BEE304

ELECTRO MAGNETIC THEORY

2^{ND} YEAR 3^{RD} SEM

UNIT I

ELECTROSTATICS

ELECTROSTATICS :

Study of Electricity in which electric charges are static i.e. not moving, is called electrostatics • STATIC CLING • An electrical phenomenon that accompanies dry weather, causes these pieces of papers to stick to one another and to the plastic comb. • Due to this reason our clothes stick to our body. • ELECTRIC CHARGE : Electric charge is characteristic developed in particle of material due to which it exert force on other such particles. It automatically accompanies the particle wherever it goes. • Charge cannot exist without material carrying it • It is possible to develop the charge by rubbing two solids having friction. • Carrying the charges is called electrification. • Electrification due to friction is called frictional electricity. Since these charges are not flowing it is also called static electricity. There are two types of charges. +ve and –ve. • Similar charges repel each other, • Opposite charges attract each other. • Benjamin Franklin made this nomenclature of charges being +ve and –ve for mathematical calculations because adding them together cancel each other. • Any particle has vast amount of charges. • The number of positive and negative charges are equal, hence matter is basically neutral. • Inequality of charges give the material a net charge which is equal to the difference of the two type of charges.

Electrostatic series :If two substances are rubbed together the former in series acquires the positive charge and later, the –ve. (i) Glass (ii) Flannel (iii) Wool (iv) Silk (v) Hard Metal (vi) Hard rubber (vii) Sealing wax (viii) Resin (ix) Sulphur Electron theory of Electrification • Nucleus of atom is positively charged. • The electron revolving around it is negatively charged. • They are equal in numbers, hence atom is electrically neutral. • With friction there is transfer of electrons, hence net charge is developed in the particles. • It also explains that the charges are compulsorily developed in pairs equally . +vein one body and –ve in second. • It establish conservation of charges in the universe. • The loss of electrons develops +ve charge. While excess of electrons develop –ve charge • A proton is 1837 times heavier than electron hence it cannot be transferred. Transferring lighter electron is easier. • Therefore for electrification of matter, only electrons are active and responsible. Charge and Mass relation • Charge cannot exist without matter. • One carrier of charge is electron which has mass as well. • Hence if there is charge transfer, mass is also transferred. • Logically, negatively charged body is heavier then positively charged body. Conductors, Insulators and Semiconductors • Conductors : Material in which electrons can move easily and freely. Ex. Metals, Tap water, human body. Brass rod in our hand, if charged by rubbing the charge will move easily to earth. Hence Brass is a conductor. The flow of this excess charge is called discharging • Insulator : Material in which charge cannot move freely. Ex . Glass, pure water, plastic etc.

It has been found experimentally that in classical electrostatics the interaction between stationary, electrically charged bodies can be described in terms of a mechanical force. Let us consider the simple case described by Figure 1.1 on page 3. Let $\mathbf{F}$ denote the force acting on a electrically charged particle with charge q located at $\mathbf{x}$, due to the presence of a charge q' located at $\mathbf{x}'$. According to *Coulomb's law* this force is, in vacuum, given by the expression

$$\mathbf{F}(\mathbf{x}) = \frac{qq'}{4\pi\varepsilon_0}\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3} = -\frac{qq'}{4\pi\varepsilon_0}\boldsymbol{\nabla}\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right) = \frac{qq'}{4\pi\varepsilon_0}\boldsymbol{\nabla}'\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right)$$

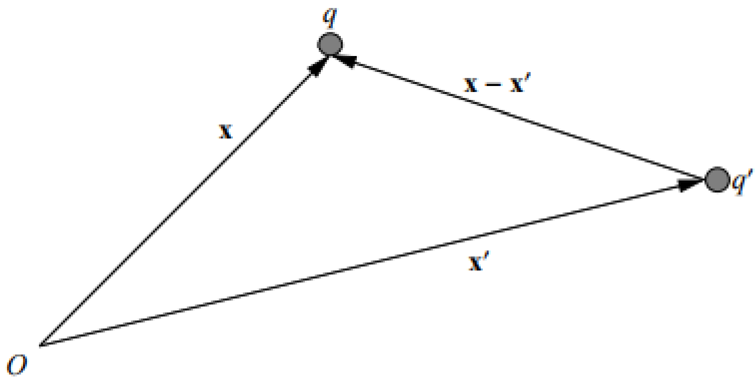


FIGURE 1.1: Coulomb's law describes how a static electric charge q, located at a point $\mathbf{x}$ relative to the origin O, experiences an electrostatic force from a static electric charge q' located at $\mathbf{x}'$.

1.1.2 The electrostatic field

Instead of describing the electrostatic interaction in terms of a 'force action at a distance', it turns out that it is for most purposes more useful to introduce the concept of a field and to describe the electrostatic interaction in terms of a static vectorial *electric field* $\mathbf{E}^{\text{stat}}$ defined by the limiting process

$$\mathbf{E}^{\text{stat}} \stackrel{\text{def}}{\equiv} \lim_{q\to 0}\frac{\mathbf{F}}{q} \tag{1.2}$$

where $\mathbf{F}$ is the electrostatic force, as defined in Equation (1.1) on the preceding page, from a net electric charge q' on the test particle with a small electric net electric charge q. Since the purpose of the limiting process is to assure that the test charge q does not distort the field set up by q', the expression for $\mathbf{E}^{\text{stat}}$ does not depend explicitly on q but only on the charge q' and the relative radius vector $\mathbf{x}-\mathbf{x}'$. This means that we can say that any net electric charge produces an electric field in the space that surrounds it, regardless of the existence of a second charge anywhere in this space.[2]

Using (1.1) and Equation (1.2) on the preceding page, and Formula (F.70) on page 160, we find that the electrostatic field $\mathbf{E}^{\text{stat}}$ at the *field point* $\mathbf{x}$ (also known as the *observation point*), due to a field-producing electric charge q' at the *source point* $\mathbf{x}'$, is given by

$$\mathbf{E}^{\text{stat}}(\mathbf{x}) = \frac{q'}{4\pi\varepsilon_0} \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} = -\frac{q'}{4\pi\varepsilon_0} \boldsymbol{\nabla} \left(\frac{1}{|\mathbf{x} - \mathbf{x}'|} \right) = \frac{q'}{4\pi\varepsilon_0} \boldsymbol{\nabla}' \left(\frac{1}{|\mathbf{x} - \mathbf{x}'|} \right) \tag{1.3}$$

In the presence of several field producing discrete electric charges q'_i, located at the points $\mathbf{x}'_i$, $i = 1, 2, 3, \ldots$, respectively, in an otherwise empty space, the assumption of linearity of vacuum[3] allows us to superimpose their individual electrostatic fields into a total electrostatic field

$$\mathbf{E}^{\text{stat}}(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0} \sum_i q'_i \frac{\mathbf{x} - \mathbf{x}'_i}{\left|\mathbf{x} - \mathbf{x}'_i\right|^3} \tag{1.4}$$

If the discrete electric charges are small and numerous enough, we introduce the *electric charge density* ρ, measured in $\mathrm{C/m^3}$ in SI units, located at $\mathbf{x}'$ within a volume V' of limited extent and replace summation with integration over this volume. This allows us to describe the total field as

$$\begin{aligned} \mathbf{E}^{\text{stat}}(\mathbf{x}) &= \frac{1}{4\pi\varepsilon_0} \int_{V'} \mathrm{d}^3x'\, \rho(\mathbf{x}') \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} = -\frac{1}{4\pi\varepsilon_0} \int_{V'} \mathrm{d}^3x'\, \rho(\mathbf{x}') \boldsymbol{\nabla} \left(\frac{1}{|\mathbf{x} - \mathbf{x}'|} \right) \\ &= -\frac{1}{4\pi\varepsilon_0} \boldsymbol{\nabla} \int_{V'} \mathrm{d}^3x'\, \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \end{aligned} \tag{1.5}$$

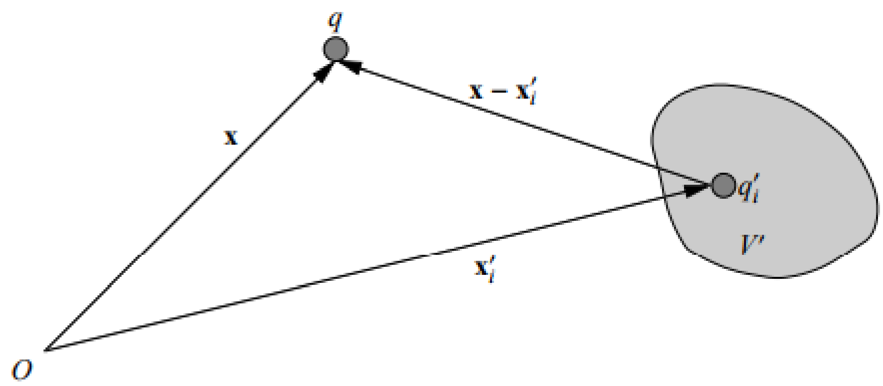
where we used Formula (F.70) on page 160 and the fact that $\rho(\mathbf{x}')$ does not depend on the unprimed (field point) coordinates on which $\boldsymbol{\nabla}$ operates.

We emphasise that under the assumption of linear superposition, Equation (1.5) above is valid for an arbitrary distribution of electric charges, including discrete charges, in which case ρ is expressed in terms of Dirac delta distributions:

$$\rho(\mathbf{x}') = \sum_i q'_i\, \delta(\mathbf{x}' - \mathbf{x}'_i) \tag{1.6}$$

as illustrated in Figure 1.2 on the facing page. Inserting this expression into expression (1.5) above we recover expression (1.4).

Taking the divergence of the general $\mathbf{E}^{\text{stat}}$ expression for an arbitrary electric charge distribution, Equation (1.5) above, and using the representation of the Dirac



Coulomb's law for a distribution of individual charges $\mathbf{x}'_i$ localised within a volume V' of limited extent.

delta distribution, Formula (F.73) on page 161, we find that

$$\begin{aligned}\nabla \cdot \mathbf{E}^{\text{stat}}(\mathbf{x}) &= \nabla \cdot \frac{1}{4\pi\varepsilon_0} \int_{V'} \mathrm{d}^3x'\, \rho(\mathbf{x}') \frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3} \\ &= -\frac{1}{4\pi\varepsilon_0} \int_{V'} \mathrm{d}^3x'\, \rho(\mathbf{x}') \nabla \cdot \nabla \left(\frac{1}{|\mathbf{x}-\mathbf{x}'|} \right) \\ &= -\frac{1}{4\pi\varepsilon_0} \int_{V'} \mathrm{d}^3x'\, \rho(\mathbf{x}')\, \nabla^2 \left(\frac{1}{|\mathbf{x}-\mathbf{x}'|} \right) \\ &= \frac{1}{\varepsilon_0} \int_{V'} \mathrm{d}^3x'\, \rho(\mathbf{x}')\, \delta(\mathbf{x}-\mathbf{x}') = \frac{\rho(\mathbf{x})}{\varepsilon_0}\end{aligned} \tag{1.7}$$

which is the differential form of *Gauss's law of electrostatics*.

Since, according to Formula (F.62) on page 160, $\nabla \times [\nabla \alpha(\mathbf{x})] \equiv \mathbf{0}$ for any 3D $\mathbb{R}^3$ scalar field $\alpha(\mathbf{x})$, we immediately find that in electrostatics

$$\nabla \times \mathbf{E}^{\text{stat}}(\mathbf{x}) = -\frac{1}{4\pi\varepsilon_0} \nabla \times \left(\nabla \int_{V'} \mathrm{d}^3x'\, \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} \right) = \mathbf{0} \tag{1.8}$$

i.e., that $\mathbf{E}^{\text{stat}}$ is an *irrotational* field.

To summarise, electrostatics can be described in terms of two vector partial differential equations

$$\nabla \cdot \mathbf{E}^{\text{stat}}(\mathbf{x}) = \frac{\rho(\mathbf{x})}{\varepsilon_0} \tag{1.9a}$$

$$\nabla \times \mathbf{E}^{\text{stat}}(\mathbf{x}) = \mathbf{0} \tag{1.9b}$$

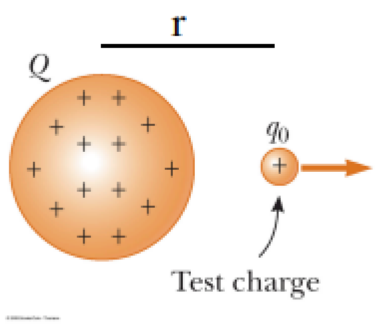
representing four scalar partial differential equations.

Electric Field due to a point charge

E-field exerts a force on other point charges

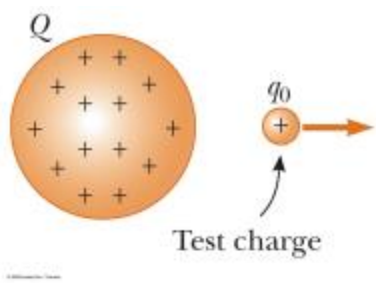
$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{\frac{k_e Q q_o}{r^2}}{q_0}$$

$$\vec{E} = \frac{k_e Q}{r^2}$$



$\vec{E}$ is a vector quantity

Magnitude & direction vary with position--but depend on object w/ charge Q setting up the field



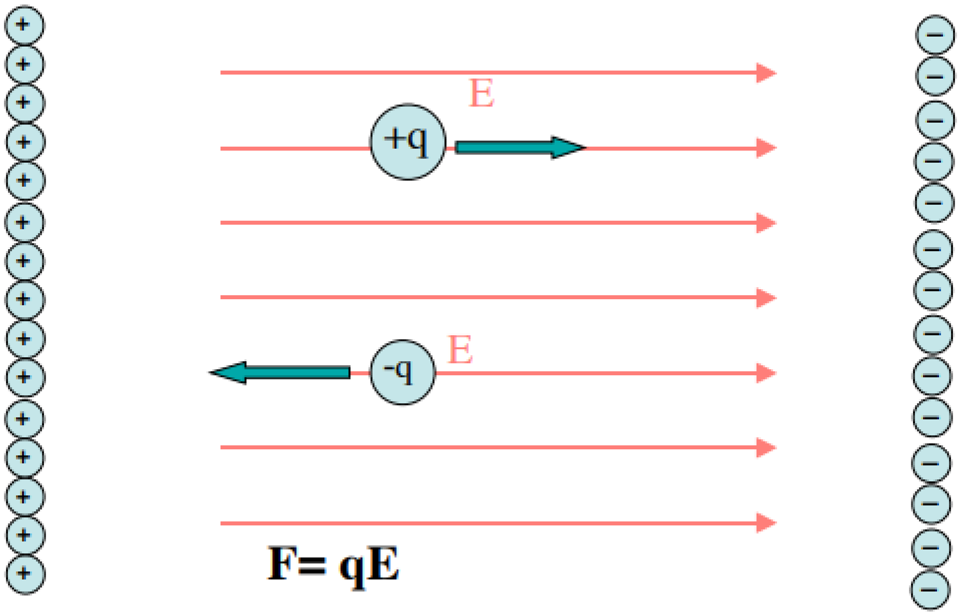
The electric field depends on Q, not q_0. It also depends on r.

If you replace q_0 with $-q_0$ or $2q_0$, the strength & magnitude of the E-field at that point in space remain the same

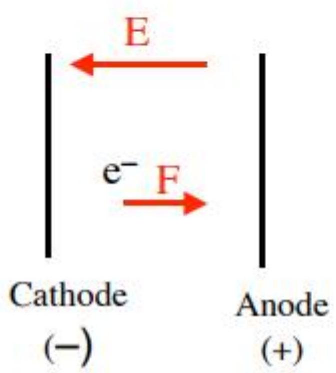
The electrostatic FORCE, however, depends on Q AND q_0 as well as r.

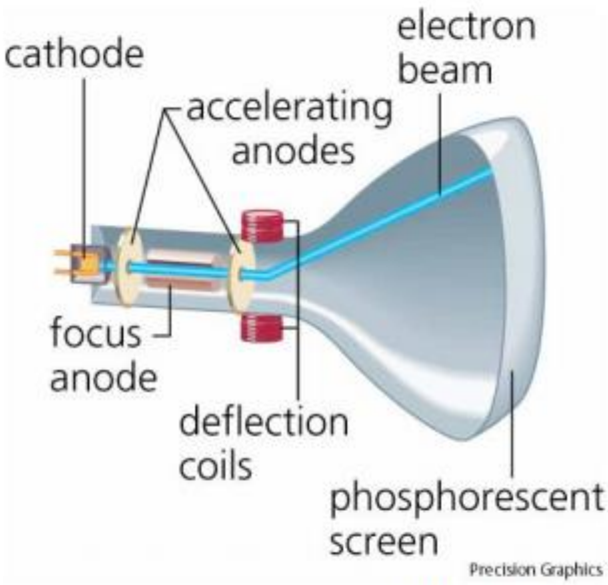
E-field exerts force on a charge

Consider an array of + charges and an array of – charges:



Cathode Ray Tube





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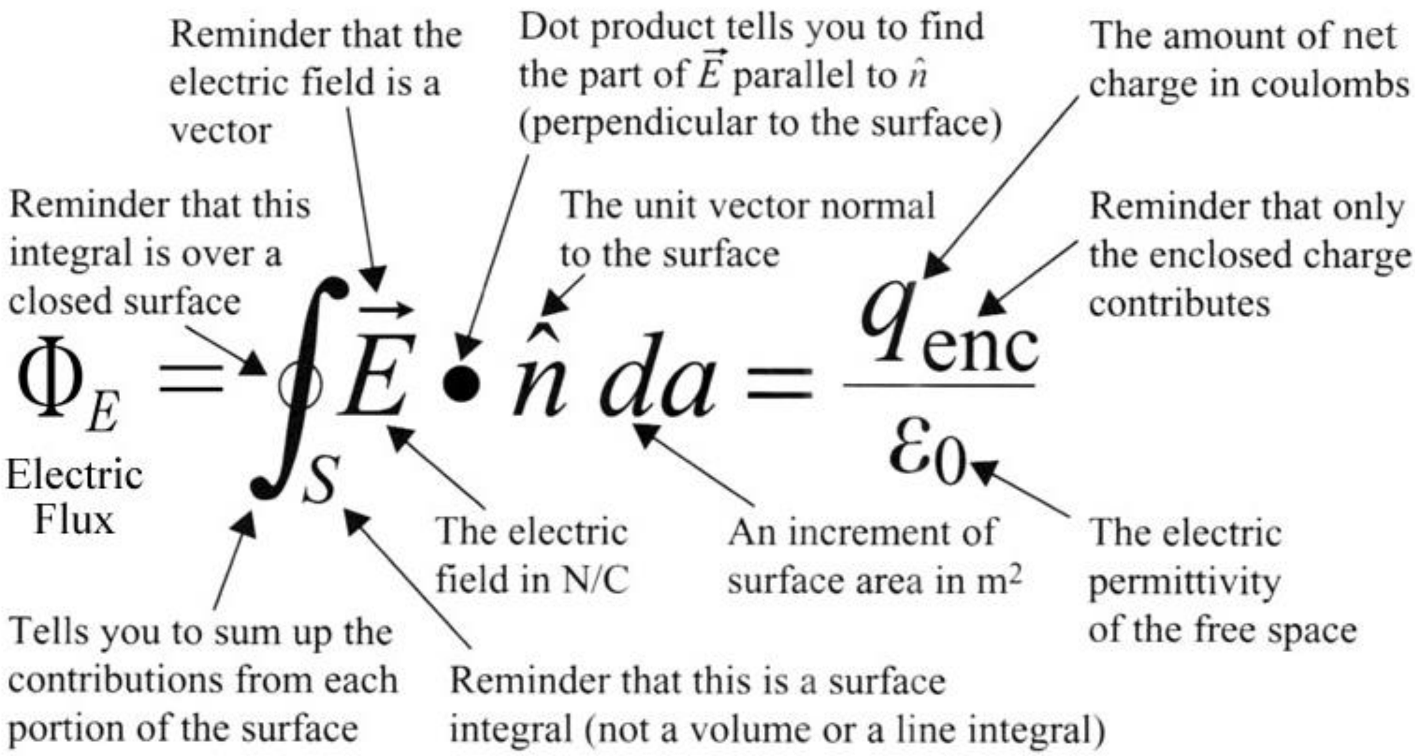
Gauss's Law.

The total of the electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity. The electric flux through an area is defined as the electric field multiplied by the area of the surface projected in a plane perpendicular to the field.

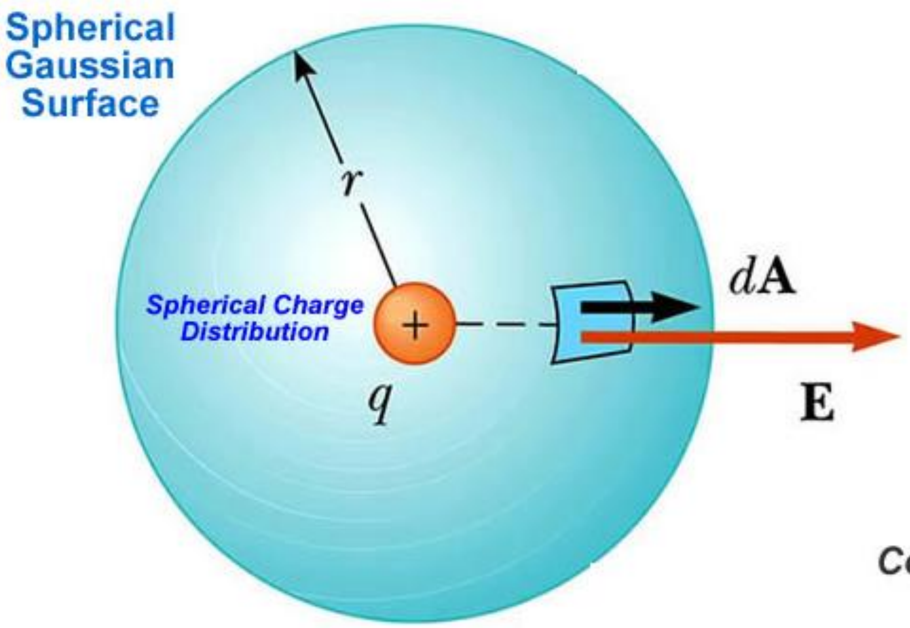


Gauss's Law to Coulomb's Law

E is constant everwhere on the surface

Electric Flux $$\Phi_E = \oint E \cdot dA = \oint E\, dA = E \oint dA = \frac{q_{in}}{\varepsilon_o}$$

E and dA are parallel everywhere on the surface



$$E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

surface area of a sphere

$$E = \frac{q_{in}}{4\pi\varepsilon_o r^2}$$

$$k = \frac{1}{4\pi\varepsilon_o}$$

Coulomb's Law

$$E = \frac{kq_{in}}{r^2}$$

the net flux through any closed surface surrounding a point charge q is given by q/ε_o and its independent of the shape of that surface

Divergence Theorem

The divergence theorem, more commonly known especially in older literature as Gauss's theorem (e.g., Arfken 1985) and also known as the Gauss-Ostrogradsky theorem, is a theorem in vector calculus that can be stated as follows. Let V be a region in space with boundary ∂V. Then the volume integral of the divergence $\nabla \cdot \mathbf{F}$ of $\mathbf{F}$ over V and the surface integral of $\mathbf{F}$ over the boundary ∂V of V are related by

$$\int_V (\nabla \cdot \mathbf{F})\, d V = \int_{\partial V} \mathbf{F} \cdot d\, \mathbf{a}. \qquad (1)$$

The divergence theorem is a mathematical statement of the physical fact that, in the absence of the creation or destruction of matter, the density within a region of space can change only by having it flow into or away from the region through its boundary.

A special case of the divergence theorem follows by specializing to the plane. Letting S be a region in the plane with boundary ∂S, equation (1) then collapses to

$$\int_S \nabla \cdot \mathbf{F}\, d A = \int_{\partial S} \mathbf{F} \cdot \hat{\mathbf{n}}\, d s. \qquad (2)$$

If the vector field $\mathbf{F}$ satisfies certain constraints, simplified forms can be used. For example, if $\mathbf{F}(x, y, z) = v(x, y, z)\, \mathbf{c}$ where $\mathbf{c}$ is a constant vector $\neq \mathbf{0}$, then

$$\int_S \mathbf{F} \cdot d\, \mathbf{a} = \mathbf{c} \cdot \int_S v\, d\, \mathbf{a}. \qquad (3)$$

But

But

$$\nabla \cdot (f\, \mathbf{v}) = (\nabla f) \cdot \mathbf{v} + f\, (\nabla \cdot \mathbf{v}),$$

so

$$\begin{aligned} \int_V \nabla \cdot (\mathbf{c}\, v)\, d V &= \int_V [(\nabla v) \cdot \mathbf{c} + v\, \nabla \cdot \mathbf{c}]\, d V \\ &= \mathbf{c} \cdot \int_V \nabla v\, d V \end{aligned}$$

and

$$\mathbf{c} \cdot \left(\int_S v\, d\, \mathbf{a} - \int_V \nabla v\, d V \right) = 0.$$

But $\mathbf{c} \neq \mathbf{0}$, and $\mathbf{c} \cdot \mathbf{f}(v)$ must vary with v so that $\mathbf{c} \cdot \mathbf{f}(v)$ cannot always equal zero. Therefore,

$$\int_S v\, d\, \mathbf{a} = \int_V \nabla v\, d V.$$

Similarly, if $\mathbf{F}(x, y, z) = \mathbf{c} \times P(x, y, z)$, where $\mathbf{c}$ is a constant vector $\neq \mathbf{0}$, then

$$\int_S d\, \mathbf{a} \times \mathbf{P} = \int_V \nabla \times \mathbf{P}\, d V.$$

Poisson's and laplace equations

A study of the previous chapter shows that several of the analogies used to obtain experimental field maps involved demonstrating that the analogous quantity satisfies Laplace's equation. This is true for small deflections of an elastic membrane, and we might have proved the current analogy by showing that the direct-current density in a conducting medium also satisfies Laplace's equation. It appears that this is a fundamental equation in more than one field of science, and, perhaps without knowing it, we have spent the last chapter obtaining solutions for Laplace's equation by experimental, graphical, and numerical methods. Now we are ready to obtain this equation formally and discuss several methods by which it may be solved analytically.

It may seem that this material properly belongs before that of the previous chapter; as long as we are solving one equation by so many methods, would it not be fitting to see the equation first? The disadvantage of this more logical order lies in the fact that solving Laplace's equation is an exercise in mathematics, and unless we have the physical problem well in mind, we may easily miss the physical significance of what we are doing. A rough curvilinear map can tell us much about a field and then may be used later to check our mathematical solutions for gross errors or to indicate certain peculiar regions in the field which require special treatment.

With this explanation let us finally obtain the equations of Laplace and Poisson.

7.1 POISSON'S AND LAPLACE'S EQUATIONS

Obtaining Poisson's equation is exceedingly simple, for from the point form of Gauss's law,

$$\nabla \cdot \mathbf{D} = \rho_v \tag{1}$$

the definition of **D**,

$$\mathbf{D} = \epsilon \mathbf{E} \tag{2}$$

and the gradient relationship,

$$\mathbf{E} = -\nabla V \tag{3}$$

by substitution we have

$$\nabla \cdot \mathbf{D} = \nabla \cdot (\epsilon \mathbf{E}) = -\nabla \cdot (\epsilon \nabla V) = \rho_v$$

or

$$\nabla \cdot \nabla V = -\frac{\rho_v}{\epsilon} \tag{4}$$

for a homogeneous region in which ϵ is constant.

Equation (4) is *Poisson's equation*, but the "double ∇" operation must be interpreted and expanded, at least in cartesian coordinates, before the equation can be useful. In cartesian coordinates,

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

and therefore

$$\nabla \cdot \nabla V = \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}\right)$$

$$= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \qquad (5)$$

Usually the operation $\nabla \cdot \nabla$ is abbreviated ∇^2 (and pronounced "del squared"), a good reminder of the second-order partial derivatives appearing in (5), and we have

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\epsilon} \tag{6}$$

in cartesian coordinates.

If $\rho_v = 0$, indicating zero *volume* charge density, but allowing point charges, line charge, and surface charge density to exist at singular locations as sources of the field, then

$$\nabla^2 V = 0 \tag{7}$$

which is *Laplace's equation*. The ∇^2 operation is called the *Laplacian of V*.

In cartesian coordinates Laplace's equation is

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \qquad \text{(cartesian)} \tag{8}$$

and the form of $\nabla^2 V$ in cylindrical and spherical coordinates may be obtained by using the expressions for the divergence and gradient already obtained in those coordinate systems. For reference, the Laplacian in cylindrical coordinates is

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2} \qquad \text{(cylindrical)} \tag{9}$$

and in spherical coordinates is

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} \qquad \text{(spherical)} \tag{10}$$

These equations may be expanded by taking the indicated partial derivatives, but it is usually more helpful to have them in the forms given above; furthermore, it is much easier to expand them later if necessary than it is to put the broken pieces back together again.

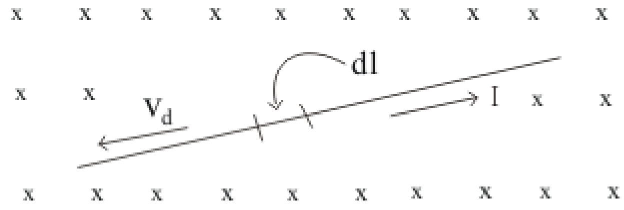
Laplace's equation is all-embracing, for, applying as it does wherever volume charge density is zero, it states that every conceivable configuration of electrodes or conductors produces a field for which $\nabla^2 V = 0$. All these fields are different, with different potential values and different spatial rates of change, yet for each of them $\nabla^2 V = 0$. Since *every* field (if $\rho_v = 0$) satisfies Laplace's equation, how can we expect to reverse the procedure and use Laplace's equation to find one specific field in which we happen to have an interest? Obviously, more

UNIT III

MAGNETOSTATICS

Force on a current element

- We know that current flowing in a conductor is nothing but the drift of free electron's from lower potential end of the conductor to the higher potential end
- when a current carrying conductor is placed in a magnetic field ,magnetic forces are exerted on the moving charges with in the conductor
- Equation -1 which gives force on a moving charge in a magnetic field can also be used for calculating the magnetic force exerted by magnetic field on a current carrying conductor (or wire)
- Let us consider a straight conducting wire carrying current I is placed in a magnetic field B(x).Consider a small element dl of the qire as shown below in the figure



or dF=I(dl X B)

Drift velocity of electrons in a conductor and current I flowing in the conductor is given by $I=neAv_d$

Where A is the area of cross-section of the wire and n is the number of free electrons per unit volume

Magnetic force experienced by each electron in presence of magnetic field is

$\mathbf{F}=e(\mathbf{v_d} \times \mathbf{B})$

where e is the amount of charge on an electron

Total number of electron in length dl of the wire

N=nAdl

Thus magnetic force on wire of length dl is

$\mathbf{dF}=(nAdl)(e\mathbf{v_d} \times \mathbf{B})$

if we denote length dl along the direction of current by the vector **dl** the above equation becomes

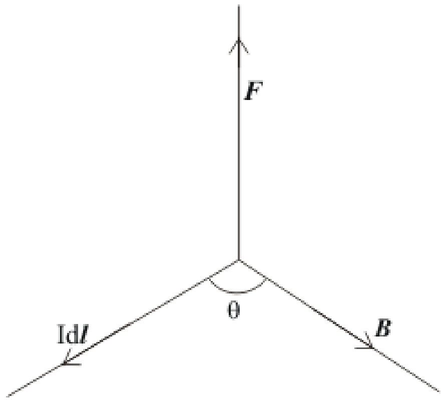
$\mathbf{dF}=(nAev_d)(\mathbf{dl} \times \mathbf{B})$

or dF=I(dl X B)

If a straight wire of length l carrying current I is placed in a uniform magnetic field then force on wire would be equal to

dF=I(L X B)

- Direction of force is always perpendicular to the plane containing the current element I**dL** and magnetic field **B**



Direction of force when current element IdL and B are perpendicular to each other can also be find using either of the following rules

Biot Savart's law:

Biot- Savarts' Law

Biot-Savarts' law provides an expression for the magnetic field due to a current segment. The field $d\vec{B}$ at a position $\vec{r}$ due to a current segment $I\vec{dl}$ is experimentally found to be perpendicular to $\vec{dl}$ and $\vec{r}$. The magnitude of the field is proportional to the length $|dl|$ and to the current I and to the sine of the angle between $\vec{r}$ and $\vec{dl}$.

inversely proportional to the square of the distance r of the point P from the current element.
Mathematically,

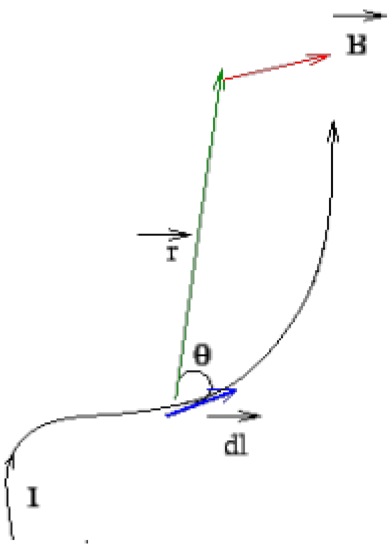
$$d\vec{B} \propto I\frac{\vec{dl} \times \hat{r}}{r^2}$$

In SI units the constant of proportionality is $\mu_0/4\pi$, where μ_0 is the permeability of the free space. The value of μ_0 is

$$\mu_0 = 4\pi \times 10^{-7}\ \text{N/amp}^2$$

The expression for field at a point P having a position vector $\vec{r}$ with respect to the current element is

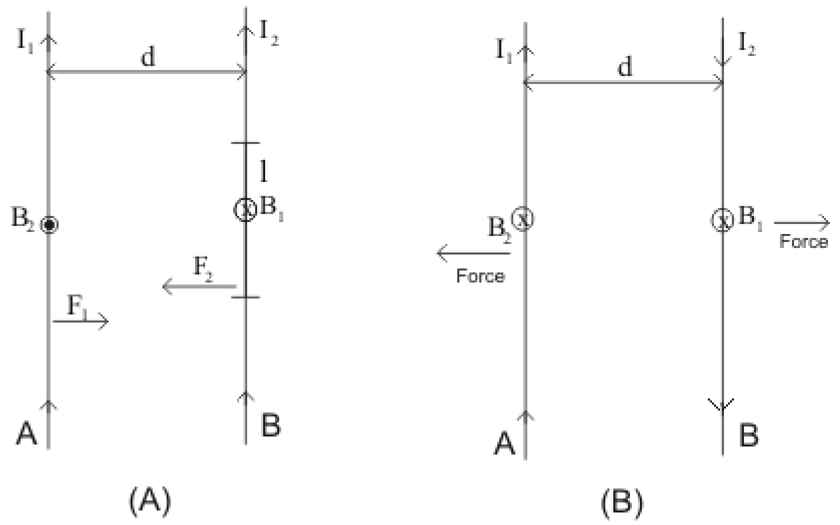
$$d\vec{B} = \frac{\mu_0}{4\pi}I\frac{\vec{dl} \times \hat{r}}{r^2}$$



For a conducting wire of arbitrary shape, the field is obtained by vectorially adding the contributions due to such current elements as per superposition principle, $\vec{B}(P) = \frac{\mu_0}{4\pi} I \int \frac{\vec{dl} \times \hat{r}}{r^2}$ where the integration is along the path of the current flow.

Force Between Current Carrying Conductors

- It is experimentally established fact that two current carrying conductors attract each other when the current is in same direction and repel each other when the current are in opposite direction
- Figure below shows two long parallel wires separated by distance d and carrying currents I_1 and I_2



Consider fig 5(a) wire A will produce a field B_1 at all near by points .The magnitude of B_1 due to current I_1 at a distance d i.e. on wire b is

$B_1=\mu_0 I_1/2\pi d$

- According to the right hand rule the direction of B_1 is in downward as shown in figure (5a)
- Consider length l of wire B and the force experienced by it will be (I_2lXB) whose magnitude is

$$F_2 = I_2 lB = \frac{\mu_0}{2\pi}\frac{lI_1I_2}{d}$$

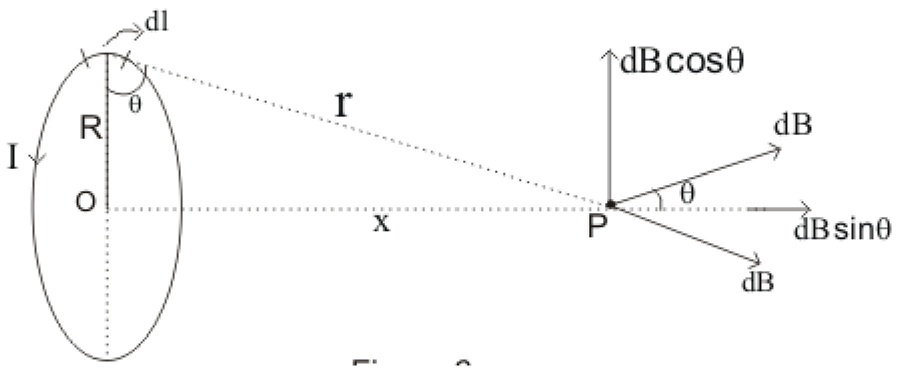
Similarly force per unit length of A due to current in B is

$$\frac{F_1}{l} = \frac{\mu_0}{2\pi}\frac{I_1I_2}{d}$$

- and is directed opposite to the force on B due to A. Thus the force on either conductor is proportional to the product of the current
- We can now make a conclusion that the conductors attract each other if the currents are in the same direction and repel each other if currents are in opposite direction

iii) Magnetic Field along axis of a circular current carrying coil

- Let there be a circular coil of radius R and carrying current I. Let P be any point on the axis of a coil at a distance x from the center and which we have to find the field
- To calculate the field consider a current element Idl at the top of the coil pointing perpendicular towards the reader
- Current element Idl and r is the vector joining current element and point P as shown below in the figure



From Biot Savart law, the magnitude of the magnetic field due to this current element at P is

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2}$$

- where Φ is the angle between the length element dl and r
- Since Idl and r are perpendicular to each other so Φ=90.Therefore

$$dB = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}$$

- Resolving dB into two components we have dBsinθ along the axis of the loop and another one is dBcosθ at right angles to the x-axis
- Since coil is symmetrical about x-axis the contribution dB due to the element on opposite side (along -y axis) will be equal in magnitude but opposite in direction and cancel out. Thus we only have dBsinθ component
- The resultant B for the complete loop is given by,

$$B=\int dB$$

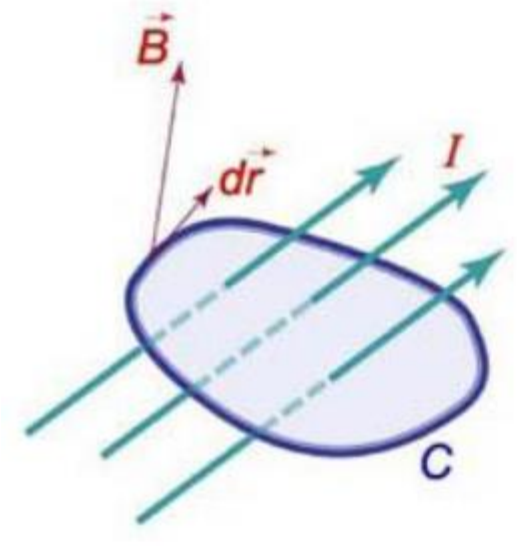
$$B = \int \frac{\mu_0}{4\pi} \frac{Idl \sin\theta}{r^2}$$

Ampere's law

Ampere's Law states that for any closed loop path, the sum of the length elements times the magnetic field in the direction of the length element is equal to the permeability times the electric current enclosed in the loop.

$$\oint \vec{B}.\overrightarrow{ds} = \mu_0 I$$

$$\oint B_{||}.ds = \mu_0 I$$



Important Notes

• In order to apply Ampère's Law all currents have to be steady (i.e. do not change with time)

• Only currents crossing the area inside the path are taken into account and have some contribution to the magnetic field

• Currents have to be taken with their algebraic signs (those going "out" of the surface are positive, those going "in" are negative)- use right hand's rule to determine directions and signs

Magnetic potential

The term **magnetic potential** can be used for either of two quantities in classical electromagnetism: the *magnetic vector potential*, **A**, (often simply called the *vector potential*) and the *magnetic scalar potential*, ψ. Both quantities can be used in certain circumstances to calculate the magnetic field.

The more frequently used magnetic vector potential, **A**, is defined such that the curl of **A** is the magnetic field **B**. Together with the electric potential, the magnetic vector potential can be used to specify the electric field, **E** as well. Therefore, many equations of electromagnetism can be written either in terms of the **E** and **B**, *or* in terms of the magnetic vector potential and electric potential. In more advanced theories such as quantum mechanics, most equations use the potentials and not the **E** and **B** fields.

The magnetic scalar potential ψ is sometimes used to specify the magnetic **H**-field in cases when there are no free currents, in a manner analogous to using the electric potential to determine the electric field in electrostatics. One important use of ψ is to determine the magnetic field due to permanent magnets when their magnetization is known. With some care the scalar potential can be extended to include free currents as well.

Historically, Lord Kelvin first introduced the concept of magnetic vector potential in 1851. He also showed the formula relating magnetic vector potential and magnetic field

The magnetic vector potential **A** is a vector field defined along with the electric potential ϕ (a scalar field) by the equations

$$\mathbf{B} = \nabla \times \mathbf{A}\,, \quad \mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}\,,$$

where **B** is the magnetic field and **E** is the electric field. In magnetostatics where there is no time-varying charge distribution, only the first equation is needed. (In the context of electrodynamics, the terms *vector potential* and *scalar potential* are used for *magnetic vector potential* and *electric potential*, respectively. In mathematics, vector potential and scalar potential have more general meanings.)

Defining the electric and magnetic fields from potentials automatically satisfies two of Maxwell's equations: Gauss's law for magnetism and Faraday's Law. For example, if **A** is continuous and well-defined everywhere, then it is guaranteed not to result in magnetic monopoles. (In the mathematical theory of magnetic monopoles, **A** is allowed to be either undefined or multiple-valued in some places; see magnetic monopole for details).

Starting with the above definitions:

$$\nabla \cdot \mathbf{B} = \nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$\nabla \times \mathbf{E} = \nabla \times \left(-\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times \mathbf{A}) = -\frac{\partial \mathbf{B}}{\partial t}.$$

Alternatively, the existence of **A** and ϕ is guaranteed from these two laws using the Helmholtz's theorem. For example, since the magnetic field is divergence-free (Gauss's law for magnetism), i.e. $\nabla \cdot \mathbf{B} = 0$, **A** always exists that satisfies the above definition.

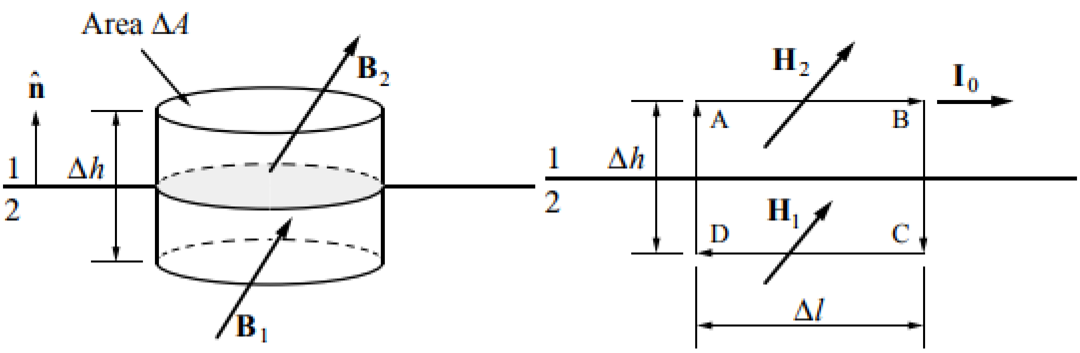
The vector potential **A** is used when studying the Lagrangian in classical mechanics and in quantum mechanics (see Schrödinger equation for charged particles, Dirac equation, Aharonov–Bohm effect).

In the SI system, the units of **A** are $V \cdot s \cdot m^{-1}$ and are the same as that of momentum per unit charge.

Although the magnetic field **B** is a pseudovector (also called axial vector), the vector potential **A** is a polar vector.[3] This means that if the right-hand rule for cross products were replaced with a left-hand rule, but without changing any other equations or definitions, then **B** would switch signs, but **A** would not change. This is an example of a general theorem: The curl of a polar vector is a pseudovector, and vice versa.[3]

Boundary conditions at the magnetic surfaces

Consider a Gaussian pill-box at the interface between two different media, arranged as in the figure above. The net enclosed (free) magnetic charge density is zero so as the height of the pill-box Δh tends to zero so the integral form of Gauss's law tells us that



$$(\mathbf{B}_2 \cdot \hat{\mathbf{n}})\Delta A - (\mathbf{B}_1 \cdot \hat{\mathbf{n}})\Delta A \approx 0$$

which becomes exact in the limit $\Delta A \rightarrow 0$ when

$$(\mathbf{B}_2 - \mathbf{B}_1) \cdot \hat{\mathbf{n}} = 0$$

therefore the component of **B** normal to the interface is continuous.

To find the **H**-field boundary condition we apply Ampère's circuital law to the path ABCD shown in the diagram above. $\mathbf{I}_0$ is the unit vector in the direction AB parallel to the surface so

$$\text{as } \Delta h \to 0 \quad \text{so} \quad (\mathbf{H}_2 - \mathbf{H}_1)\cdot \mathbf{I}_0 \Delta l = \mathbf{j}_c \cdot (\hat{\mathbf{n}} \times \mathbf{I}_0) = (\mathbf{j}_c \times \hat{\mathbf{n}})\cdot \mathbf{I}_0$$

or equivalently

$$(\mathbf{H}_2 - \mathbf{H}_1)_{\parallel} = \mathbf{j}_c \times \hat{\mathbf{n}}.$$

One can take the cross-product of this expression to obtain a form that is useful for deducing $\mathbf{j}_c$ if **H** is known on each side of the boundary.

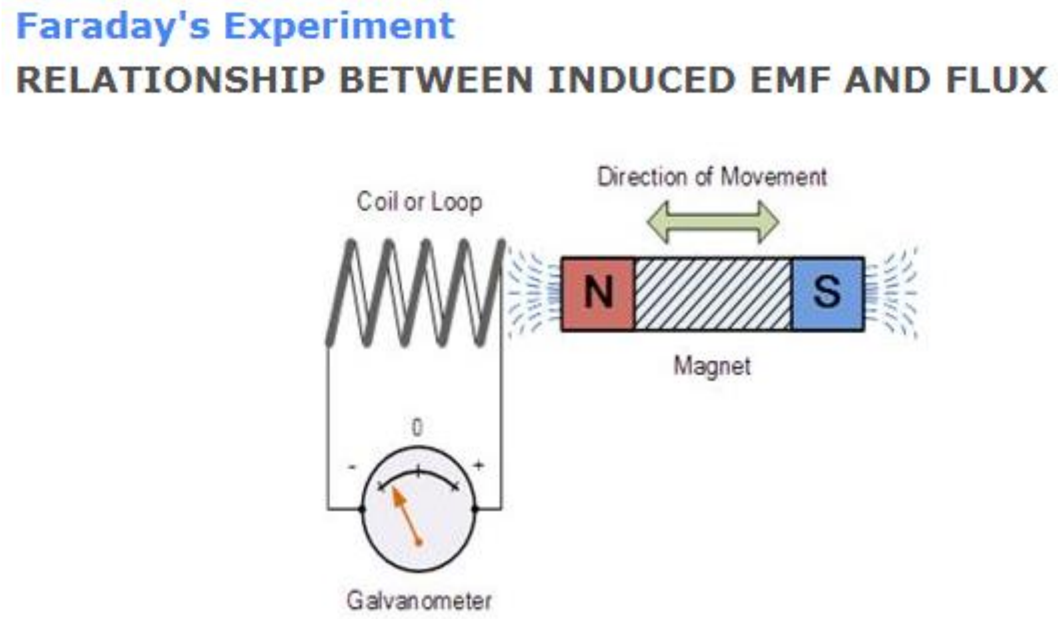
$$\hat{\mathbf{n}} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{j}_c.$$

To summarise, the component of **H** tangential to the interface is continuous across the interface unless there is a conduction surface current density $\mathbf{j}_c$.

UNIT IV

MAGNETOSTATICS APPLICATIONS

In 1831, Michael Faraday, an English physicist gave one of the most basic laws of electromagnetism called **Faraday's law of electromagnetic induction**. This law explains the working principle of most of the electrical motors, generators, electrical transformers and inductors. This law shows the relationship between electric circuit and magnetic field. Faraday performs an experiment with a magnet and coil. During this experiment, he found how emf is induced in the coil when flux linked with it changes. He has also done experiments in electro-chemistry and electrolysis.



In this experiment, Faraday takes a magnet and a coil and connects a galvanometer across the coil. At starting, the magnet is at rest, so there is no deflection in the galvanometer i.e needle of galvanometer is at the center or zero position. When the magnet is moved towards the coil, the needle of galvanometer deflects in one direction. When the magnet is held stationary at that position, the needle of galvanometer returns back to zero position. Now when the magnet is moved away from the coil, there is some deflection in the needle but in opposite direction and again when the magnet becomes stationary, at that point with respect to coil, the needle of the galvanometer returns back to the zero position. Similarly, if magnet is held stationary and the coil is moved away and towards the magnet, the galvanometer shows deflection in similar manner. It is also seen that, the faster the change in the magnetic field, the greater will be the induced emf or voltage in the coil.

Faraday's Laws

Faraday's First Law

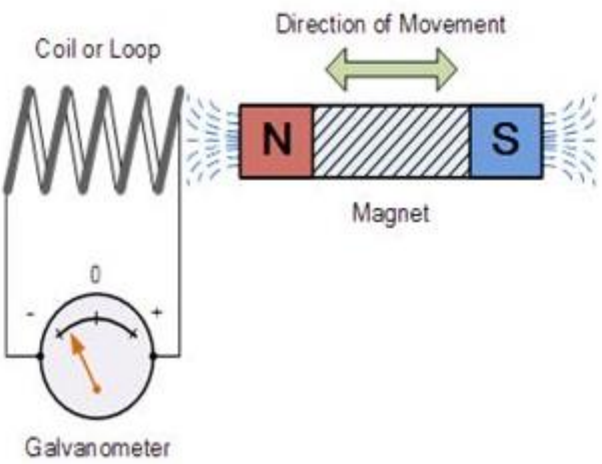
Any change in the magnetic field of a coil of wire will cause an emf to be induced in the coil. This emf induced is called induced emf and if the conductor circuit is closed, the current will also circulate through the circuit and this current is called induced current.
Method to change magnetic field:

1. By moving a magnet towards or away from the coil
2. By moving the coil into or out of the magnetic field.
3. By changing the area of a coil placed in the magnetic field
4. By rotating the coil relative to the magnet.

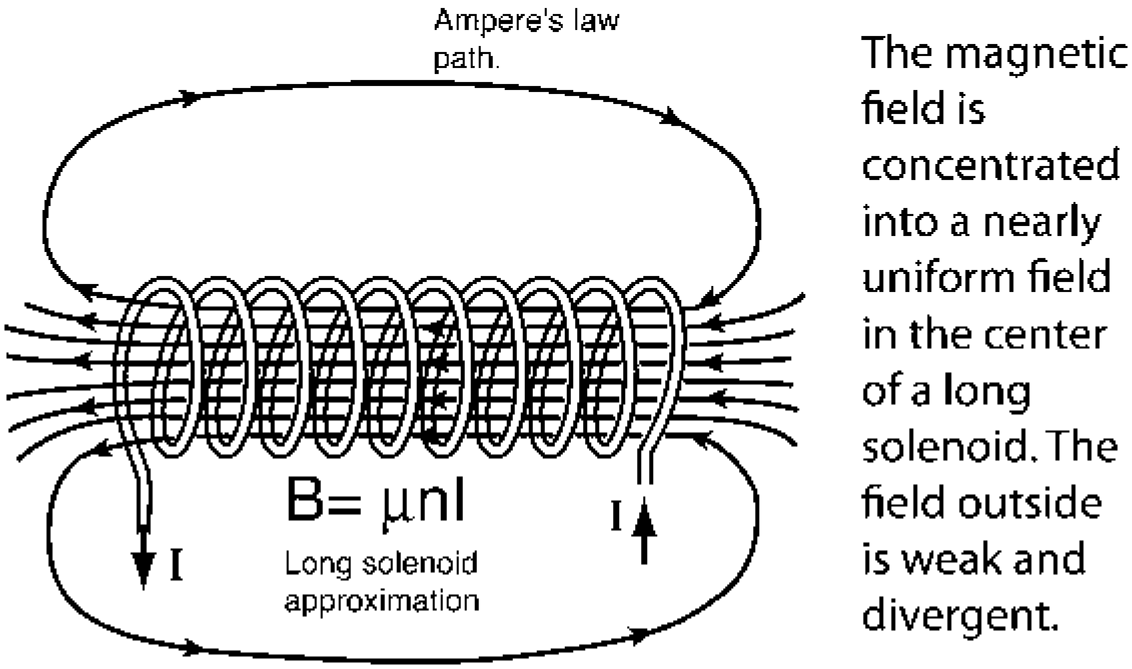
Faraday's Second Law

It states that the magnitude of emf induced in the coil is equal to the rate of change of flux that linkages with the coil. The flux linkage of the coil is the product of number of turns in the coil and flux associated with the coil.

Faraday Law Formula

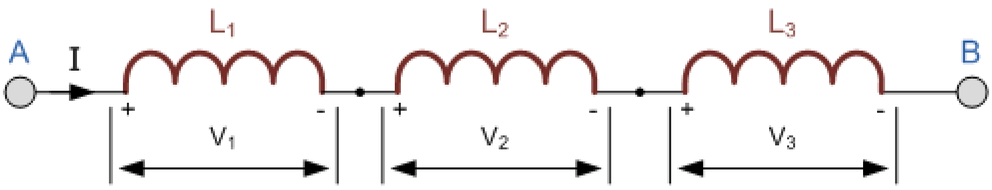


Inductance of a Solenoid



$$L = \frac{\mu N^2 A}{\ell}$$ ℓ = length of solenoid, A = cross-sectional area

Inductor in Series Circuit



The current, (I) that flows through the first inductor, L_1 has no other way to go but pass through the second inductor and the third and so on. Then, series inductors have a **Common Current** flowing through them, for example:

$$I_{L1} = I_{L2} = I_{L3} = I_{AB} \ldots \text{etc.}$$

In the example above, the inductors L_1, L_2 and L_3 are all connected together in series between points A and B. The sum of the individual voltage drops across each inductor can be found using Kirchoff's Voltage Law (KVL) where, $V_T = V_1 + V_2 + V_3$ and we know from the previous tutorials on inductance that the self-induced emf across an inductor is given as: V = L di/dt.

So by taking the values of the individual voltage drops across each inductor in our example above, the total inductance for the series combination is given as:

$$L_T \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt}$$

By dividing through the above equation by di/dt we can reduce it to give a final expression for calculating the total inductance of a circuit when connecting inductors together in series and this is given as:

Inductors in Series Equation

$$L_{total} = L_1 + L_2 + L_3 + \ldots.. + L_n \text{ etc.}$$

Then the total inductance of the series chain can be found by simply adding together the individual inductances of the inductors in series just like adding together resistors in series. However, the above equation only holds true when there is "NO" mutual inductance or magnetic coupling between two or more of the inductors, (they are magnetically isolated from each other).

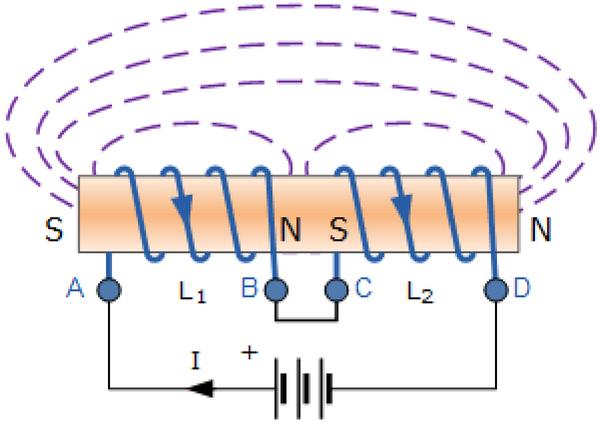
One important point to remember about inductors in series circuits, the total inductance (L_T) of any two or more inductors connected together in series will always be **GREATER** than the value of the largest inductor in the series chain

Mutually Connected Inductors in Series

When inductors are connected together in series so that the magnetic field of one links with the other, the effect of mutual inductance either increases or decreases the total inductance depending upon the amount of magnetic coupling. The effect of this mutual inductance depends upon the distance apart of the coils and their orientation to each other.

Mutually connected series inductors can be classed as either "Aiding" or "Opposing" the total inductance. If the magnetic flux produced by the current flows through the coils in the same direction then the coils are said to be **Cumulatively Coupled**. If the current flows through the coils in opposite directions then the coils are said to be **Differentially Coupled** as shown below.

Cumulatively Coupled Series Inductors



While the current flowing between points A and D through the two cumulatively coupled coils is in the same direction, the equation above for the voltage drops across each of the coils needs to be modified to take into account the interaction between the two coils due to the effect of mutual inductance. The self inductance of each individual coil, L_1 and L_2 respectively will be the same as before but with the addition of M denoting the mutual inductance.

Then the total emf induced into the cumulatively coupled coils is given as:

$$L_T \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + 2\left(M \frac{di}{dt}\right)$$

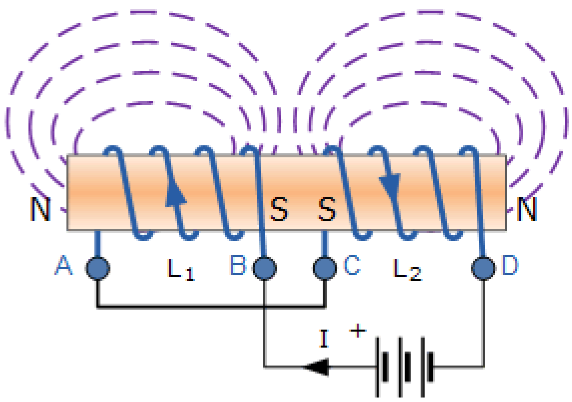
Where: 2M represents the influence of coil L_1 on L_2 and likewise coil L_2 on L_1.

By dividing through the above equation by di/dt we can reduce it to give a final expression for calculating the total inductance of a circuit when the inductors are cumulatively connected and this is given as:

$$L_{total} = L_1 + L_2 + 2M$$

If one of the coils is reversed so that the same current flows through each coil but in opposite directions, the mutual inductance, M that exists between the two coils will have a cancelling effect on each coil as shown below.

Differentially Coupled Series Inductors



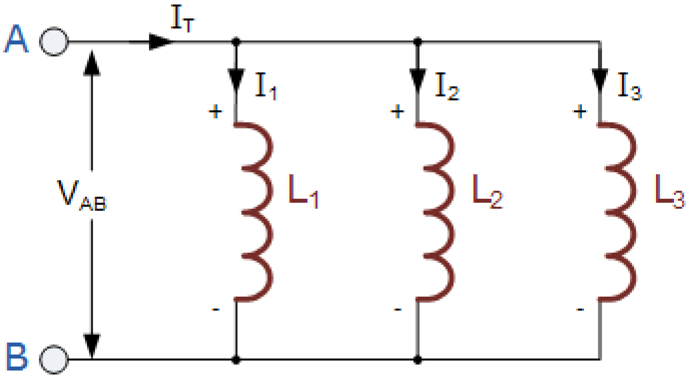
The emf that is induced into coil 1 by the effect of the mutual inductance of coil 2 is in opposition to the self-induced emf in coil 1 as now the same current passes through each coil in opposite directions. To take account of this cancelling effect a minus sign is used with M when the magnetic field of the two coils are differentially connected giving us the final equation for calculating the total inductance of a circuit when the inductors are differentially connected as:

$$L_{total} = L_1 + L_2 - 2M$$

Then the final equation for inductively coupled inductors in series is given as:

$$L_T = L_1 + L_2 \pm 2M$$

Inductors in Parallel Circuit



In the previous series inductors tutorial, we saw that the total inductance, L_T of the circuit was equal to the sum of all the individual inductors added together. For inductors in parallel the equivalent circuit inductance L_T is calculated differently.

The sum of the individual currents flowing through each inductor can be found using Kirchoff's Current Law (KCL) where, $I_T = I_1 + I_2 + I_3$ and we know from the previous tutorials on inductance that the self-induced emf across an inductor is given as: V = L di/dt

Then by taking the values of the individual currents flowing through each inductor in our circuit above, and substituting the current i for $i_1 + i_2 + i_3$ the voltage across the parallel combination is given as:

$$V_{AB} = L_T \frac{d}{dt}\left(i_1 + i_2 + i_3\right) = L_T\left(\frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt}\right)$$

By substituting di/dt in the above equation with v/L gives:

$$V_{AB} = L_T\left(\frac{V}{L_1} + \frac{V}{L_2} + \frac{V}{L_3}\right)$$

We can reduce it to give a final expression for calculating the total inductance of a circuit when connecting inductors in parallel and this is given as:

Parallel Inductor Equation

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \ldots\ldots + \frac{1}{L_N}$$

Here, like the calculations for parallel resistors, the reciprocal (1/Ln) value of the individual inductances are all added together instead of the inductances themselves. But again as with series connected inductances, the above equation only holds true when there is "NO" mutual inductance or magnetic coupling between two or more of the inductors, (they are magnetically isolated from each other). Where there is coupling between coils, the total inductance is also affected by the amount of coupling.

This method of calculation can be used for calculating any number of individual inductances connected together within a single parallel network. If however, there are only two individual inductors in parallel then a much simpler and quicker formula can be used to find the total inductance value, and this is:

$$L_T = \frac{L_1 \times L_2}{L_1 + L_2}$$

One important point to remember about inductors in parallel circuits, the total inductance (L_T) of any two or more inductors connected together in parallel will always be **LESS** than the value of the smallest inductance in the parallel chain.

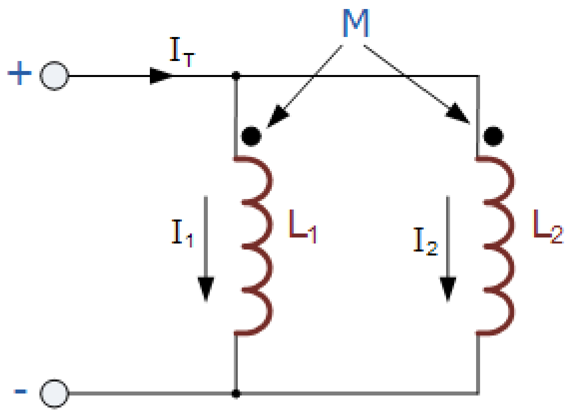
Mutually Coupled Inductors in Parallel

When inductors are connected together in parallel so that the magnetic field of one links with the other, the effect of mutual inductance either increases or decreases the total inductance depending upon the amount of magnetic coupling that exists between the coils. The effect of this mutual inductance depends upon the distance apart of the coils and their orientation to each other.

Mutually connected inductors in parallel can be classed as either "aiding" or "opposing" the total inductance with parallel aiding connected coils increasing the total equivalent inductance and parallel opposing coils decreasing the total equivalent inductance compared to coils that have zero mutual inductance.

Mutual coupled parallel coils can be shown as either connected in an aiding or opposing configuration by the use of polarity dots or polarity markers as shown below.

Parallel Aiding Inductors



The voltage across the two parallel aiding inductors above must be equal since they are in parallel so the two currents, i_1 and i_2 must vary so that the voltage across them stays the same. Then the total inductance, L_T for two parallel aiding inductors is given as:

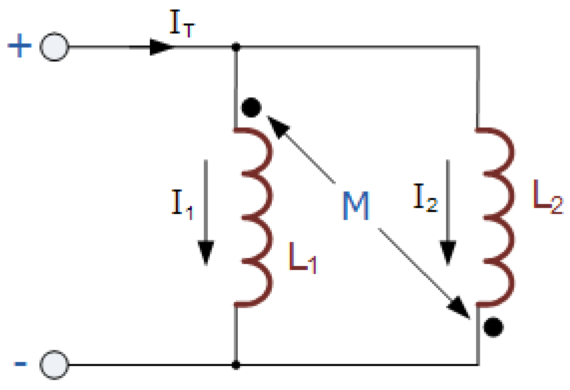
$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

Where: 2M represents the influence of coil L_1 on L_2 and likewise coil L_2 on L_1.

If the two inductances are equal and the magnetic coupling is perfect such as in a toroidal circuit, then the equivalent inductance of the two inductors in parallel is L as $L_T = L_1 = L_2 = M$. However, if the mutual inductance between them is zero, the equivalent inductance would be L ÷ 2 the same as for two self-induced inductors in parallel.

If one of the two coils was reversed with respect to the other, we would then have two parallel opposing inductors and the mutual inductance, M that exists between the two coils will have a cancelling effect on each coil instead of an aiding effect as shown below.

Parallel Opposing Inductors



Then the total inductance, L_T for two parallel opposing inductors is given as:

$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

This time, if the two inductances are equal in value and the magnetic coupling is perfect between them, the equivalent inductance and also the self-induced emf across the inductors will be zero as the two inductors cancel each other out.

This is because as the two currents, i_1 and i_2 flow through each inductor in turn the total mutual flux generated between them is zero because the two flux's produced by each inductor are both equal in magnitude but in opposite directions.

Then the two coils effectively become a short circuit to the flow of current in the circuit so the equivalent inductance, L_T becomes equal to (L ± M) ÷ 2.

Mutual inductance of series and parallel circuits

- Total number of turns: N
- Magnetic field inside toroid: $B = \frac{\mu_0 I}{2\pi r}$
- Magnetic flux through each turn (loop):
  $\Phi_B = \int_a^b BH\,dr = \frac{\mu_0 INH}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 INH}{2\pi} \ln\frac{b}{a}$
- Inductance: $L \equiv \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 H}{2\pi} \ln\frac{b}{a}$

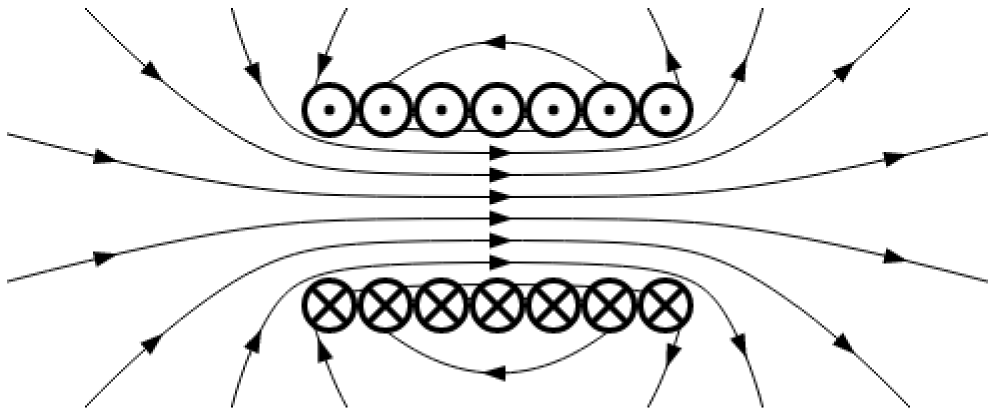
- Inductance: $L \equiv \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 H}{2\pi} \ln\frac{b}{a}$
- Narrow toroid: $s \equiv b - a \ll a$
  $\ln\frac{b}{a} = \ln\left(1 + \frac{s}{a}\right) \simeq \frac{s}{a}$
- Inductance: $L = \frac{\mu_0 N^2 (sH)}{2\pi a}$

Energy stored in magnetic fields

When a conductor carries a current, a magnetic field surrounding the conductor is produced. The resulting magnetic flux is proportional to the current. If the current changes, the change in magnetic flux is proportional to the time-rate of change in current by a factor called inductance (L). Since nature abhors rapid change, a voltage(*electromotive force, EMF*) produced in the conductor opposes the change in current, which is also proportional to the change in magnetic flux. Thus, inductors oppose change in current by producing a voltage that,in turn, creates a current to oppose the change in magnetic flux; the voltage is proportional to the change in current.

Energy Stored in Magnetic Field

Let's consider Fig 1 , an example of a solenoid (ℓ: length, N: number of turns, I: current, A: cross-section area) that works as an inductor. From Eq. 1, the energy stored in the magnetic field created by the solenoid is:



Energy Stored in Inductor

Due to energy conservation, the energy needed to drive the original current must have an outlet. For an inductor, that outlet is the magnetic field—the energy stored by an inductor is equal to the work needed to produce a current through the inductor. The formula for this energy is given as:

Energy is "stored" in the magnetic field is

$$E = \frac{1}{2}LI^2$$

Forces and torques on closed circuits

In the presence of material motion $\vec{v}$, electric field $\vec{E}'$ in a "moving" frame is related to electric field $\vec{E}$ in a "stationary" frame and to magnetic field $\vec{B}$ by:

$$\vec{E}' = \vec{E} + \vec{v} \times \vec{B}$$

This is an experimental result obtained by observing charged particles moving in combined electric and magnetic fields. It is a relatavistic expression, so that the qualifiers "moving" and "stationary" are themselves relative. The electric fields are what would be observed in either frame. In MQS systems, the magnetic flux density $\vec{B}$ is the same in both frames.

The term relating to current density becomes:

$$\vec{E} \cdot \vec{J} = \left(\vec{E}' - \vec{v} \times \vec{B}\right) \cdot \vec{J}$$

We can interpret $\vec{E}' \cdot \vec{J}$ as dissipation, but the second term bears a little examination. Note that it is in the form of a vector triple (scalar) product:

$$-\vec{v} \times \vec{B} \cdot \vec{J} = -\vec{v} \cdot \vec{B} \times \vec{J} = -\vec{v} \cdot \vec{J} \times \vec{B}$$

This is in the form of velocity times force density and represents power conversion from electromagnetic to mechanical form. This is consistent with the Lorentz force law (also experimentally observed):

$$\vec{F} = \vec{J} \times \vec{B}$$

This last expression is yet another way of describing energy conversion processes in electric machinery, as the component of apparent electric field produced by material motion through a magnetic field, when reacted against by a current, produces energy conversion to mechanical form rather than dissipation.

UNIT V

We have already obtained two of Maxwell's equations for time-varying fields,

and

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

The remaining two equations are unchanged from their non-time-varying form:

$$\nabla \cdot \mathbf{D} = \rho_v$$

$$\nabla \cdot \mathbf{B} = 0$$

Maxwell's Equation's in integral form

$$\oiint_A \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_o} = \frac{1}{\varepsilon_o} \iiint_V \rho dV$$ Gauss's Law

$$\oiint_A \vec{B} \cdot d\vec{A} = 0$$ Gauss's Law for Magnetism

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \iint_A \vec{B} \cdot d\vec{A}$$ Faraday's Law

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_o I_{encl} + \mu_o \varepsilon_o \frac{d\Phi_E}{dt} = \mu_o \iint_A \left(\vec{J} + \varepsilon_o \frac{d\vec{E}}{dt} \right) \cdot d\vec{A}$$

Ampere's Law

wave equation

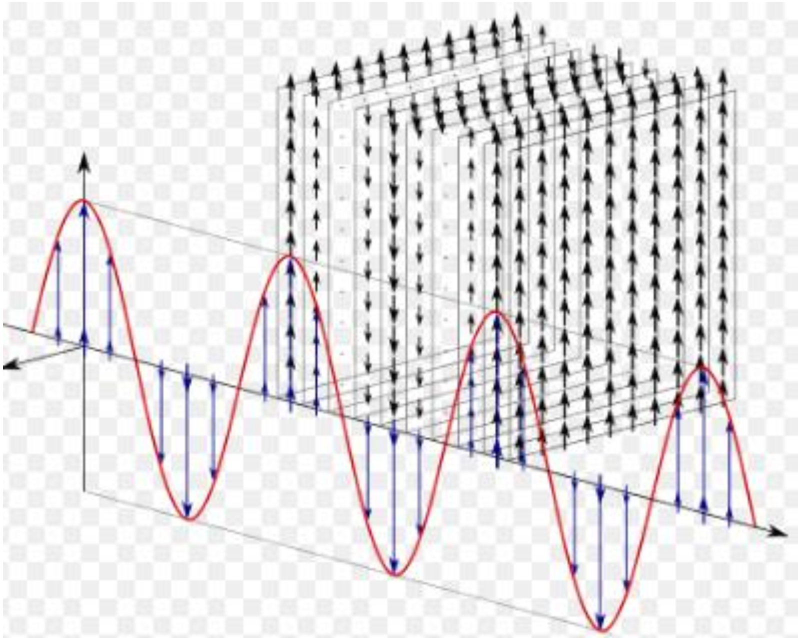
The **wave equation** is an important second-order linear hyperbolic partial differential **equation** for the description of**waves**—as they occur in classical physics—such as sound**waves**, light **waves** and water **waves**. It arises in fields like acoustics, electromagnetics, and fluid dynamics.

$$\frac{\partial^2 y}{\partial t^2} = \frac{1}{v_w^2}\frac{\partial^2 y}{dt^2}$$

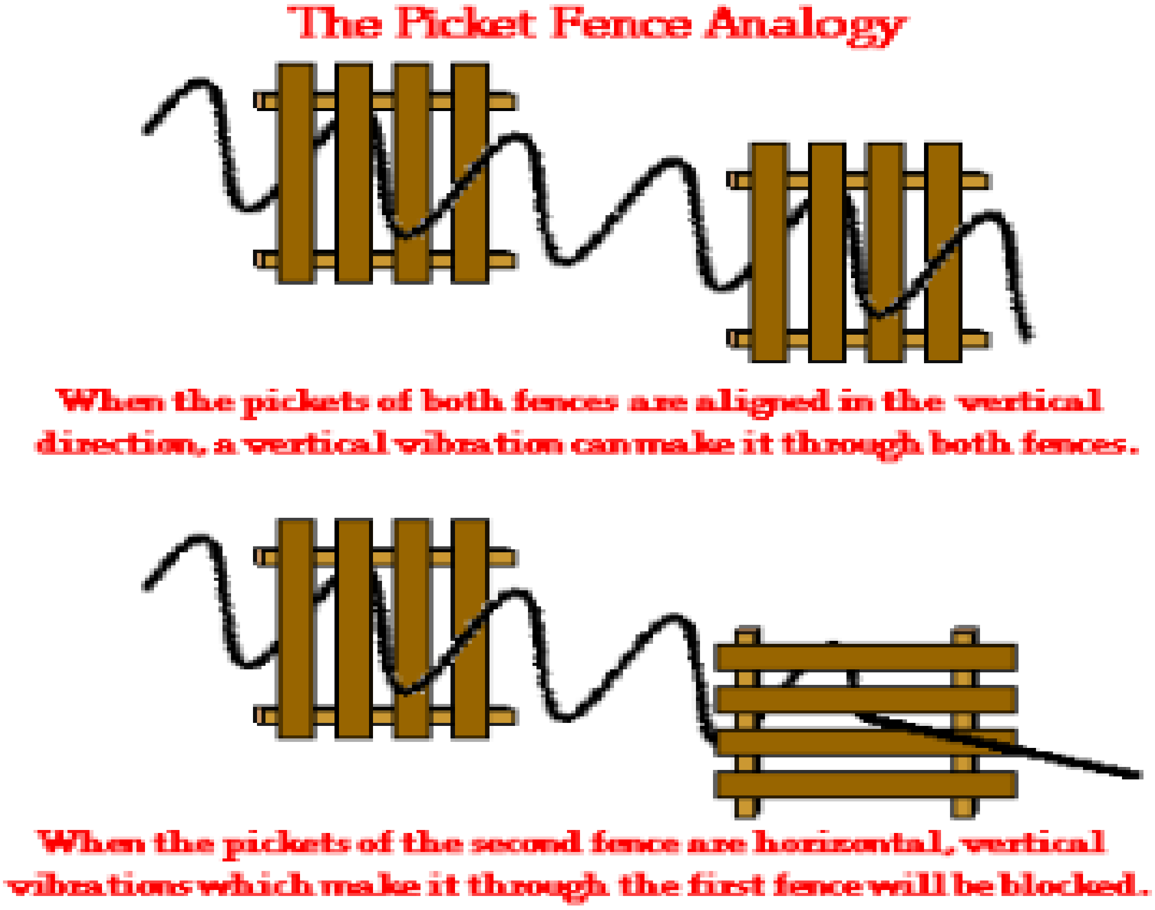
Plane waves in free space

comprise the **plane** with normal vector . Thus, the points of equal field value of always form a **plane** in**space**. ... A homogeneous **plane wave** is one in which the **planes** of constant phase are perpendicular to the direction of propagation .

plane waves in free space



Polarization

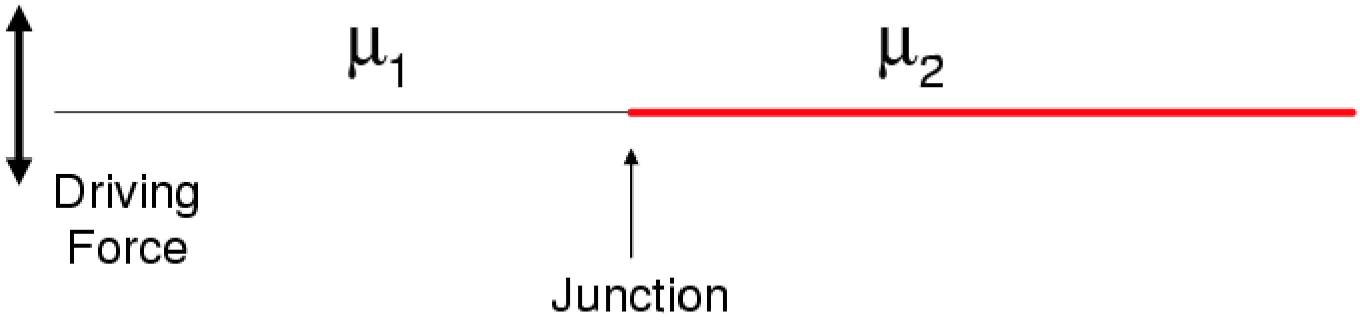


It is possible to transform unpolarized light into **polarized** light.**Polarized** light waves are light waves in which the vibrations occur in a single plane. The process of transforming unpolarized light into **polarized** light is known as **polarization**. There are a variety of methods of **polarizing** light.

reflection and transmission of waves

When the medium through which a wave travels suddenly changes, the wave often experiences partial transmission and partial refection at the interface. Reflection is a wave phenomenon that changes the direction of a wavefront at an interface between two different media so that the wavefront returns into the medium from which it originated. Transmission permits the passage of wave, with some or none of the incident wave being absorbed. Reflection and transmission often occur at the same time .

Consider a long string made by connecting two sub-strings with different density $\mu1,\mu2$. When the string is driven by an external force, partial reflection and transmission occurs as in Figure 18426. For the incoming, reflected, and transmitted waves, we can try a solution of the following forms:



$$y_{inc} = A\ cos(k_1 x - \omega t)$$
$$y_{ref} = B\ cos(k_1 x + \omega t)$$
$$y_{trans} = C\ cos(k_2 x - \omega t)$$

k_1 and k_2 are determined by the speed of the wave in each medium. We choose our coordinates such that the junction of two sub-strings is located at x=0. In choosing a trial solution for the waves, we assumed that the incident and transmitted waves travel to the right, while the reflected waves travel to the left. (This is why the '+' sign is chosen before ωt in the reflected wave. On the left side of the junction, we have

$$y_l = y_{inc} + y_{ref} = A\ cos(k_1 x - \omega t).$$

On the right side, we have

$$y_r = y_{trans} = C\ cos(k_2 x - \omega t).$$

We will impose additional restriction on the waves by applying "boundary conditions" at x=0. At the boundary x=0, the wave must be continuous and there should be no kinks in it. Thus we must have

Energy in electromagnetic fields

Energy in Electromagnetic Waves

From Sect. 233, the energy stored per unit volume in an electromagnetic wave is given by

$$w = \frac{\epsilon_0\,E^2}{2} + \frac{B^2}{2\,\mu_0}.$$

Since, $B = E/c$, for an electromagnetic wave, and $c = 1/\sqrt{\mu_0\,\epsilon_0}$, the above expression yields

$$w = \frac{\epsilon_0\,E^2}{2} + \frac{E^2}{2\,\mu_0\,c^2} = \frac{\epsilon_0\,E^2}{2} + \frac{\epsilon_0\,E^2}{2},$$

or

$$w = \epsilon_0\,E^2.$$

It is clear, from the above, that half the energy in an electromagnetic wave is carried by the electric field, and the other half is carried by the magnetic field.

As an electromagnetic field propagates it transports energy. Let P be the power per unit area carried by an electromagnetic wave: *i.e.*, P is the energy transported per unit time across a unit cross-sectional area perpendicular to the direction in which the wave is traveling. Consider a plane electromagnetic wave propagating along the z-axis. The wave propagates a distance $c\,dt$ along the z-axis in a time interval dt. If we consider a cross-sectional area A at right-angles to the z-axis, then in a time dt the wave sweeps through a volume dV of space, where $dV = A\,c\,dt$. The amount of energy filling this volume is

$$dW = w\,dV = \epsilon_0\,E^2\,A\,c\,dt.$$

It follows, from the definition of P, that the power per unit area carried by the wave is given by

$$P = \frac{dW}{A\,dt} = \frac{\epsilon_0\,E^2\,A\,c\,dt}{A\,dt},$$

so that

$$P = \epsilon_0\,E^2\,c.$$

Since half the energy in an electromagnetic wave is carried by the electric field, and the other half is carried by the magnetic field, it is conventional to convert the above expression into a form involving both the electric and magnetic field strengths. Since, $E = c\,B$, we have

$$P = \epsilon_0\,c\,E\,(c\,B) = \epsilon_0\,c^2\,E\,B = \frac{E\,B}{\mu_0}.$$

Thus,

$$P = \frac{E\,B}{\mu_0}.$$

specifies the power per unit area transported by an electromagnetic wave at any given instant of time. The *peak* power is given by

$$P_0 = \frac{E_0\,B_0}{\mu_0},$$

where E_0 and B_0 are the peak amplitudes of the oscillatory electric and magnetic fields, respectively. It is easily demonstrated that the *average* power per unit area transported by an electromagnetic wave is *half*the peak power, so that

$$S = \bar{P} = \frac{E_0\,B_0}{2\,\mu_0} = \frac{\epsilon_0\,c\,{E_0}^2}{2} = \frac{c\,{B_0}^2}{2\,\mu_0}.$$

The quantity S is conventionally termed the *intensity* of the wave.