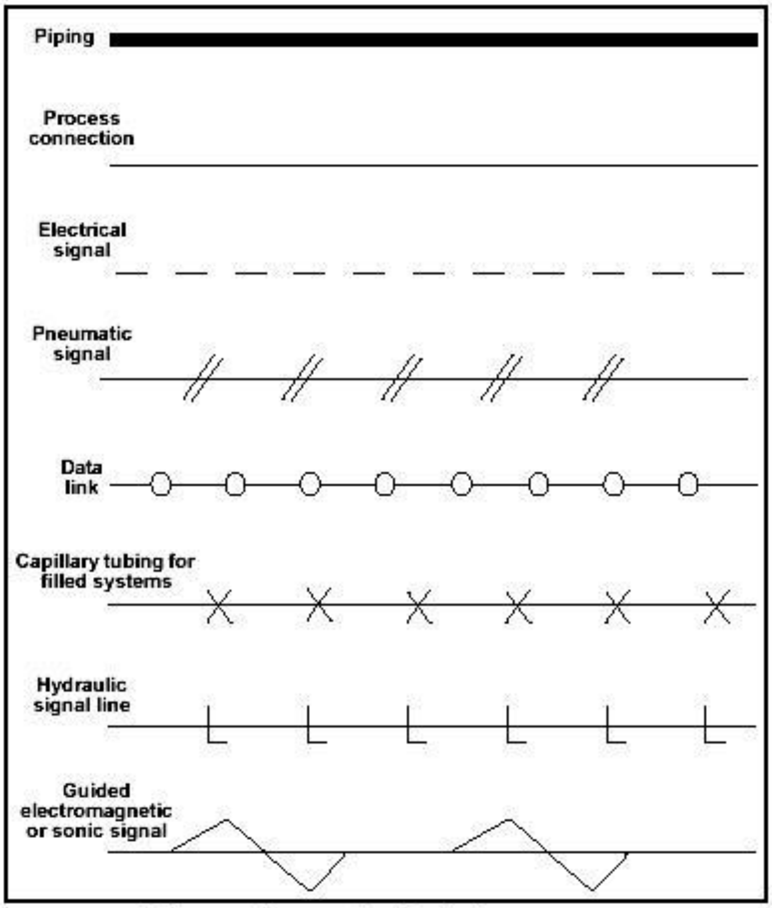
Some instruments are part of a Distributed Control System (DCS) where a specific controller or indicator can be selected from many others but shown in one location (like a terminal screen)



Summary of instrument type & location .



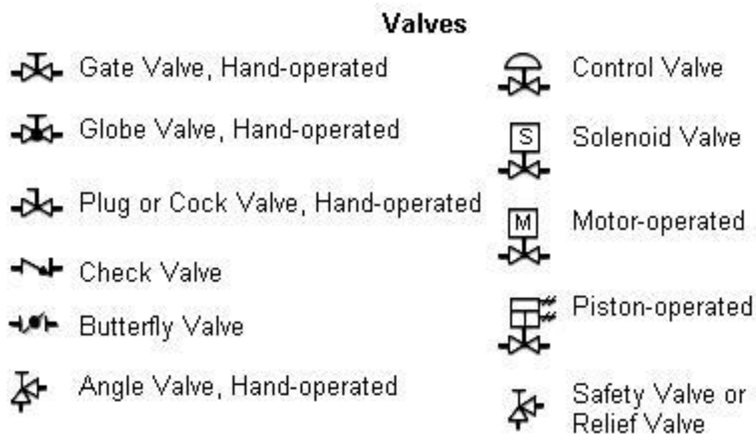
Piping and Connection Symbols



Piping and Connection Symbols

These symbols are used to identify how the instruments in the process connect to each other.and what type of signal is being *used. (electrical, pneumatic, data, etc)*

Valve Symbols



Interaction between control loops

Consider a process with two inputs and two outputs:

$$\bar{y}_1 = G_{11}\bar{u}_1 + G_{12}\bar{u}_2 \qquad (1)$$
$$\bar{y}_2 = G_{21}\bar{u}_1 + G_{22}\bar{u}_2 \qquad (2)$$

The schematic of such process is given by the Fig. 1

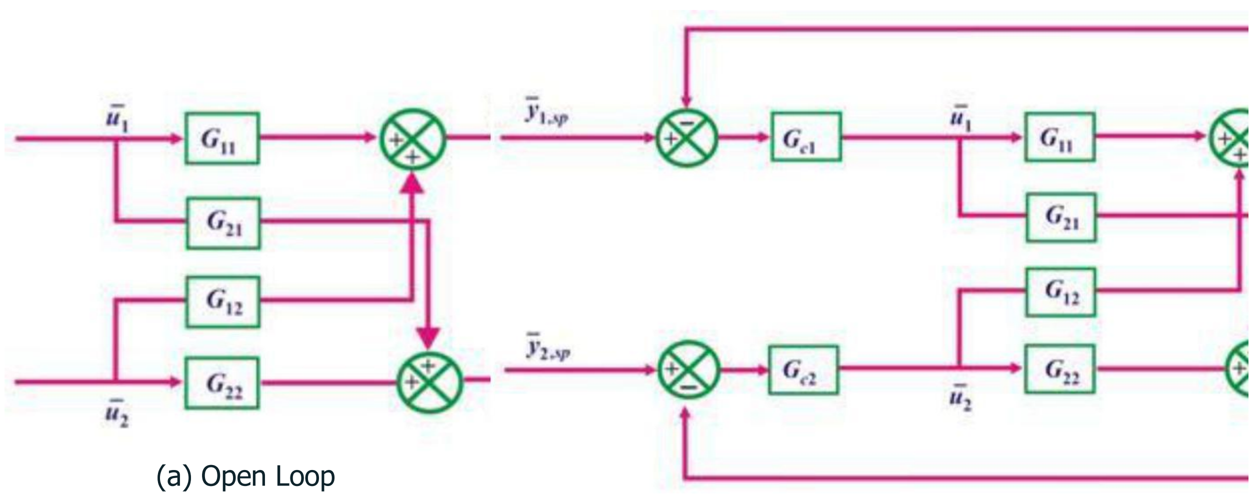


Fig. 1 Schematic of a two input and two output system

Any change in either of the inputs will lead to change in the values of both the outputs.

This phenomenon persists even when two control loops are formed as in Fig. VI.3(b). This is termed as control loop interaction. There are two types of effects of an input on an output, viz . direct and indirect. The effects are best explained by the Fig.2 where blue line indicates direct effect of input $\bar{u}_1$ on output $\bar{y}_1$ within its own loop, i.e. , loop 1 whereas green line indicates indirect effect of input $\bar{u}_1$ on output that $\bar{y}_1$ yields through loop 2.

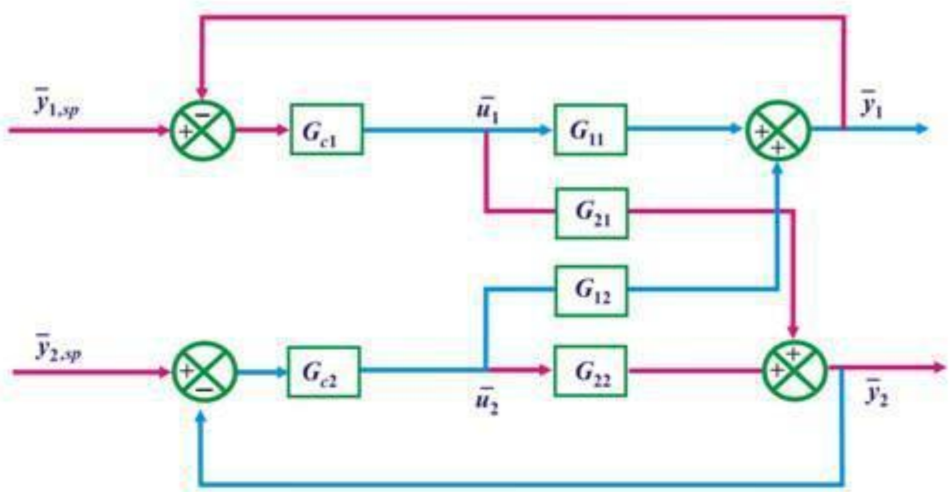


Fig. 2 Direct and indirect effects of control loop interaction

The concept of this loop interaction will be used for understanding Relative Gain Array (RGA).

Relative Gain Array and selection of control loops

The RGA provides a quantitative criterion for selection of control loops that would lead to minimum interaction among the loops. Consider the following two experimentations (or simulations):

- Let us open the loops and detach the controllers from the process. Let us keep $\bar{u}_2$ constant and introduce a step input in $\bar{u}_1$. That would yield a static gain $K = \frac{\Delta \bar{y}_1}{\Delta \bar{u}_1}\Big|_{\bar{u}_2 = constant}$ that would indicate the direct effect of input on output .

• Let us now close only the loop 2 and attach the corresponding controller with the process. Let us now introduce a step input in $\bar{u}_1$ while maintaining $\bar{y}_2$ at its desired setpoint through the loop 2 controller. That would yield another open loop gain $K' = \left.\frac{\Delta \bar{y}_1'}{\Delta \bar{u}_1}\right|_{\bar{y}_2 = at\ setpoint}$ that would indicate the direct as well as indirect effect of $\bar{u}_1$ input on output $\bar{y}_1$.

The ratio of above two open loop gains is defined as the relative gain

$$\lambda_{11} = \frac{K}{K'} = \frac{\left.\frac{\Delta \bar{y}_1}{\Delta \bar{u}_1}\right|_{\bar{u}_2 = constant}}{\left.\frac{\Delta \bar{y}_1'}{\Delta \bar{u}_1}\right|_{\bar{y}_2 = at\ setpoint}} \tag{3}$$

In the similar manner, relative gains between other input-output combinations can be calculated as $\lambda_{12}, \lambda_{21}, \lambda_{22}$. The relative gain array is expressed in the matrix form as

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \tag{4}$$

It should be noted that sum of relative gains along any row or column should be 1.

The relative gain provides useful information on interaction

• If $\lambda_{11} = 0$, input $\bar{u}_1$ does not have any effect on output and thus they should not be paired.

• If $\lambda_{11} = 1$, input $\bar{u}_2$ does not have any effect on output and hence the system is completely decoupled . Thus pairing of input and output is ideal.

• If $0 < \lambda_{11} < 1$, interaction exists. Smaller the value of relative gain, larger will be the interaction.

• If $\lambda_{11} < 0$, input $\bar{u}_2$ has strong effect on output $\bar{y}_1$ and that also in the opposite direction of that of input $\bar{u}_1$. Here the interaction effect is dangerous an detrimental for the system.

Let us now analyze the following conditions for RGA:

- When $\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the system is completely decoupled. Pairing of $\{u_1 vs\ y_1, u_2 vs\ y_2\}$ is ideal.

- When $\Lambda = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, the system is completely decoupled in the reverse manner. Pairing of $\{u_1 vs\ y_2, u_2 vs\ y_1\}$. is ideal.

- When $\Lambda = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$, strong interaction exists in the system and it does not matter which ever pairing is resorted to.

- When $\Lambda = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$, mild interaction exists in the system. However, pairing of $\{u_1 vs\ y_1, u_2 vs\ y_2\}$ is favourable in this case.

Quantitative analysis with RGA can be extended to systems with more number of inputs and outputs in the similar manner, however, the situation might not be straightforward in all such cases. Needless to mention, RGA is very much sensitive to model uncertainty if it is produced through simulation. Reassessment of modeling equations is required if RGA analysis is too poor to be conclusive.

Non Interaction of control loops

The relative gain array indicates how the inputs should be paired with the outputs. However, if the interaction between the loops is beyond acceptable limit, then a control designer ought to seek a solution whereby he/she can implement some technique that would decouple the loops from one another and make several non-interacting loops in result. For the present 2 x 2 system, it is evident that $\bar{u}_2$ does affect the $\bar{y}_1$ however it is possible to negate the effect by appropriately manipulating $\bar{u}_1$ in addition to whatever guidance it obtains from the controller G_{c1}. From eq. (VI.6), it can be concluded that if output is maintained static at its nominal point then deviation $\bar{y}_1 = 0.$ Hence, the steady state relationship between two inputs is

$$\bar{u}_1 = \left(-\frac{G_{12}}{G_{11}}\right)\bar{u}_2 = D_1\bar{u}_2 \tag{5}$$

Similarly, from eq. (VI.7), it can be concluded that if output is maintained static at its nominal point then deviation $\bar{y}_2 = 0.$ Hence, the steady state relationship between two inputs would be

$$\bar{u}_2 = \left(-\frac{G_{21}}{G_{22}}\right)\bar{u}_1 = D_2\bar{u}_1 \tag{6}$$

Now let us introduce two transfer functions, viz ., decouplers, D_1 and D_2 in the following fashion

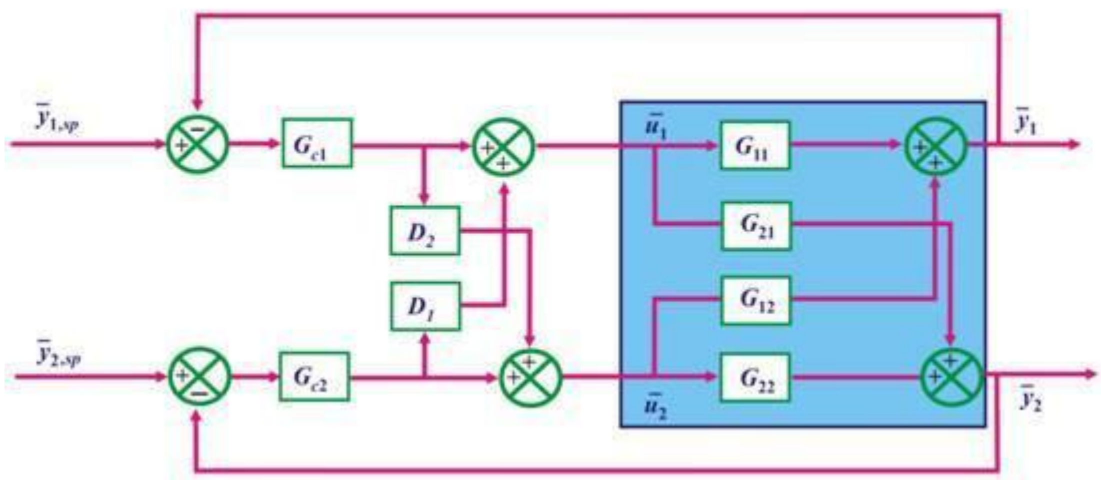


Fig. 3 Introduction of decouplers

Now the manipulated inputs of the renewed loops are:

$$\bar{u}_1 = G_{c1}\left(\bar{y}_{1,sp} - \bar{y}_1\right) + D_1 G_{c2}\left(\bar{y}_{2,sp} - \bar{y}_2\right) \tag{7}$$

$$\bar{u}_2 = G_{c2}\left(\bar{y}_{2,sp} - \bar{y}_2\right) + D_2 G_{c1}\left(\bar{y}_{1,sp} - \bar{y}_1\right) \tag{8}$$

Using eqs. (7) and (8), in eq. (1) one obtains,

$$\begin{aligned}
\bar{y}_1 &= G_{11} G_{c1}\left(\bar{y}_{1,sp} - \bar{y}_1\right) + G_{11} D_1 G_{c2}\left(\bar{y}_{2,sp} - \bar{y}_2\right) + G_{12} G_{c2}\left(\bar{y}_{2,sp} - \bar{y}_2\right) \\
&\quad + G_{12} D_2 G_{c1}\left(\bar{y}_{1,sp} - \bar{y}_1\right) \\
&= \left(G_{11} G_{c1} + G_{12} D_2 G_{c1}\right)\left(\bar{y}_{1,sp} - \bar{y}_1\right) + \left(G_{11} D_1 G_{c2} + G_{12} G_{c2}\right)\left(\bar{y}_{2,sp} - \bar{y}_2\right) \\
&= \left(G_{11} G_{c1} - G_{12} \frac{G_{21}}{G_{22}} G_{c1}\right)\left(\bar{y}_{1,sp} - \bar{y}_1\right) + \left(-G_{11} \frac{G_{12}}{G_{11}} G_{c2} + G_{12} G_{c2}\right)\left(\bar{y}_{2,sp} - \bar{y}\right. \\
&= G_{c1}\left(G_{11} - G_{12} \frac{G_{21}}{G_{22}}\right)\left(\bar{y}_{1,sp} - \bar{y}_1\right)
\end{aligned} \tag{9}$$

or rearranging the above equation we obtain,

$$\bar{y}_1 = \left\{ \frac{G_{c1}\left(G_{11} - G_{12} \frac{G_{21}}{G_{22}}\right)}{1 + G_{c1}\left(G_{11} - G_{12} \frac{G_{21}}{G_{22}}\right)} \right\} \bar{y}_{1,sp} \tag{10}$$

Similarly using eqs. (7) and (8), in eq. (2) one obtains,

$$\bar{y}_2 = \left\{ \frac{G_{c2}\left(G_{22} - G_{12} \frac{G_{21}}{G_{11}}\right)}{1 + G_{c2}\left(G_{22} - G_{12} \frac{G_{21}}{G_{11}}\right)} \right\} \bar{y}_{2,sp} \tag{11}$$

The eqs. (10) and (11) are the results of the following system

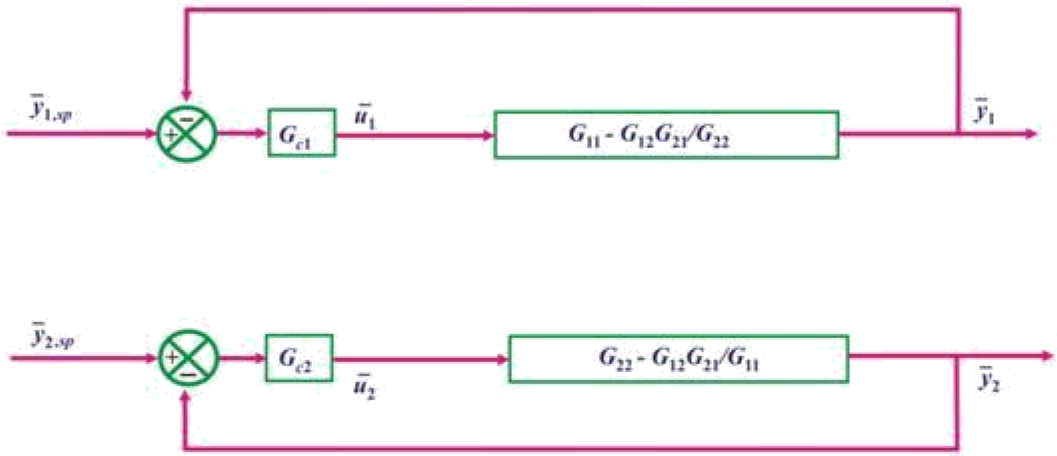


Fig. 4 Decoupled process

Where the 2 x 2 process loops are completely decoupled from each other. It should be noted that decouplers are essentially feedforward control elements. These elements are very much sensitive to operating conditions of the process. As the transfer functions are developed at the operating conditions, any substantial shift in the nominal operating region would deteriorate decoupling effect. Adaptive decouplers are useful for this purpose, however, they are beyond the scope of this course. Again process dead time plays an important role in assigning the decoupler transfer functions. In case the resultant decoupler transfer function obtains a time lead (rather than a time delay), it yields a physically unrealizable decoupler.

Brief Introduction to Distillation Control

All distillation columns have to be carefully operated in order to achieve the required production rates and product quality. The 3 main objectives of column control can be stated as:

To set stable conditions for column operation

To regulate conditions in the column so that the product(s) always meet the required specifications

To achieve the above objective most efficiently, e.g. by maximizing product yield, minimizing energy consumption, etc.

Distillation Control Philosophy

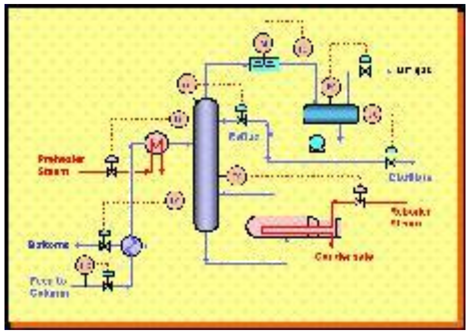
Some of the general guidelines are noted below:

1.Column pressure normally controlled at a constant value
2.Feed flow rate often set by the level controller on a preceding column
3.Feed flow rate is independently controlled if fed from storage tank or surge tank
4.Feed temperature controlled by a feed preheater. Prior to preheater, feed may be heated by bottom product via feed/bottom exchanger
5.Top temperature usually controlled by varying the reflux
6.Bottom temperature controlled by varying thesteam to reboiler
7.Differential pressure control used in packed columns to monitor packing condition, also used in tray columns to indicate foaming 8.The compositions controlled by regulating the reflux flow and boiled-up (reboiler vapour)

Pressure is often considered the prime distillation control variable, as it affects temperature, condensation, vapourisation, compositions, volatilities and almost any process that takes place inside the column. Column pressure control is frequently integrated with the condenser control system.Reboilers and condensers are integral part of a distillation system. They regulate the energy inflow and outflow in a distillation column.

An Example of Distillation Column Control

A typical distillation column has a combination of different control loops. The control system of a particular column is designed to meet that column's particular process requirements. An example is shown in the Figure below.



There are several control loops associated with the distillation column:

Temperature:

1. Overhead condensation (Fin-fan)
2. Overhead column (Reflux)
3. Feed preheat
4. Column bottom (Reboiler steam)

Pressure:

1. Overhead accumulator (Off gas)

Level:

1. Overhead accumulator (Distillate product)
2. Column bottom (Bottoms product)

Flow:

1. Column feed

HEAT EXCHANGERS

Heat exchangers transfer thermal energy between fluids. Although heat transfer is typically efficient, controlling the temperature of the fluid being heated at a specific and stable setpoint can be challenging. However, these challenges can be overcome by understanding heat exchanger control schemes implemented in industry.

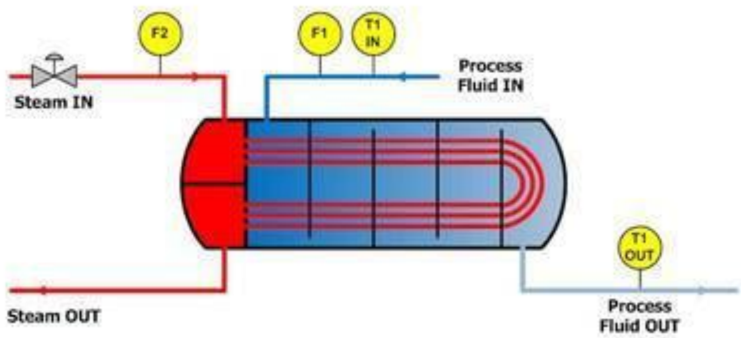
Shell-and-tube heat exchanger at a glance

By far, the shell-and-tube is the most common type of heat exchanger used in petrochemical industries because it is suitable for low and high pressure applications (see Figure 1). It consists of an outer shell with a bundle of tubes inside. The tubes are oriented in a straight or in a "U" shape. One fluid runs through the tubes, and another fluid flows through the shell surrounding the tubes to transfer heat between the two fluids (see Figure 2). The set of tubes is known as a "tube bundle."

Heat is transferred from one fluid to the other through the walls of the tubes.

Heat is transferred from the tube fluid to shell fluid to remove heat, or from the shell fluid to the tube fluid to heat the material inside. Fluids can be liquids or gases on either the shell or the tube side. To transfer heat efficiently, many tubes are used, which increases the heat-transfer surface area between the two fluids.

Control objective



To develop a comprehensive control strategy for any control loop, it's important to identify the process variable of interest—called the "controlled variable," the manipulated variable, and the different disturbance variables that directly affect the controlled variable.

Consider the heat exchanger shown in Figure 3. The shell side fluid is the process fluid that is required to be heated to a certain temperature setpoint. The resulting temperature is measured at the outlet of the heat exchanger $T1_{Out}$ (controlled variable).

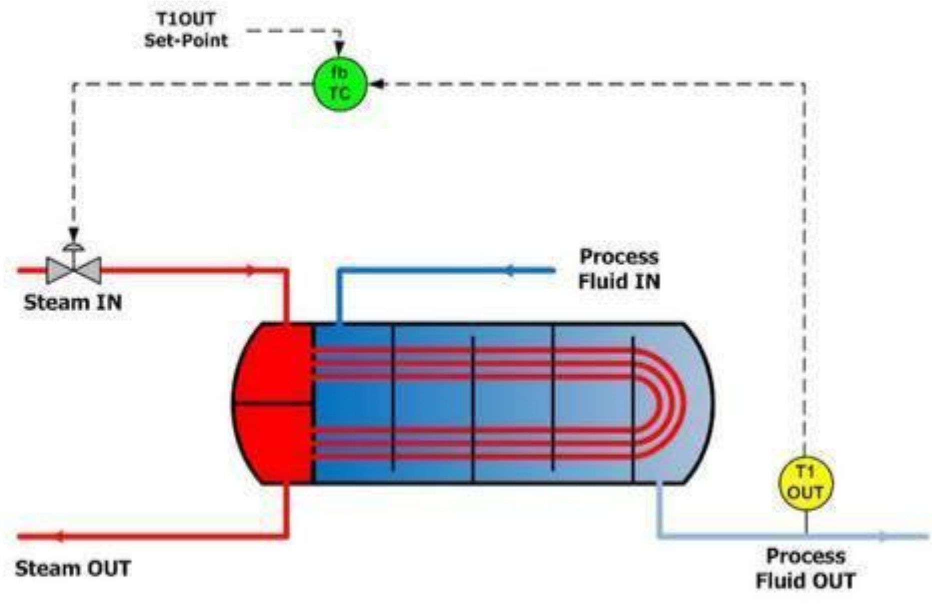
Heating is achieved by passing steam through the tube side. The more steam passing through the tubes, the more heat is transferred to the process fluid, and vice versa. Control of the steam flow F2 (manipulated variable) is achieved by throttling a modulating valve installed on the steam inlet side.

Three major disturbances can affect the process fluid outlet temperature:

- Changes in process fluid flow rate, F1
- Changes in process fluid inlet temperature, $T1_{In}$
- Changes in steam pressure, causing a change in steam flow rate, F2.

The control objective is to maintain process fluid outlet temperature $T1_{Out}$ at the desired setpoint—regardless of disturbances—by manipulating the steam flow rate F2.

Feedback control

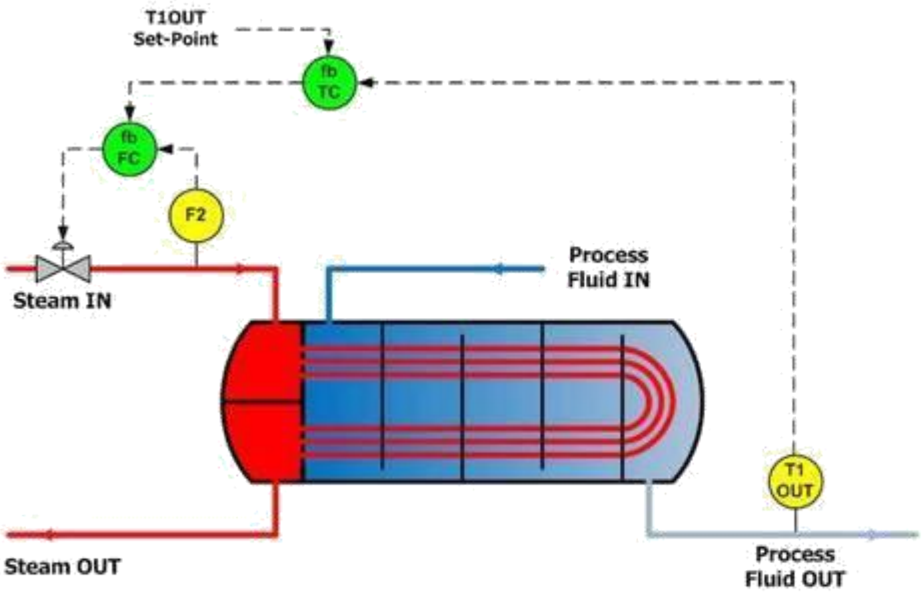


In the feedback control scheme, the process variable, $T1_{Out}$, is measured and applied to a proportional-integral-derivative (PID)-based feedback temperature controller (fbTC), which compares the process variable with the desired temperature setpoint and in turn calculates and generates the control action required to open or close the steam control valve (see Figure 4).

The most important advantage of the feedback control scheme is that regardless of the disturbance source, the controller will take corrective action. Employing feedback control requires very little knowledge of the process. Therefore, a process model is not necessary to set up and tune the feedback scheme, although it would be an advantage.

The major disadvantage of feedback control is its incapability to respond to disturbances—even major ones—until the controlled variable is already affected. Also, if too many disturbances occur with significant magnitude, they can create unrecoverable process instability.

Cascade control



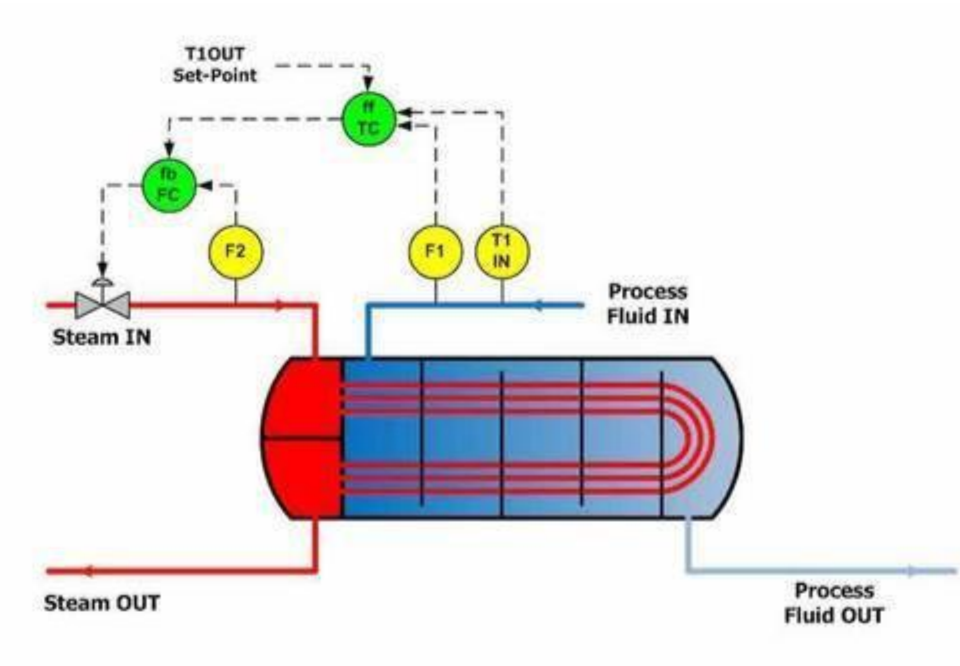
In the cascade control scheme, instead of feeding the output of the PID temperature controller directly to the control valve, it is fed as a setpoint to a feedback PID-based, steam-flow controller (fbFC). This second loop is responsible for ensuring the flow rate of the steam doesn't change due to uncontrollable factors, such as steam pressure changes or valve problems.

To understand how this works, consider that the heat exchanger is in steady-state operation, the outlet temperature matches the setpoint, and the controller output of fbTC is constant. A sudden increase in steam pressure will cause steam flow rate F2 to ramp up (see Figure 5). This will cause a gradual change in the controlled variable. Without the flow control loop, fbTC will not take corrective action until the outlet temperature is already affected.

By implementing the cascade strategy, the feedback flow control loop fbFC will adjust the valve position immediately when the steam flow rate has changed to bring the flow back to the value of the previous steady-state condition (because the flow setpoint given by the temperature controller didn't change as the outlet temperature did not yet change), preventing a change in the outlet temperature before it happens.

Note that the flow control loop must be tuned to run much faster than the temperature control loop, therefore cancelling the effect of flow variance before it affects the process fluid outlet temperature.

Feedforward control



Unlike feedback control, feedforward takes a corrective action when a disturbance occurs. Feedforward control doesn't see the process variable. It sees only the disturbances and responds to them as they occur. This enables a feedforward controller to quickly and directly compensate for the effect of a disturbance (see Figure 6).

To implement feedforward control, an understanding of the process model and the direct relationship between disturbances and the process variables is necessary. For heat exchangers, a derivation from the steady-state model will lead to the following equation, which determines the amount of steam flow required:

$F2sp = F1 \times (T1_{OUT}sp - T1_{IN}) \times (Cp/\Delta H)$

Where:

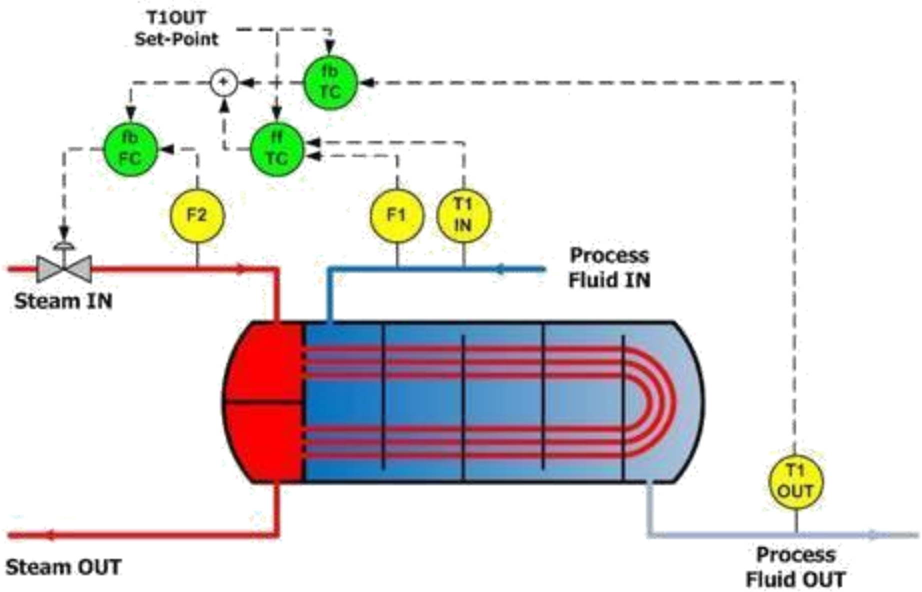
- *F2sp* = steam flow rate calculated setpoint to be applied to fbFC

- $F1$ = process fluid flow rate measured disturbance
- $T1_{OUT}sp$ = process fluid temperature setpoint at the heat exchanger outlet
- $T1_{IN}$= process fluid inlet temperature measured disturbance
- Cp = process fluid specific heat (known)
- ΔH = latent heat of vaporization for steam (known).

Applying this equation to calculate the required steam flow rate is sufficient to cancel the effects of changes of the process fluid flow rate and temperature. In a perfect world with few enhancements to the process model, this feedforward controller is enough to perfectly control the process. Unfortunately, it's not a perfect world.

The obvious advantage of using feedforward control is that it takes corrective action before the process is upset. A disadvantage is that it mandates a high initial capital cost because every disturbance must be measured, increasing the number of instruments and the associated engineering costs. In addition, this approach requires deeper knowledge of the process. It's not always realistic to depend on feedforward control only without taking into account the measured process variable.

Integrated approach



An integrated approach that uses feedback, feedforward, and cascade control is shown in Figure 7. This approach is more than capable of accommodating heat exchanger control requirements:

- A feedforward loop will handle major disturbances in the process fluid
- A cascaded-flow control loop will handle issues related to steam pressure and valve problems
- A feedback loop will handle everything else.

Combining the three techniques to optimize heat exchanger temperature control is necessary to minimize process variance, maximize product quality, and ensure energy efficiency in petrochemical industries.