UNIT - I
Network Theory

- **Graph of a Network**
- **Concept of Tree and Co-tree**
- **Incidence Matrix**
- **Tie-set**
- **Tie-set Schedule**
- **Cut-set**
- **Cut-set Schedule**
- **Formation of Equilibrium Equations in Matrix Form**
- **Solution of Resistive Networks**
- **Principle of Duality**

I. Basic definitions:

Network Topology:

- Is another method of solving electric circuits
- Is generalized approach

Network:

A combination of two or more network elements is called a network.

Topology:

Topology is a branch of geometry which is concerned with the properties of a geometrical figure, which are not changed when the figure is physically distorted, provided that, no parts of the figure are cut open or joined together.

The geometrical properties of a network are independent of the types of elements and their values.

Every element of the network is represented by a line segment with dots at the ends irrespective of its nature and value.

Circuit:

If the network has at least one closed path it is a circuit.

Note that every circuit is a network but every network is not a circuit.

Branch:

Representation of each element (component) of a electric network by a line segment is a branch.

Node:

A point at which two or more elements are joined is a node. End points of the branches are called nodes.

Graph:

It is collection of branches and nodes in which each branch connects two nodes.

Graph of a Network:

The diagram that gives network geometry and uses lines with dots at the ends to represent network element is usually called a graph of a given network. For example,

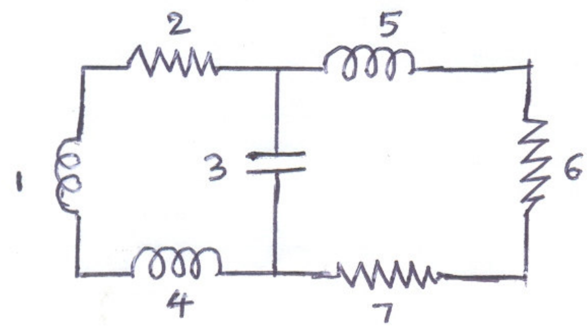


Fig.1 Network

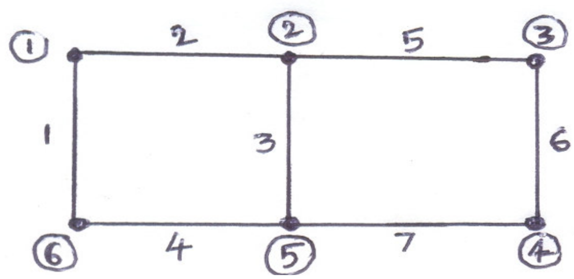


Fig.2 Graph

Sub-graph:

A sub-graph is a subset of branches and nodes of a graph for example branches 1, 2, 3 & 4 forms a sub-graph. The sub-graph may be connected or unconnected. The sub- graph of graph shown in figure 2 is shown in figure 3.

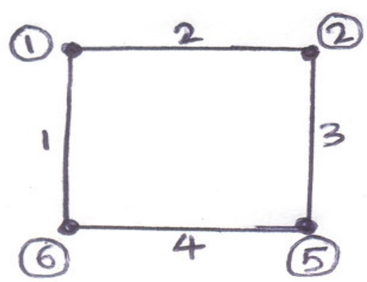


Fig.3 Sub-graph

Connected Graph:

If there exists at least one path from each node to every other node, then graph is said to be connected. Example,

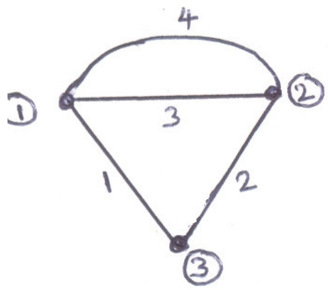


Fig.4 Connected Graph

Un-connected Graph:

If there exists no path from each node to every other node, the graph is said to be un-connected graph. For example, the network containing a transformer (inductively coupled parts) its graph could be un-connected.

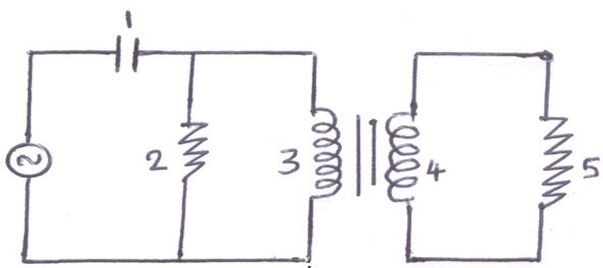


Fig.5 Network

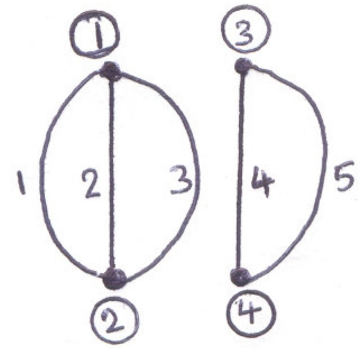


Fig.6 Un-connected Graph

Path (Walk):

A sequence of branches going from one node to other is called path. The node once considered should not be again considered the same node.

Loop (Closed Path):

Loop may be defined as a connected sub-graph of a graph, which has exactly two branches of the sub-graph connected to each of its node.

For example, the branches1, 2 & 3 in figure 7 constitute a loop.

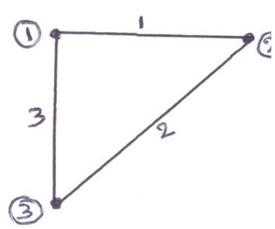


Fig.7 Loop

Planar and Non-planar Graphs:

A planar graph is one where the branches do not cross each other while drawn on a plain sheet of paper. If they cross, they are non-planar.

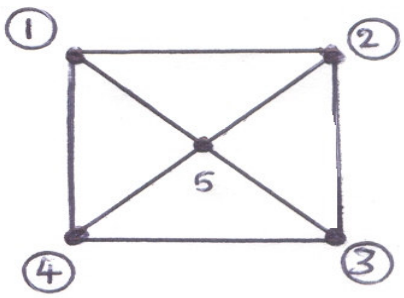


Fig.8 Planar Graph

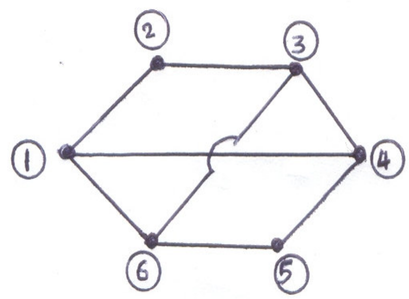


Fig.9 Non-planar graph

Oriented Graph:

The graph whose branches carry an orientation is called an oriented graph

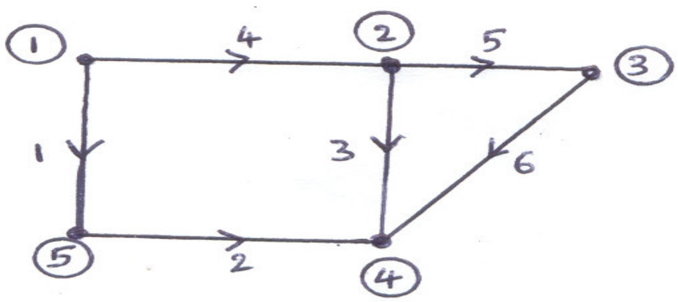
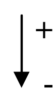


Fig.10 Oriented Graph

The current and voltage references for a given branches are selected with a +ve sign at tail side and –ve sign at head



Tree:

Tree of a connected graph is defined as any set of branches, which together connect all the nodes of the graph without forming any loops. The branches of a tree are called **Twigs.**

Co-tree:

Remaining branches of a graph, which are not in the tree form a co-tree. The branches of a co-tree are called **links** or **chords.**

The tree and co-tree for a given oriented graph shown in figure11 is shown in figure12 and figure13.

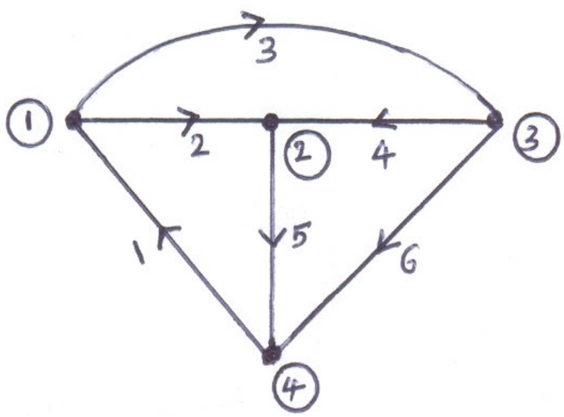


Fig. 11 Oriented Graph

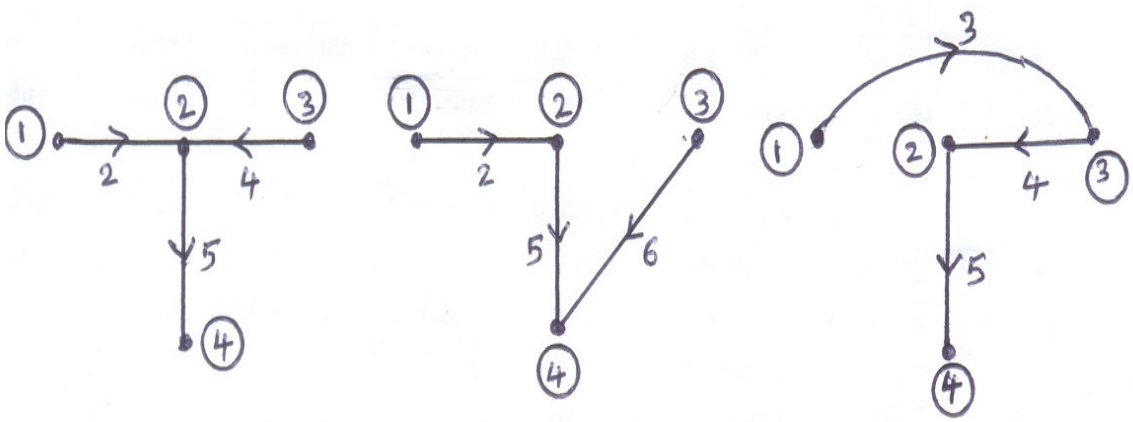


Fig.12 Trees

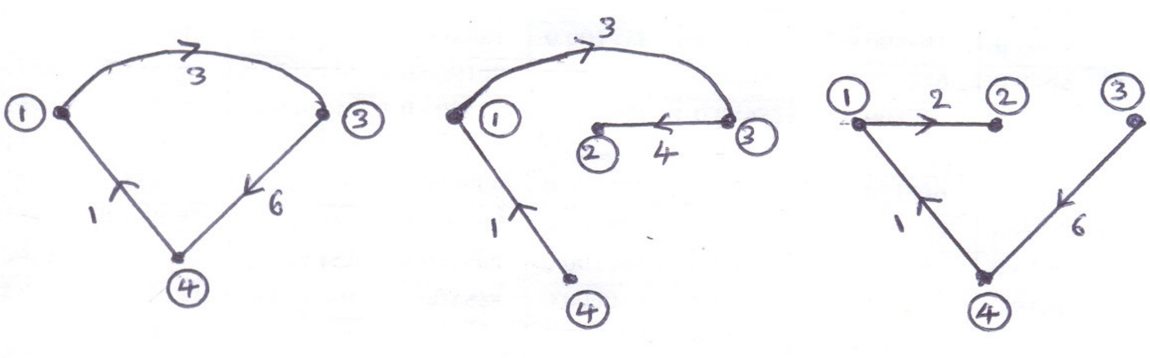


Fig.13 Co-trees

Tree	Twigs	Links (Chords)
1	2, 4 & 5	1, 3 & 6
2	3, 4 & 5	1, 2 & 6
3	2, 5 & 6	1, 3 & 4

Properties of Tree:

i) It contains all the nodes of the graph.

ii) It contains (n_t-1) branches. Where 'n_t' is total number of nodes in the given graph.

iii) There are no closed paths.

Total number of tree branches, n = (n_t-1)

Where n_t = Total number of nodes

Total number of links, l = (b-n)

Where b = Total number of branches in the graph.

Degree of Node:

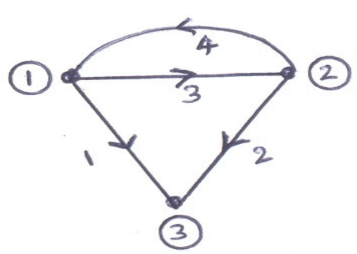
The number of branches attached to the node is degree of node.

II. Complete Incidence Matrix (A_a):

Incidence matrix gives us the information about the branches, which are joined to the nodes and the orientation of the branch, which may be towards a node or away from it.

Nodes of the graph form the rows and branches form the columns. If the branch is not connected to node, corresponding element in the matrix is given the value **'0'**. If a branch is joined, it has two possible orientations. If the orientation is away from the node, the corresponding matrix element is written as **'+1'**. If it is towards the node, the corresponding matrix element is written as **'-1'**.

Example: 1) Obtain complete incidence matrix for the graph shown



Solution:

A_a =

Nodes	**Branches**			
	1	2	3	4
1	1	0	1	-1
2	0	1	-1	1
3	-1	-1	0	0

$$\mathbf{A_a} = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ -1 & -1 & 0 & 0 \end{bmatrix}$$

Properties of Incidence Matrix:

i) Each column has only two non-zero elements and all other elements are zero.

ii) If all the rows of **'A_a'** are added, the sum will be a row whose elements equal zero.

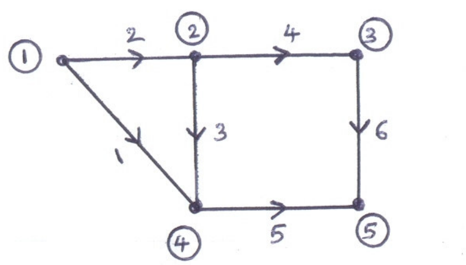
If the graph has **'b'** branches and **'n_t'** nodes, the complete incidence matrix is of the order (n_t x b).

III. Reduced Incidence Matrix (A):

When one row is eliminated from the complete incidence matrix, the remaining matrix is called **reduced incidence matrix**

If the graph has **'b'** branches and **'n_t'** nodes, the reduced incidence matrix is of the order (n_t-1) x b.

Example: 2) Write the complete and reduced incidence matrix for the given graph shown



Solution:

A_a =

Nodes	Branches					
	1	2	3	4	5	6
1	1	1	0	0	0	0
2	0	-1	1	1	0	0
3	0	0	0	-1	0	1
4	-1	0	-1	0	1	0
5	0	0	0	0	-1	-1

Complete Incidence Matrix, $\mathbf{A_a} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$

Reduced Incidence Matrix, $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 & 1 & 0 \end{bmatrix}$

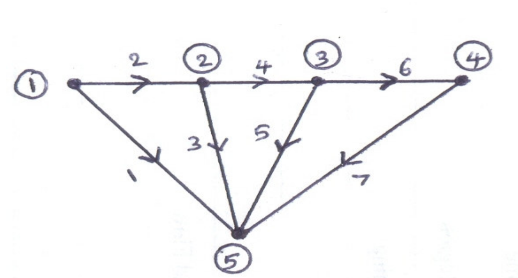
Example: 3) Draw the oriented graph of incidence matrix shown below

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Solution: The given matrix is a reduced incidence matrix. Obtain the complete incidence matrix in order to draw the oriented graph.

$$\mathbf{A_a} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & -1 & 0 & -1 & 0 & -1 \end{bmatrix}$$

Total number of nodes = $n_t = 5$
Total number of branches = b = 7



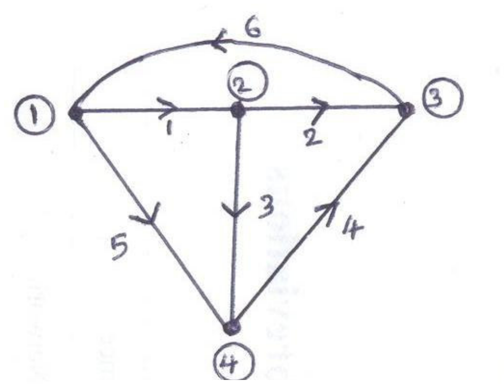
Oriented Graph

Example: 4) Draw the oriented graph of incidence matrix shown below

$$A_a = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 & -1 & 0 \end{bmatrix}$$

Solution:

Total number of nodes = n_t = 4
Total number of branches = b = 6



Oriented Graph

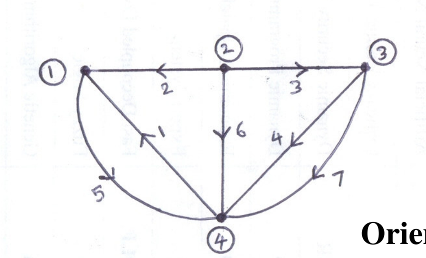
Example: 5) Draw the oriented graph of incidence matrix shown below

$$A = \begin{bmatrix} -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Solution: The given matrix is a reduced incidence matrix. Obtain the complete incidence matrix in order to draw the oriented graph.

$$A_a = \begin{bmatrix} -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & -1 & -1 & -1 \end{bmatrix}$$

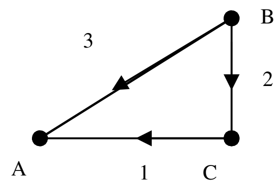
Total number of nodes = n_t = 4
Total number of branches = b = 7



Oriented Graph

Example: 6) Show that determinant of the incidence matrix of a closed loop is zero.

Proof: let us consider a closed path ABC



Total number of nodes = $n_t = 3$
Total number of branches = b = 3

The complete incidence matrix is

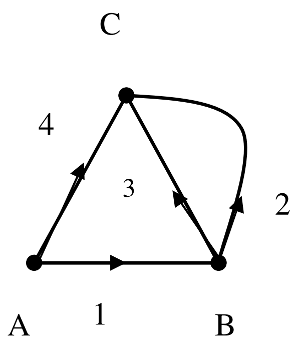
$$\mathbf{A_a} = \begin{pmatrix} -1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$

The determinant of complete incidence matrix of the closed loop is

$$\begin{vmatrix} -1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = -1\,(0 + 1) - 0 - 1\,(0 - 1) = -1 + 1 = 0$$

IV. Number of Possible Trees of a Graph:

For the given network graph, it is possible to write several trees. The number of possible trees is equal to determinant of $[A]\,[A]^t$. Where [A] is the reduced incidence matrix obtained by removing any one row from complete incidence matrix and $[A]^t$ is the transpose of [A].



A_a =

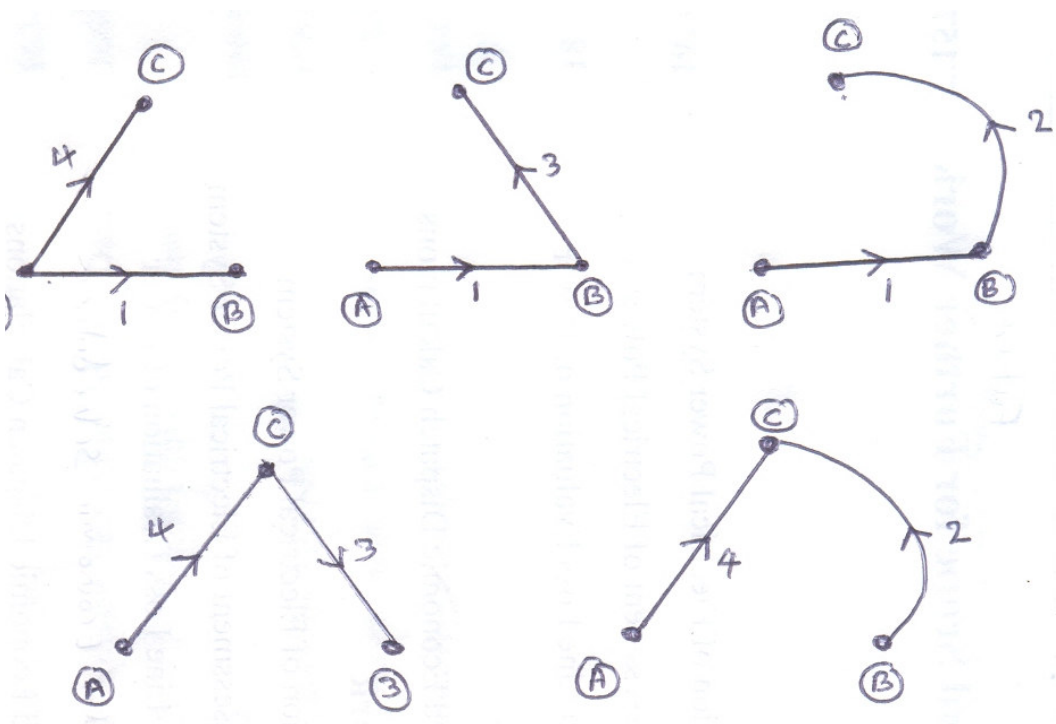
Node No.	Branches 1	2	3	4
A	1	0	0	1
B	-1	1	1	0
c	0	-1	-1	-1

$$[A_a] = \begin{pmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & -1 & -1 \end{pmatrix}$$

$$[A] = \begin{pmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 \end{pmatrix}$$

$$[A]\,[A]^t = \begin{pmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1+1 & -1 \\ -1 & 1+1+1 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$$

$$\left|\,[A]\,[A]^t\,\right| = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = (6-1) = 5$$



Node Pair Voltages:

The voltage between any two nodes of a network is known as the node-pair voltages. In general all branch voltages are node-pair voltages.

Network Variables:

In Loop analysis, the loop currents are unknown parameters. Once, they are evaluated, all branch currents can be determined in terms of these loop currents.

Similarly, in nodal analysis, the node-pair voltages are the unknown parameters. Once, they are evaluated, the voltages across any two nodes of the network can be found. Hence, the **node-pair voltages** and **loop currents** are called network variables.

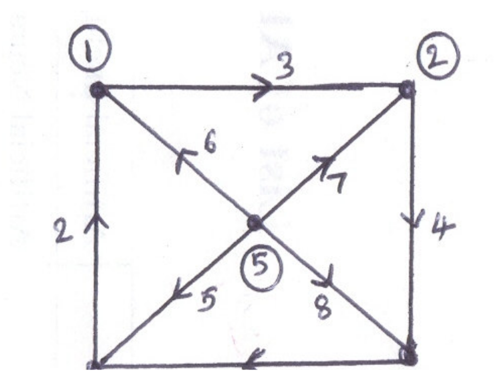
The network variables are independent variables and all other quantities depend on these values.

V. Tie-set:

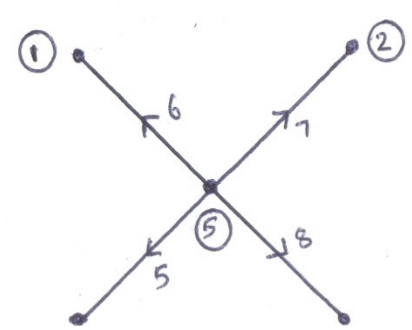
A tie-set is a set of branches contained in a loop such that each loop contains one link or chord and remainder are tree branches.

Or

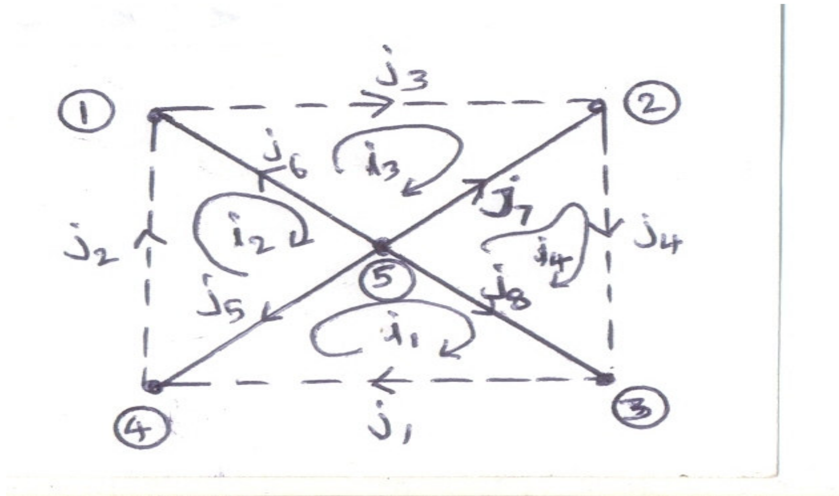
The set of branches forming the closed loop in which link or loop current circulate is called a **Tie-set.**



Oriented Graph



Tree



Let the branch currents in the network graph denoted by the symbol **‘j’** and various loop currents by symbol **‘i’.**

The orientation of a closed loop will be chosen to be the same as that of its connecting link.

For the given network graph,

Number of branches, b = 8

Number of nodes, n_t= 5

Number of closed loops = [b–(n_t - 1)]

Where (n_t-1) = Number of tree branches.

VI. Tie-set Schedule:

For a given network tree, a systematic way of indicating the links is through use of a schedule called **Tie-set Schedule**

The tie-set schedule for the given network oriented graph is shown below

Link Current or Number	**Branches**							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	**1**	**0**	**0**	**0**	**-1**	**0**	**0**	**1**
2	**0**	**1**	**0**	**0**	**1**	**-1**	**0**	**0**
3	**0**	**0**	**1**	**0**	**0**	**1**	**-1**	**0**
4	**0**	**0**	**0**	**1**	**0**	**0**	**1**	**-1**

The tie-set schedule can be written in matrix form is known as **Tie-set matrix (B).**

$$\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 \end{pmatrix}$$

After writing the schedule, the columns of a schedule or matrix gives branch currents in terms of link currents. Thus column 1 gives equation for j_1 in terms of link currents

i.e. $j_1 = i_1 \quad j_5 = i_2 - i_1$
Similarly, $j_2 = i_2 \quad j_6 = i_3 - i_2$
$j_3 = i_3 \quad j_7 = i_4 - i_3$
$j_4 = i_4 \quad j_8 = i_1 - i_4$ (1)

$$\left.\begin{array}{ll} j_1 = i_1 & j_5 = i_2 - i_1 \\ j_2 = i_2 & j_6 = i_3 - i_2 \\ j_3 = i_3 & j_7 = i_4 - i_3 \\ j_4 = i_4 & j_8 = i_1 - i_4 \end{array}\right\} \quad (1)$$

The rows of the schedule give KVL equations in terms of coefficients of the schedule or matrix. i. e.

$$\left.\begin{array}{l} e_1 - e_5 + e_8 = 0 \\ e_2 + e_5 - e_6 = 0 \\ e_3 + e_6 - e_7 = 0 \\ e_4 + e_7 - e_8 = 0 \end{array}\right\} \quad (2)$$

The set of equation (1) can be expressed in the matrix form as

$$\begin{bmatrix} j_1 \\ j_2 \\ j_3 \\ j_4 \\ j_5 \\ j_6 \\ j_7 \\ j_8 \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \quad (3)$$

In compact form

$$[\mathbf{I_b}] = [\mathbf{B}]^{\mathbf{T}} [\mathbf{I_l}] \quad (4)$$

Where

$[\mathbf{I_b}]$ = is a column matrix of branch currents of the order (bx1).

$[\mathbf{B}]^{\mathbf{T}}$ = is the transpose of the fundamental tie-set matrix B.

$[\mathbf{I_l}]$ = is a column matrix of loop currents or link currents of the order (lx1).

Where 'l' is the number of independent loops.

From set of equation (2),

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \\ e_8 \end{pmatrix} = 0 \qquad (5)$$

In compact form

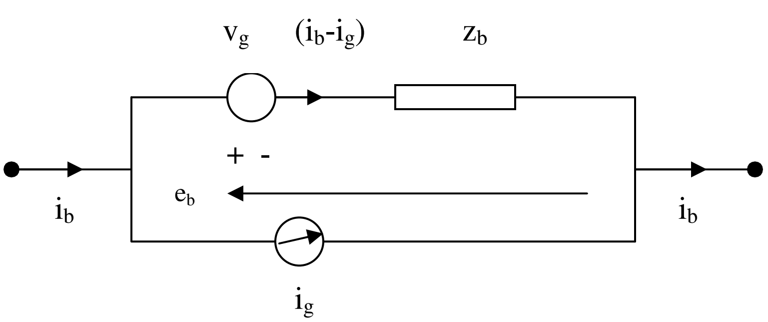
$$[\mathbf{B}] \; [\mathbf{E_b}] = \mathbf{0} \qquad (6)$$

$[\mathbf{E_b}]$ = is a column matrix of branch voltages of the order (bx1).

Equations (4) & (6) are the basic equations required to deriving equilibrium equations on **loop current basis**

VII. Equilibrium Equations with Loop Currents as Variables:

Consider a general branch of a network as shown in figure



Where v_g = Total series voltage in the branch

i_g = Total current source connected across the branch

z_b = Total impedance of the branch

i_b = Branch current

The voltage current relation for the branch can be written as

$$e_b = v_g + z_b\,(i_b - i_g) \qquad \text{(i)}$$

and $$i_b = i_g + y_b\,(i_b - i_g) \qquad \text{(ii)}$$

Where y_b = Total admittance of the branch.

For a network with more number of branches, equation (i) & (ii) may be written as

$[E_b] = [V_g] + [Z_b]\ ([I_b] - [I_g])$ (iii)

$[I_b] = [I_g] + [Y_b]\ ([E_b] - [V_g])$ (iv)

Where

$[E_b]$, $[V_g]$, $[I_b]$, & $[I_g]$ are (bx1) matrices of branch voltages, source voltages in the branches, branch currents and source currents in the branches respectively.

$[Z_b]$ & $[Y_b]$ are branch impedance and branch admittance matrices of the order (bxb).

We have

$[E_b] = [V_g] + [Z_b]\ ([I_b] - [I_g])$ (1)

$[I_b] = [B]^T\ [I_l]$ (2)

$[B]\ [E_b] = 0$ (3)

Substituting (1) in (3), we get

$[B]\ [V_g] + [B][Z_b]\ ([I_b] - [I_g]) = 0$

$[B][Z_b]\ [I_b] = [B][Z_b]\ [I_g] - [B]\ [V_g]$ (4)

Substituting (2) in (4)

$[B][Z_b]\ [B]^T\ [I_l] = [B]\ [Z_b]\ [I_g] - [B]\ [V_g]$ (5)

Let $[B]\ [Z_b]\ [I_g] - [B]\ [V_g] = [V_l]$

and $[B]\ [Z_b]\ [B]^T = [Z_l]$ = Loop Impedance matrix

Then equation (5) may be written as

$[Z_l]\ [I_l] = [V_l]$ or

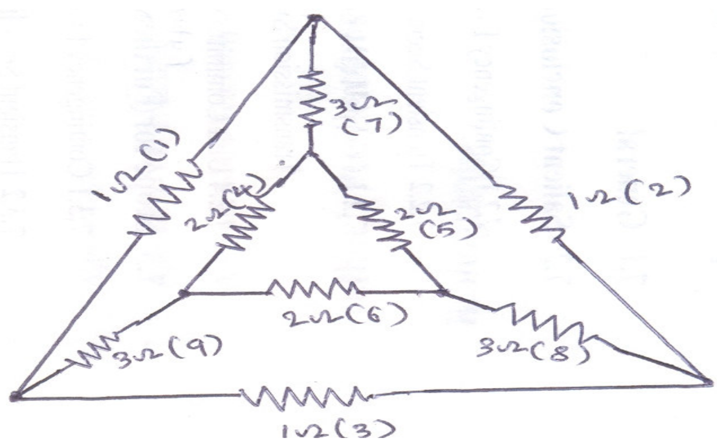
$[I_l] = [Z_l]^{-1}\ [V_l]$ (6)

Equations (5) and (6) represent a set of **Equilibrium Equations** with loop currents as independent variables. On solving these equations, loop currents are obtained. Once the loop currents are known, all the branch currents can be found. If the elements of the branches are known, then all branch voltages also can be found.

Example: 1)

(a) For the given network shown. Draw the graph, select a tree with branches 9, 4, 7, 5, & 8 and write the tie-set matrix. The number inside the brackets indicates branch numbers.

(b) Using the above tie-set matrix formulate equilibrium equations.

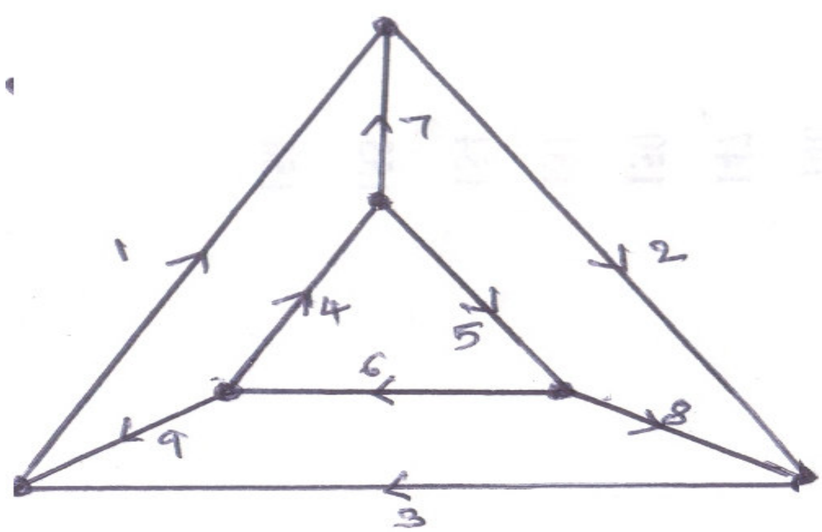


Solution: Total number of branches, b = 9;

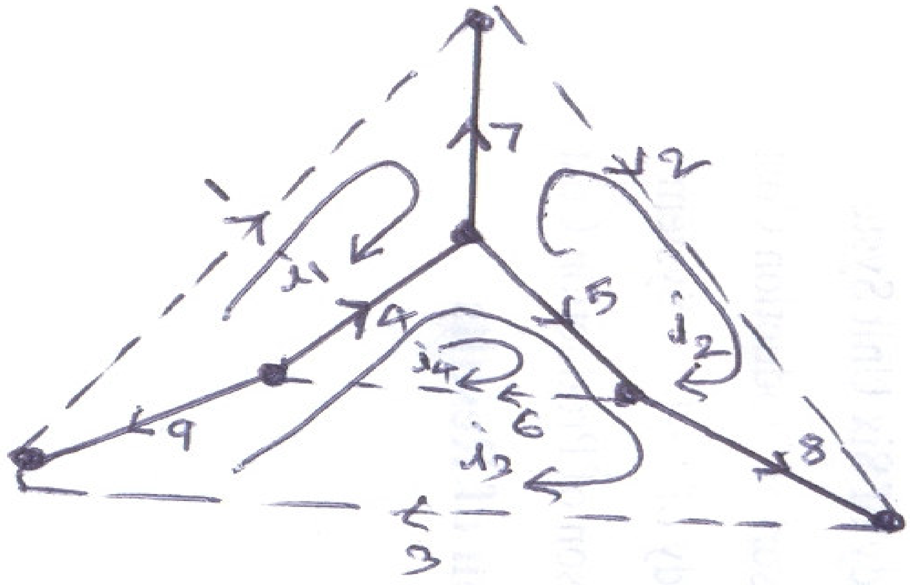
Total number of nodes n_t =6;

Total number of tree branches, n = (n_t-1) = (6-1) = 5;

Total number of links, l = (b– n) = (9 – 5) = 4



Oriented Graph



i_1, i_2, i_3 and i_4 are the loop currents

$$\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Rows of above matrix give KVL Equations

$$\left.\begin{aligned} & e_1 - e_4 - e_7 + e_9 = 0 \\ & e_2 - e_5 + e_7 - e_8 = 0 \\ & e_3 + e_4 + e_5 + e_8 - e_9 = 0 \\ & e_4 + e_5 + e_6 = 0 \end{aligned}\right\} \qquad (1)$$

Columns of above matrix give Branch Currents

$$\left.\begin{aligned} & j_1 = i_1 \\ & j_2 = i_2 \\ & j_3 = i_3 \\ & j_4 = -i_1 + i_3 + i_4 \\ & j_5 = -i_2+i_3+i_4 \\ & j_6 = i_4 \\ & j_7 = -i_1+i_2 \\ & j_8 = -i_2+i_3 \\ & j_9 = i_1 - i_3 \end{aligned}\right\} \qquad (2)$$

We have, branch voltage is given by $e_k = r_k\, j_k$. Where k=1, 2, 3 …9.

$$\begin{aligned} e_1 &= r_1\, j_1 \\ e_2 &= r_2\, j_2 \\ e_3 &= r_3\, j_3 \\ e_4 &= r_4\, j_4 \\ e_5 &= r_5\, j_5 \\ e_6 &= r_6\, j_6 \\ e_7 &= r_7\, j_7 \\ e_8 &= r_8\, j_8 \\ e_9 &= r_9\, j_9 \end{aligned} \tag{3}$$

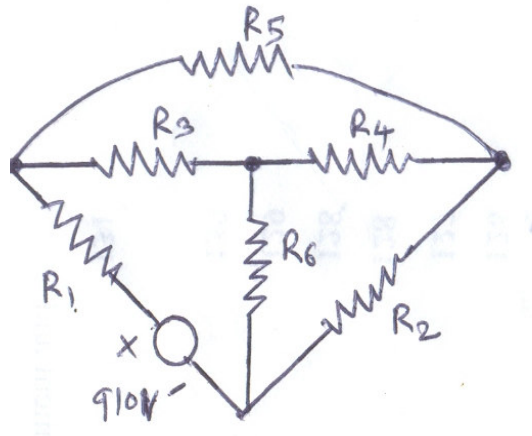
Substituting equation (2) in (3) and resulting equations are substituting in equation (1)
We get,

$$\begin{aligned} 9i_1 - 3i_2 - 5i_3 - 2i_4 &= 0 \\ -3i_1 + 9i_2 - 5i_3 - 2i_4 &= 0 \\ -5i_1 - 5i_2 + 11i_3 + 4i_4 &= 0 \\ -2i_1 - 2i_2 + 4i_3 + 6i_4 &= 0 \end{aligned} \tag{4}$$

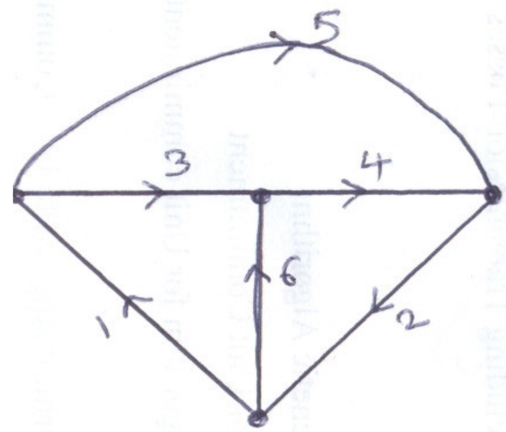
Set of equations (4) are called **Equilibrium Equations.** Solutions of these equations will give the **Link currents or Loop currents**. Then using these link currents, we can find out the branch currents

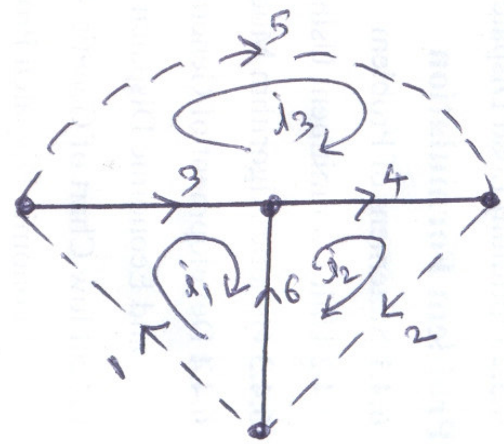
Example: 2)

For the given resistive network, write a tie-set schedule and equilibrium equations on the current basis. Obtain values of branch current and branch voltages. Given that R_1=5 Ω; R_2=5 Ω; R_3=R_4=R_6= 10 Ω and R_5= 2 Ω.



Solution:





Total number of branches, b = 6;
Total number of nodes $n_t = 4$;
Total number of tree branches, $n = (n_t-1) = (4-1) = 3$;
Total number of links, $l = (b - n) = (6 - 3) = 3$

Tie-set Schedule:

Link Currents	**Branches**					
	1	**2**	**3**	**4**	**5**	**6**
$\mathbf{i_1}$	**1**	**0**	**1**	**0**	**0**	**-1**
$\mathbf{i_2}$	**0**	**1**	**0**	**1**	**0**	**1**
$\mathbf{i_3}$	**0**	**0**	**-1**	**-1**	**1**	**0**

The columns of the above schedule give branch currents in terms of link currents

$$\left.\begin{aligned} j_1 &= i_1 \\ j_2 &= i_2 \\ j_3 &= (i_1 - i_3) \\ j_4 &= (i_2 - i_3) \\ j_5 &= i_3 \\ j_6 &= (-i_1 + i_2) \end{aligned}\right\} \qquad (1)$$

The rows of above schedule, give the KVL Equations

$$\left.\begin{aligned} e_1+e_3-e_6 &= 0 \\ e_2+e_4+e_6 &= 0 \\ -e_3-e_4+e_5 &= 0 \end{aligned}\right\} \qquad (2)$$

We have, branch voltage is given by $e_k = r_k\, j_k$. Where k=1, 2, 3 …6.

$$\left.\begin{aligned} e_1 &= r_1\, j_1 - 910 = 5i_1 - 910 \\ e_2 &= r_2\, j_2 = 5i_2 \\ e_3 &= r_3\, j_3 = 10i_1 - 10i_3 \\ e_4 &= r_4\, j_4 = 10i_2 - 10i_3 \\ e_5 &= r_5\, j_5 = 2i_3 \\ e_6 &= r_6\, j_6 = -10i_1 + 10i_2 \end{aligned}\right\} \qquad (3)$$

Substituting equation (3) in (2) we get,

$$\left.\begin{aligned} 25i_1 - 10i_2 - 10i_3 &= 910 \\ -10i_1 + 25i_2 - 10i_3 &= 0 \\ -10i_1 - 10i_2 + 22i_3 &= 0 \end{aligned}\right\} \qquad (4)$$

The set of equations (4) are called **Equilibrium Equations.**

$$\begin{pmatrix} 25 & -10 & -10 \\ -10 & 25 & -10 \\ -10 & -10 & 22 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 910 \\ 0 \\ 0 \end{pmatrix}$$

$i_1 = \Delta_1/\Delta = 40950/4550 = 90A$

Where

$$\Delta = \begin{vmatrix} 25 & -10 & -10 \\ -10 & 25 & -10 \\ -10 & -10 & 22 \end{vmatrix} \text{ and } \Delta_1 = \begin{vmatrix} 910 & -10 & -10 \\ 0 & 25 & -10 \\ 0 & -10 & 22 \end{vmatrix}$$

$$i_2 = \Delta_2/\Delta = 64A;\text{ where } \Delta_2 = \begin{vmatrix} 25 & 910 & -10 \\ -10 & 0 & -10 \\ -10 & 0 & 22 \end{vmatrix}$$

$$i_3 = \Delta_3/\Delta = 70A;\text{ where } \Delta_3 = \begin{vmatrix} 25 & -10 & 910 \\ -10 & 25 & 0 \\ -10 & -10 & 0 \end{vmatrix}$$

From equation (1), Branch currents in terms of link currents are given by

$j_1 = i_1 = 90A$
$j_2 = i_2 = 64A$
$j_3 = (i_1 – i_3) = (90-70) = 20A$
$j_4 = (i_2 - i_3) = (64-70) = -6A$
$j_5 = i_3 = 70A$
$j_6 = (-i_1 + i_2) = (-90+64) = -26A$

From equation (3), Branch Voltages in terms of branch currents or link currents are given by

$e_1 = r_1 j_1 - 910 = 5i_1 - 910 = 460V$
$e_2 = r_2 j_2 = 5i_2 = 320V$
$e_3 = r_3 j_3 = 10i_1 - 10i_3 = 200V$
$e_4 = r_4 j_4 = 10i_2 - 10i_3 = -60V$
$e_5 = r_5 j_5 = 2i_3 = 140V$
$e_6 = r_6 j_6 = -10i_1 + 10i_2 = -260V$

VIII. Cut-set:

Tree branches connect all the nodes in the network graph. Hence, it is possible to trace the path from one node to any other node by traveling along the tree branch only. Therefore, potential difference between any two nodes called node-pair voltage can be expressed in terms of tree branch voltages.

The cut set is a minimal set of branches of the graph, removal of which cuts the graph into two parts. It separates the nodes of the graph into two groups. The cut-set consists of only one tree branch and remainders are links. Each branch of the cut-set has one of its terminal incident at a node in one group and its other end at a node in the other group and its other end at a node in the other group. The orientation of the cut-set is same as orientation of tree branch.

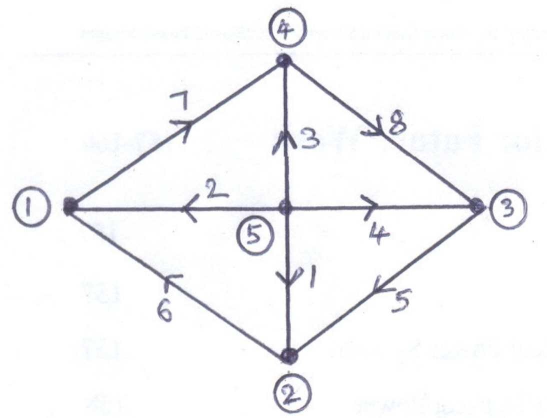
The number of cut-sets is equal to number of tree branches [i.e. (nt-1) = n where n_t is total number of nodes in the network graph].

IX. Cut-set schedule:

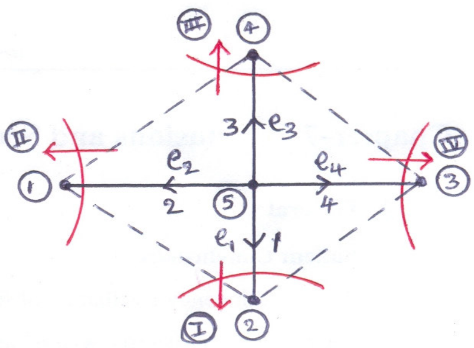
For a given network tree, a systematic way of indicating the tree branch voltage through use of a schedule called **cut-set schedule**

To write the cut-set schedule for network graph,

(i) Consider an oriented network graph
(ii) Write any one possible tree of the network graph
(iii) Assume tree branch voltages as (e1, e2…en) independent variables.
(iv) Assume the independent voltage variable is same direction as that of a tree branch voltage
(v) Mark the cut-sets (recognize) in the network graph.



Oriented Graph



Tree and Cut-sets

The tree branch voltages e_1, e_2, e_3, & e_4 entered in the first column of the schedule correspond to 4 branches 1, 2, 3 & 4. In order to fill the first row corresponding to the tree branch voltages e_1, by looking into the direction of currents in the branches connected to the cut-set under consideration. If the direction of current in the cut-set branch is towards the cut-set node, write **'+1'** in the branch column of concerned cut-set branch. If the direction of current in the cut-set branch is away from the cut-set node, write **'-1'** in that particular cut-set branch column. Write **'0'** in the branch columns, which are not in that particular, cut-set.

Cut-set schedule:

Tree Branch Voltages	Branches							
	1	2	3	4	5	6	7	8
e_1	1	0	0	0	1	-1	0	0
e_2	0	1	0	0	0	1	-1	0
e_3	0	0	1	0	0	0	1	-1
e_4	0	0	0	1	-1	0	0	1

The columns of the cut-set schedule give branch voltage equations;

i.e.
$$\left.\begin{aligned}
v_1&=e_1\\
v_2&=e_2\\
v_3&=e_3\\
v_4&=e_4\\
v_5&=(e_1-e_4)\\
v_6&=(-e_1+e_2)\\
v_7&=(-e_2+e_3)\\
v_8&=(-e_3+e_4)
\end{aligned}\right\} \qquad (1)$$

The rows of cut-set schedule give KCL equations;

$$\left.\begin{aligned}
j_1+j_5-j_6&=0\\
j_2+j_6-j_7&=0\\
j_3+j_7-j_8&=0\\
j_4-j_5+j_8&=0
\end{aligned}\right\} \qquad (2)$$

The cut-set schedule can be written in matrix form is known as **cut-set matrix**. This matrix can be represented by **Q.**

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1\\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix}$$

From set of equations (1)

$$\begin{bmatrix} v_1\\ v_2\\ v_3\\ v_4\\ v_5\\ v_6\\ v_7\\ v_8 \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 0 & -1\\ -1 & 1 & 0 & 0\\ 0 & -1 & 1 & 0\\ 0 & 0 & -1 & 1 \end{pmatrix} \begin{bmatrix} e_1\\ e_2\\ e_3\\ e_4 \end{bmatrix}$$

In compact form;

$$[\mathbf{V_b}] = [\mathbf{Q}]^{\mathbf{T}}[\mathbf{V_T}] \qquad (3)$$

Where
$[V_b]$ = Branch Voltage Column Matrix of order **bx1**
$[V_T]$ = Tree Branch Voltage Column Matrix of order **nx1**
$[Q]$ = Cut-set Matrix of order **nxb**

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} j_1 \\ j_2 \\ j_3 \\ j_4 \\ j_5 \\ j_6 \\ j_7 \\ j_8 \end{pmatrix} = 0$$

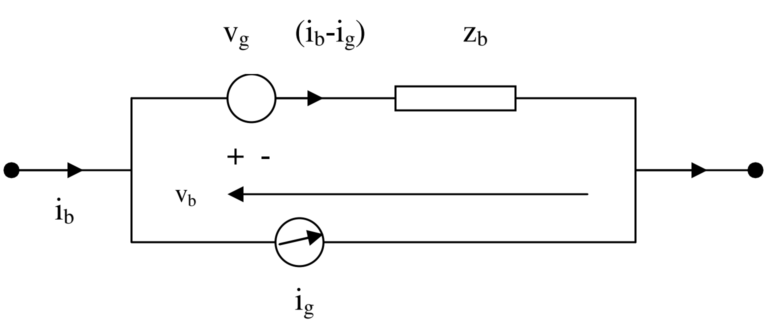
In compact form;

$[Q]\,[I_b] = 0$ **(4)**

Where
$[I_b]$ = Branch Current Column Matrix of order **bx1**
$[Q]$ = Cut-set Matrix of order **nxb**

X. Formation of Equilibrium Equation in matrix form with tree branch voltages as Variables

Consider a general branch of a network as shown in figure



Where v_g = Total series voltage in the branch

i_g = Total current source connected across the branch

z_b = Total impedance of the branch

i_b = Branch current

The voltage current relation for the branch can be written as

$$v_b = v_g + z_b (i_b - i_g) \qquad (1)$$

and $$i_b = i_g + y_b (v_b - v_g) \qquad (2)$$

Where y_b = Total admittance of the branch.
For a network with more number of branches, equation (i) & (ii) may be written as

$$[V_b] = [V_g] + [Z_b] ([I_b] - [I_g]) \qquad (3)$$
$$[I_b] = [I_g] + [Y_b] ([V_b] - [V_g]) \qquad (4)$$

Where
$[V_b]$, $[V_g]$, $[I_b]$, & $[I_g]$ are (bx1) matrices of branch voltages, source voltages in the branches, branch currents and source currents in the branches respectively.
$[Z_b]$ & $[Y_b]$ are branch impedance and branch admittance matrices of the order (bxb).

We have

$$[V_b] = [V_g] + [Z_b] ([I_b] - [I_g])$$

$$[V_b] = [Q]^T [V_T] \qquad (5)$$

$$[Q] [I_b] = 0 \qquad (6)$$

Substituting (4) in (6), we get
$$[Q] [I_g] + [Q] [Y_b] \{[V_b] - [V_g]\} = 0$$
$$[Q] [I_g] + [Q] [Y_b] [V_b] - [Q] [Y_b][V_g] = 0 \qquad (7)$$

Substituting (5) in (7), we get
$$[Q] [I_g] + [Q] [Y_b] [Q]^T [V_T] - [Q] [Y_b][V_g] = 0 \qquad (8)$$

Let,
$[Q] [Y_b] [Q]^T = [Y_c]$ = Cut-set admittance matrix

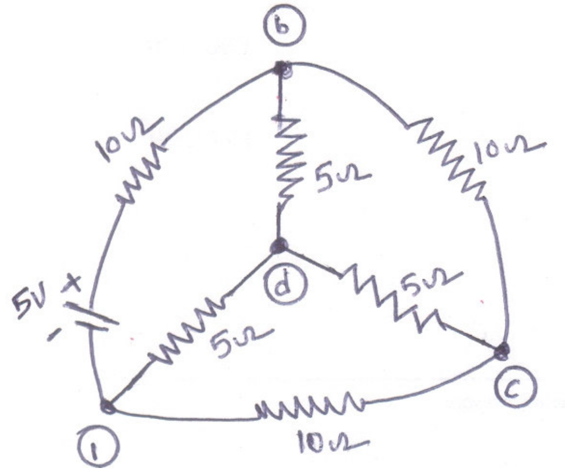
Equation (8) becomes,
$$[Y_c] [V_T] = [Q] [Y_b][V_g] - [Q] [I_g]$$
Or
$$[V_T] = [Y_c]^{-1} \{[Q] [Y_b][V_g] - [Q] [I_g]\} \qquad (9)$$

This equation (9) is known as **Equilibrium Equation**.

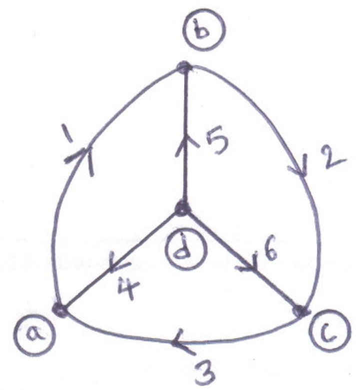
The solutions of equation (9) give the tree branch voltages and branch currents can be find using equation (5) and (4).

Example: 1)

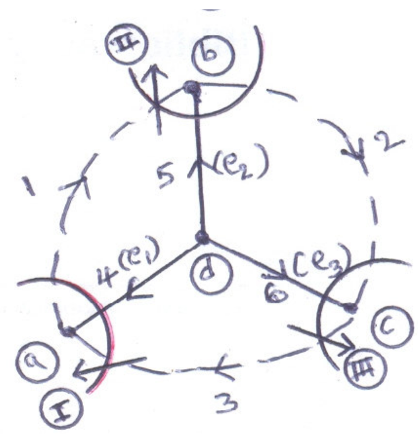
For the given resistive network, write a cut-set schedule and obtain equilibrium equations on the voltage basis. Solve these equations and hence calculate values of branch voltages and branch currents.



Solution:



Oriented Graph



Tree and Cut-sets

Cut-set Schedule:

Cut-sets or Tree branch Voltages	Branches					
	1	**2**	**3**	**4**	**5**	**6**
1 or (e_1)	-1	0	1	1	0	0
2 or (e_2)	1	-1	0	0	1	0
3 or (e_3)	0	1	-1	0	0	1

Columns of the above schedule give branch voltages in terms of tree branch voltages

$$\left.\begin{aligned} v_1 &= -e_1 + e_2 \\ v_2 &= -e_2+e_3 \\ v_3 &= e_1-e_3 \\ v_4 &= e_1 \\ v_5 &= e_2 \\ v_6 &= e_3 \end{aligned}\right\} \quad (1)$$

Rows of the above schedule give KCL equations

$$\left.\begin{aligned} -j_1+ j_3 + j_4 &= 0 \\ j_1- j_2 + j_5 &= 0 \\ j_2- j_3 + j_6 &= 0 \end{aligned}\right\} \quad (2)$$

We have from given network, $(v_{k\pm} v_{g)} = j_k \times r_k$ Where k=1, 2, 3 …6.

Branch currents $j_k = (v_{k\,\pm}\, v_{g)} /r_{k.}$ Hence,

$$\left.\begin{aligned} j_1 &= (v_1 + 5)/10 = (-e_1 + e_2 + 5)/10 = -0.1e_1 + 0.1e_2 + 0.5 \\ j_2 &= v_2/10 = (-e_2 + e_3) /10 = -0.1e_2 + 0.1e_3 \\ j_3 &= v_3/10 = (e_1- e_3)/10 = 0.1e_1 - 0.1e_3 \\ j_4 &= v_4/5 = e_1/5 = 0.2e_1 \\ j_5 &= v_5/5 = e_2/5 = 0.2e_2 \\ j_6 &= v_6/5 = e_3/5 = 0.2e_3 \end{aligned}\right\} \quad (3)$$

Substituting set of equations (3) in (2), we get

$$\left.\begin{aligned} 0.4e_1 -0.1e_2 -0.1e_3 &= 0.5 \\ -0.1e_1 +0.4e_2 -0.1e_3 &= - 0.5 \\ -0.1e_1 -0.1e_2 +0.4e_3 &= 0 \end{aligned}\right\} \quad (4)$$

The set of equations (4) are called **Equilibrium Equations**

$$\begin{pmatrix} \mathbf{0.4} & \mathbf{-0.1} & \mathbf{-0.1} \\ \mathbf{-0.1} & \mathbf{0.4} & \mathbf{-0.1} \\ \mathbf{-0.1} & \mathbf{-0.1} & \mathbf{0.4} \end{pmatrix} \begin{pmatrix} \mathbf{e_1} \\ \mathbf{e_2} \\ \mathbf{e_3} \end{pmatrix} = \begin{pmatrix} \mathbf{0.5} \\ \mathbf{-0.5} \\ \mathbf{0} \end{pmatrix}$$

$$\Delta = \begin{vmatrix} 0.4 & -0.1 & -0.1 \\ -0.1 & 0.4 & -0.1 \\ -0.1 & -0.1 & 0.4 \end{vmatrix} = 50 \qquad \Delta_1 = \begin{vmatrix} 0.5 & -0.1 & -0.1 \\ -0.5 & 0.4 & -0.1 \\ 0 & -0.1 & 0.4 \end{vmatrix} = 50$$

$$\Delta_2 = \begin{vmatrix} 0.4 & 0.5 & -0.1 \\ -0.1 & -0.5 & -0.1 \\ -0.1 & 0 & 0.4 \end{vmatrix} = -50$$

$$\Delta_3 = \begin{vmatrix} .4 & -0.1 & 0.5 \\ -0.1 & 0.4 & -0.5 \\ -0.1 & -0.1 & 0 \end{vmatrix} = 0$$

$e_1 = v_4 = \Delta_1 / \Delta = 1V$
$e_2 = v_5 = \Delta_2 / \Delta = -1V$
$e_3 = v_6 = \Delta_3 / \Delta = 0V$
$v_1 = -e_1+e_2 = -1-1= -2V$
$v_2 = -e_2+e_3 = 1+ 0=1V$
$v_3 = e_1-e_3 = 1-0=1V$

From equation (3),

$j_1= (v_1 + 5)/10 = (-2+5)/10 = 0.3A$

$j_2= v_2/10 = 1/10 = 0.1\ A$

$j_3= v_3/10 =1/10 = 0.1\ A$

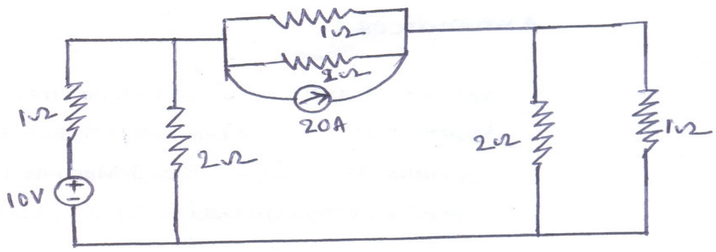
$j_4= v_4/5 = 1/5 = 0.2\ A$

$j_5= v_5/5 = 1/5 = 0.2\ A$

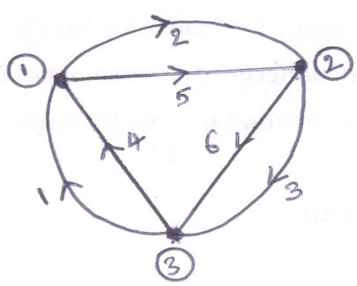
$j_6= v_6/5 = 0/5 = 0\ A$

Example: 2)

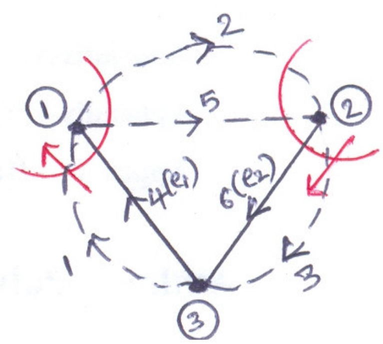
Draw the graph for the network shown in figure below. Write the cut-set schedule & obtain equilibrium equations and hence calculate values of branch voltages and branch currents



Solution:



Oriented Graph



Tree and Cut-sets

Cut-set Schedule:

Cut-sets or Tree branch Voltages	Branches					
	1	2	3	4	5	6
1 or (e_1)	1	-1	0	1	-1	0
2 or (e_2)	0	-1	1	0	-1	1

Columns of the above schedule give branch voltages in terms of tree branch voltages

$$\left.\begin{aligned} v_1 &= e_1 \\ v_2 &= -e_1 - e_2 \\ v_3 &= e_2 \\ v_4 &= e_1 \\ v_5 &= -e_1 - e_2 \\ v_6 &= e_2 \end{aligned}\right\} \qquad (1)$$

Rows of the above schedule give KCL equations

$$\left.\begin{array}{l} j_1 - j_2 + j_4 - j_5 = 0 \\ -j_2 + j_3 - j_5 + j_6 = 0 \end{array}\right\} \qquad (2)$$

We have from given network, $(v_{k \pm} v_{g)} = j_k \text{ x } r_k$
Branch currents $j_k = (v_{k \pm} v_{g)} / r_{k.}$ Hence,

$$\left.\begin{array}{l} j_1 = (v_1 + 10)/1 = (e_1+10)/1 = e_1 + 10 \\ j_2 = v_2/1 = (-e_1 - e_2)/1 = -e_1 - e_2 \\ j_3 = v_3/1 = e_2/1 = e_2 \\ j_4 = v_4/2 = e_1/2 = 0.5e_1 \\ j_5 = (v_5 + 40)/2 = [(-e_1 - e_{2)} + 40]/2 = -0.5e_1 - 0.5e_2 + 20 \\ j_6 = v_6/2 = e_2/2 = 0.5e_2 \end{array}\right\} \qquad (3)$$

Substituting set of equations (3) in (2), we get

$$\left.\begin{array}{l} 3e_1 + 1.5e_2 = 10 \\ 1.5e_1 + 3e_2 = 20 \end{array}\right\} \qquad (4)$$

The set of equations (4) are called **Equilibrium Equations**

$$\begin{pmatrix} \mathbf{3} & \mathbf{1.5} \\ \mathbf{1.5} & \mathbf{3} \end{pmatrix} \begin{pmatrix} \mathbf{e_1} \\ \mathbf{e_2} \end{pmatrix} = \begin{pmatrix} \mathbf{10} \\ \mathbf{20} \end{pmatrix}$$

$$\mathbf{e_1 = \Delta_1 / \Delta \quad where \quad \Delta = \begin{vmatrix} 3 & 1.5 \\ 1.5 & 3 \end{vmatrix} = 6.75 \text{ and } \Delta_1 = \begin{vmatrix} 10 & 1.5 \\ 20 & 3 \end{vmatrix} = 0}$$

$\mathbf{e_1 = 0/6.75 = 0\ V.}$

$$\mathbf{e_2 = \Delta_2 / \Delta \quad where \ \Delta = \begin{vmatrix} 3 & 1.5 \\ 1.5 & 3 \end{vmatrix} = 6.75 \text{ and } \Delta_2 = \begin{vmatrix} 10 & 1.5 \\ 20 & 3 \end{vmatrix} = 45}$$

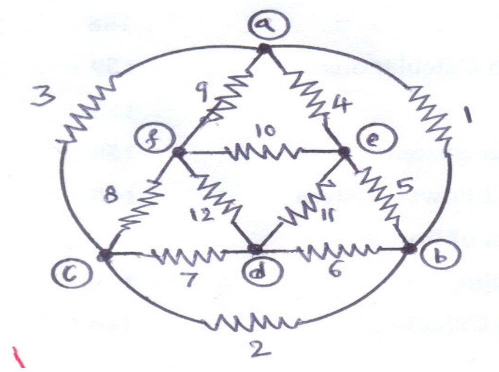
$\mathbf{e_2 = 45/6.75 = 6.667\ V.}$

The branch voltages are
$v_1 = e_1 =$ **0 V**
$v_2 = -e_1 - e_2 = 0 - 6.667 =$ **-6.667 V**
$v_3 = e_2 =$ **6.667 V**
$v_4 = e_1 =$ **0 V**
$v_5 = -e_1 - e_2 = -0 - 6.667 =$ **-6.667V**
$v_6 = e_2 =$ **6.667 V**

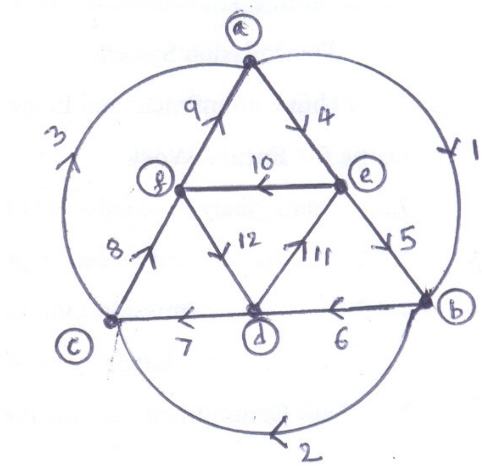
The branch currents are
$j_1 = (v_1 + 10)/1 =$ **10 A**
$j_2 = v_2/1 =$ **6.667 A**
$j_3 = v_3/1 =$ **6.667 A**
$j_4 = v_4/2 =$ **0 A**
$j_5 = (v_5 + 40)/2 = (-6.667 + 40)/2 =$ **16.66 A**
$j_6 = v_6/2 =$ **3.33 A**

Example: 3)

For the given network, draw the oriented graph and a tree. Select suitable tree branch voltages and write the cut-set schedule and also write the equations for the branch voltages in terms of the tree branch voltages.



Solution: **Oriented Graph**



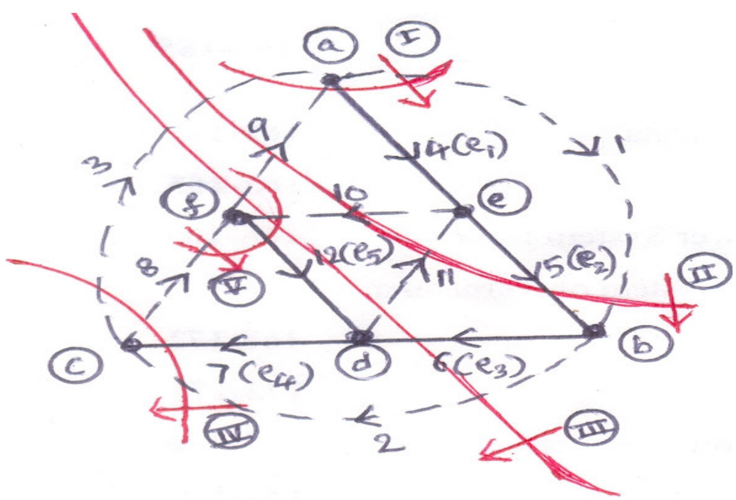
For the given network graph,

Number of branches, b = 12

Number of nodes, n_t= 6

Number of Cut-sets = [n_t - 1]

= [6-1] =5



Tree and Cut-sets

Cut-set Schedule:

Tree Branch Voltages or Cut-set	Branches											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
e_1	1	0	-1	1	0	0	0	0	-1	0	0	0
e_2	1	0	-1	0	1	0	0	0	-1	1	-1	0
e_3	0	1	-1	0	0	1	0	0	-1	1	-1	0
e_4	0	1	-1	0	0	0	1	-1	0	0	0	0
e_5	0	0	0	0	0	0	0	-1	1	-1	0	1

Columns of the above schedule will give branch voltages in terms of tree branch voltages

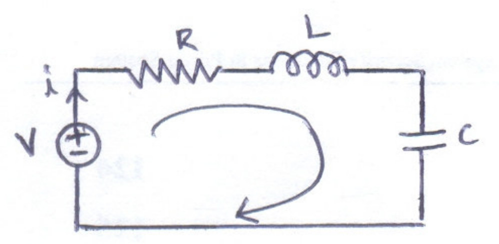
$v_1 = e_1+e_2$
$v_2 = e_3+e_4$
$v_3 = -e_1- e_2 – e_3 –e_4$
$v_4= e_1$
$v_5= e_2$
$v_6= e_3$
$v_7 = e_4$
$v_8 = -e_4 – e_5$
$v_9 = -e_1-e_2 - e_3 - e_5$
$v_{10} = e_2+e_3-e_5$
$v_{11} = -e_1- e_3$
$v_{12} = e_5$

XI. Principle of Duality:

Duality: is the mutual relationship.

We come across a number of similarities in analyzing the network on current (Loop) basis and voltage (Node) basis. The principal quantities (and concepts) involved in the two methods form pairs. Each of the quantity in such a pair thus plays a dual role. These quantities (or concepts) forming pair are called **dual quantities.**

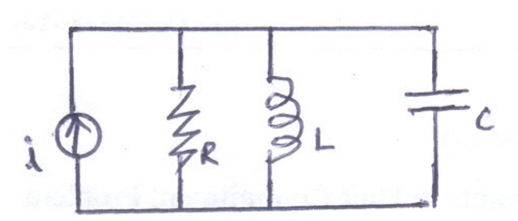
Consider a network containing R, L & C elements connected in series and excited by a voltage source as shown.



The integro- differential equations for the circuit is

$$R\, i + L\, (di/dt) + (1/C) \int i\, dt = v \qquad (1)$$

Consider a network containing R, L & C elements connected in parallel and driven by a current source as shown.



The integro- differential equations for the circuit is

$$(1/R)\ v + C\ (dv/dt) + (1/L) \int v\ dt = i \qquad (2)$$

OR

$$G\ v + C\ (dv/dt) + (1/L) \int v\ dt = i \qquad (2)$$

If we observe both the equations, the solutions of these equations are the same.

Therefore, these two networks are called **duals**

XII. Construction of a Dual of a Network:

Only planar networks without mutual inductances have **duals.**

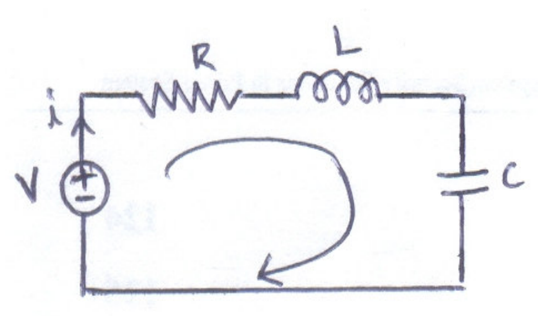
S.N.	Quantity or Concept	Dual Quantity or Concept
1	Current	Voltage
2	Resistance	Conductance
3	Inductance	Capacitance
4	Branch Current	Branch Voltage
5	Mesh	Node
6	Loop	Node-Pair
7	Number of Loops	Number of Node-Pairs
8	Loop Current	Node-Pair Voltage
9	Mesh Current	Node Voltage or Node Potential
10	Link	Tree Branch
11	Tie-set	Cut-set
12	Short Circuit	Open Circuit
13	Parallel Path	Series Path
14	Charge (Q)	Flux Linkages (ψ)

Procedure to draw a Dual Network:

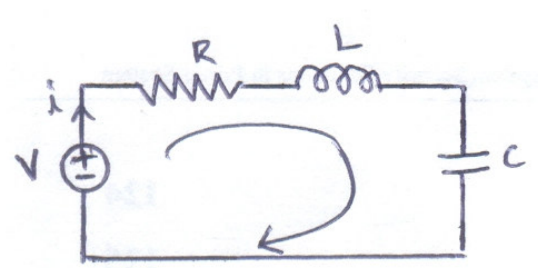
Step 1: In each Loop of a given network place a node and place an extra node called reference node outside the network.

Step 2: Draw the lines connecting adjacent nodes passing through each element and also to the reference node by placing the dual of each element in the line passing through original elements.

Example: 1) Draw the dual of a network for given network shown in figure.



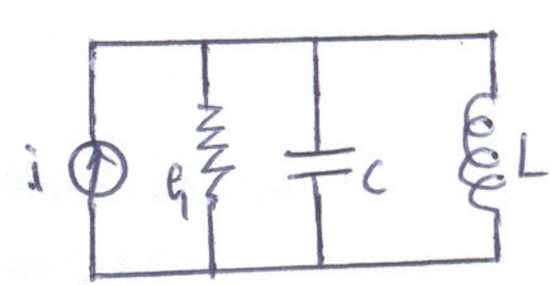
Solution:



The integro- differential equations for the circuit is

$$R\, i + L\,(di/dt) + (1/C) \int i\,dt = v \qquad (1)$$

Dual Network

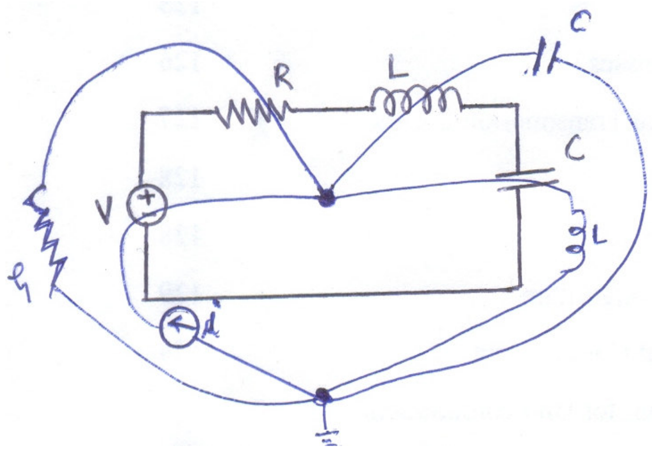


The integro- differential equations for the circuit is

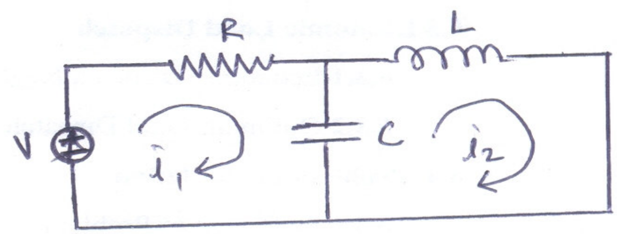
$(1/R)\ v + C\ (dv/dt) + (1/L) \int v\ dt = i$ (2)

OR

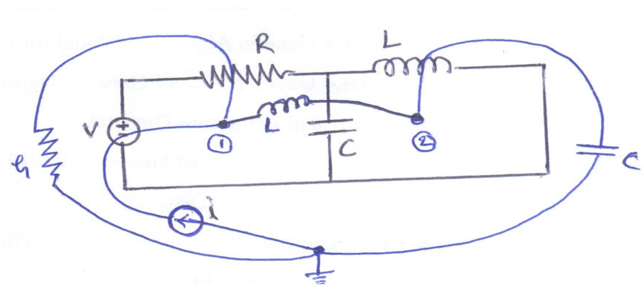
$G\ v + C\ (dv/dt) + (1/L) \int v\ dt = i$



Example: 2) Draw the dual of the network shown in figure.



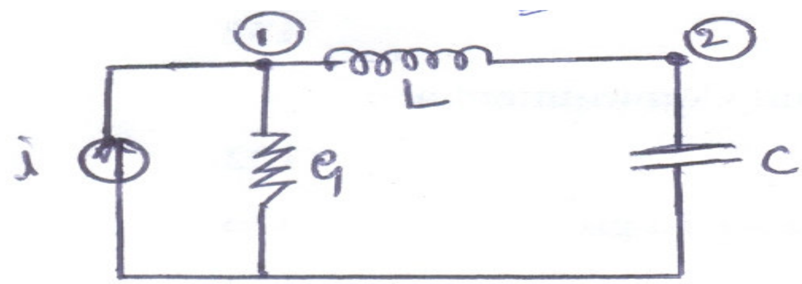
Solution:



The integro- differential equations for the network is

$R\ i_1 + (1/C) \int (i_1 - i_2)\ dt = v\ (t)$ (1)

$L\ (di_2/dt) + (1/C) \int (i_2 - i_1)\ dt = 0$ (2)

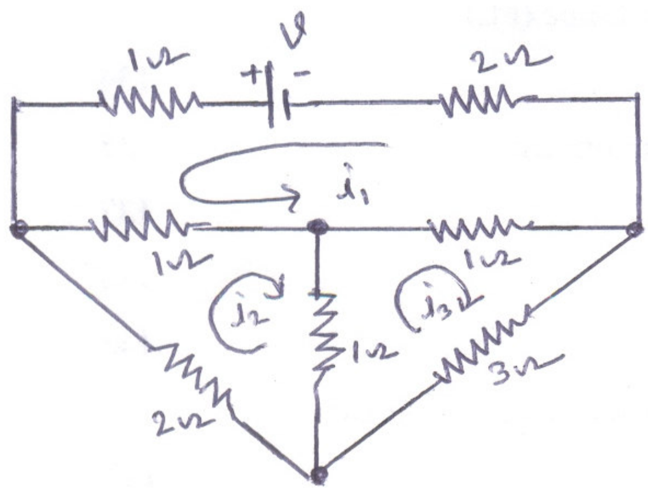


The integro- differential equations for the network is

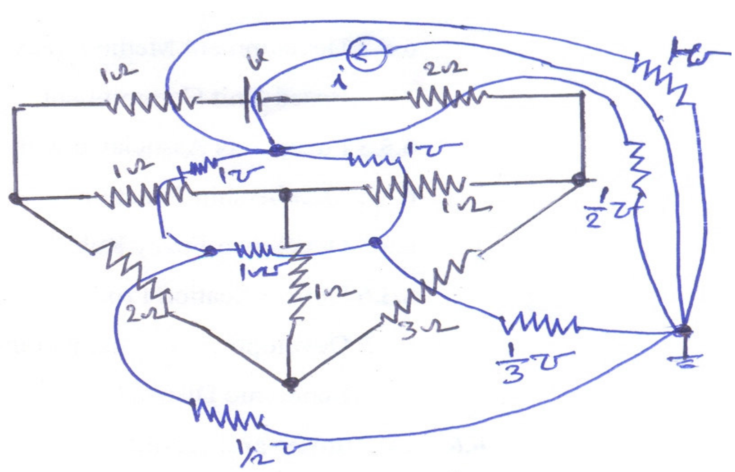
$G\, v_1 + (1/L) \int (v_1 - v_2)\, dt = i$ (1)

$C\, (dv_2/dt) + (1/L) \int (v_2 - v_1)\, dt = 0$ (2)

Example: 3) Draw the dual of the network shown in figure.

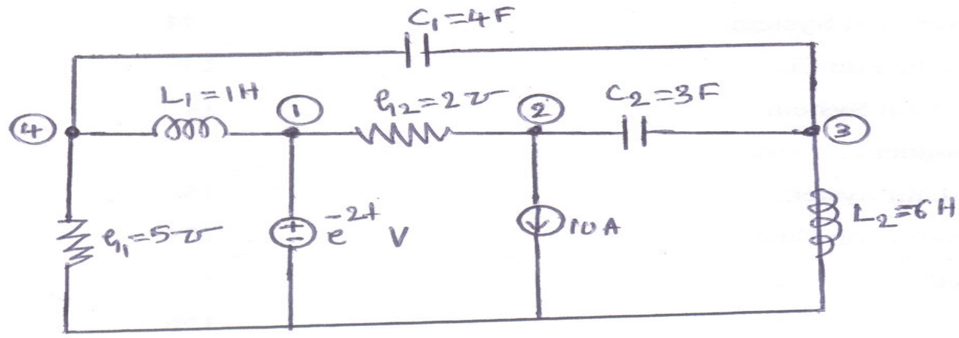


Solution:



Example: 4)

For the network shown in figure below, write the node equations. Draw the dual of this network and write mesh equations for the dual network. Verify whether these two sets of equations are dual equations



Node Equations:

At node (1);

$$(I/L_1) \int (e^{-2t} - v_4)\, dt + G_2 (e^{-2t} - v_2) = 0 \qquad (1)$$

At node (2);

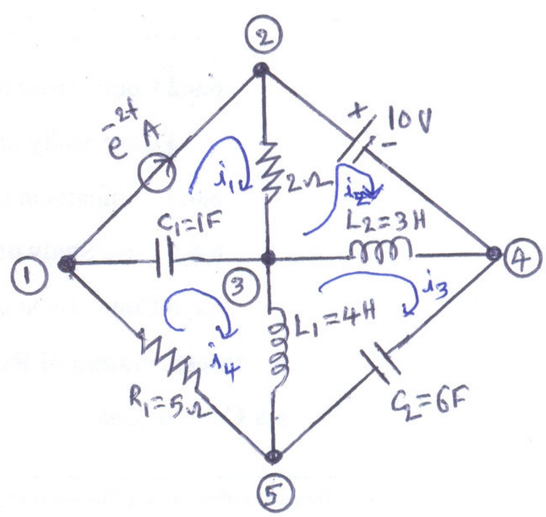
$$(G_2) \int (v_2 - e^{-2t}) + C_2 [d (v_2 - v_3) /dt] + 10 = 0 \qquad (2)$$

At node (3);

$$(I/L_2) \int v_3\, dt + C_2 [d (v_3 - v_2) /dt] + C_1 [d (v_3 - v_4) /dt] = 0 \qquad (3)$$

At node (4);

$$G_1 v_4 + (1/L_1) \int (v_4 - e^{-2t})\, dt + C_1 [d (v_4 - v_3) /dt] = 0 \qquad (4)$$



Loop Equations for dual network are

$$(I/C_1) \int (e^{-2t} - i_4)\, dt + R_2 (e^{-2t} - i_2) = 0 \quad (5)$$

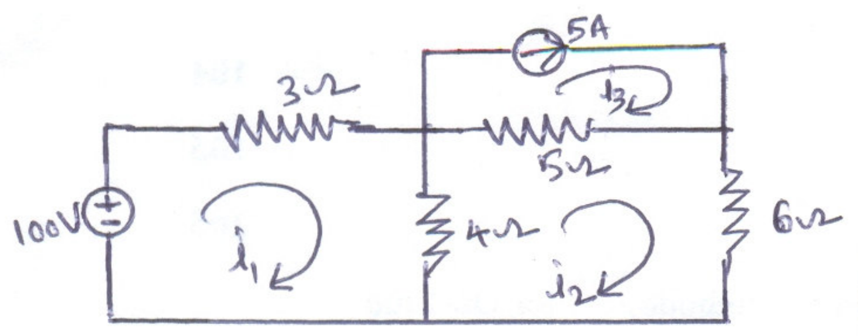
$$R_2 \int (i_2 - e^{-2t}) + L_2 [d (i_2 - i_3) /dt] + 10 = 0 \quad (6)$$

$$(I/C_2) \int i_3\, dt + L_2 [d (i_3 - i_2) /dt] + L_1 [d (i_3 - i_4) /dt] = 0 \quad (7)$$

$$R_1\, i_4 + (1/C_1) \int (i_4 - e^{-2t})\, dt + L_1 [d (i_4 - i_3) /dt] = 0 \quad (8)$$

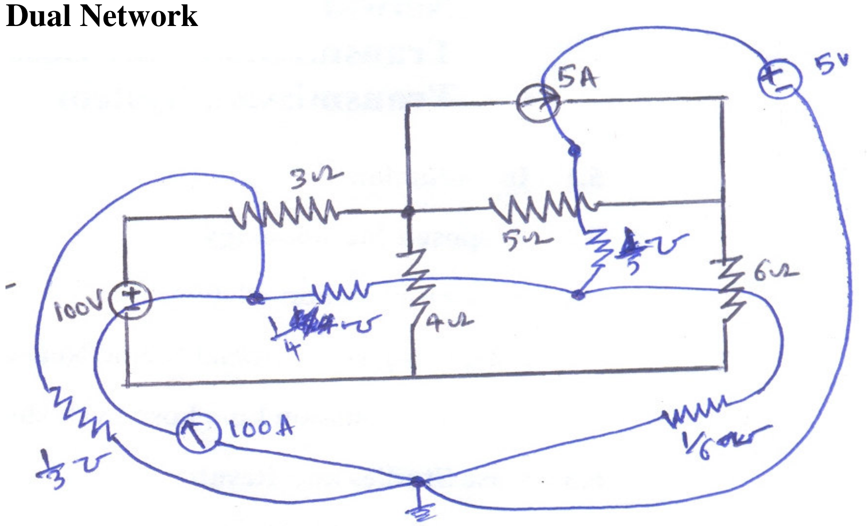
By looking into two sets of equations, we can say that two sets of equations are dual equations.

Example: 5) Draw the dual of the network shown below



Solution:

Dual Network



KVL Equations;

$-100 + 3i_1 + 4(i_1 - i_2) = 0$
$7i_1 - 4i_2 = 100$ (1)

$4(i_2 - i_1) + 5(i_2 - i_3) + 6i_2 = 0$
$-4i_1 + 15i_2 - 5i_3 = 0$ (2)

$-5i_2 + 5i_3 = 0$
$-5i_2 + 25 = 0$ (3)
Since $i_3 = 5A$

KCL Equations;

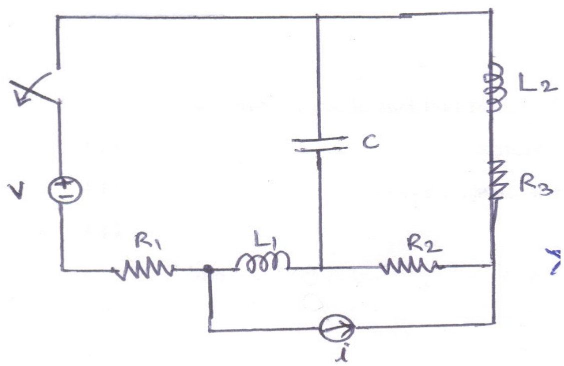
$100 = (1/3)v_1 + (1/4)(v_1 - v_2)$
$(7/12)v_1 - (1/4)v_2 = 100$ (1)

$(1/4)(v_2 - v_1) + (1/5)(v_2 - v_3) + (1/6)v_2 = 0$
$(-1/4)v_1 + (9/20)v_2 - (1/5)v_3 = 0$ (2)

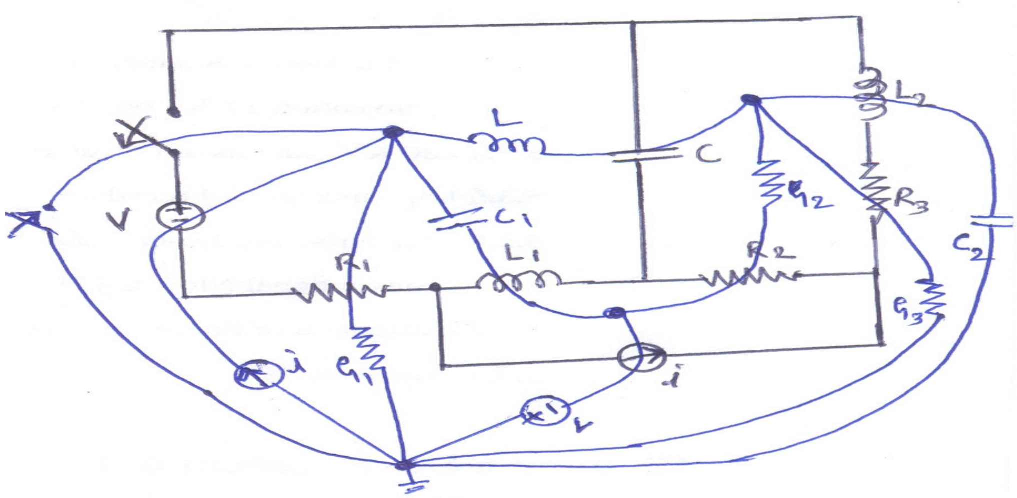
$(-1/5)v_2 + (1/5)v_3 = 0$
$(-1/5)v_2 + 1 = 0$ (3)
Since $v_3 = 5v$

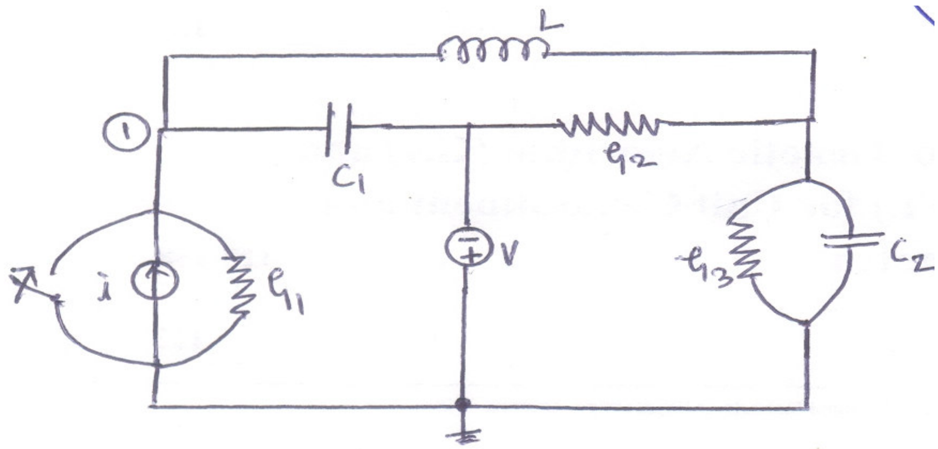
By looking into two sets of equations, we can say that two sets of equations dual equations

Example: 6) Draw the dual of the network shown below

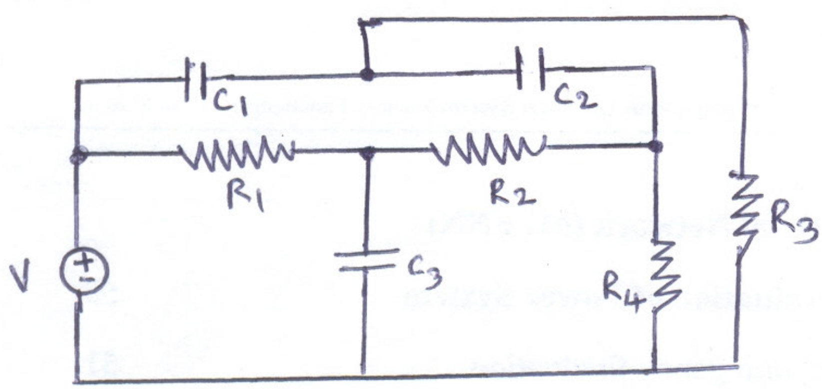


Solution:

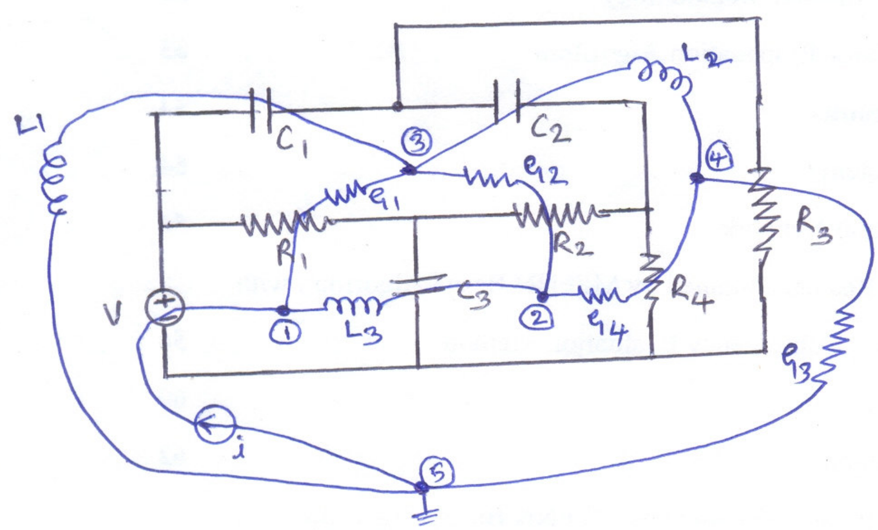




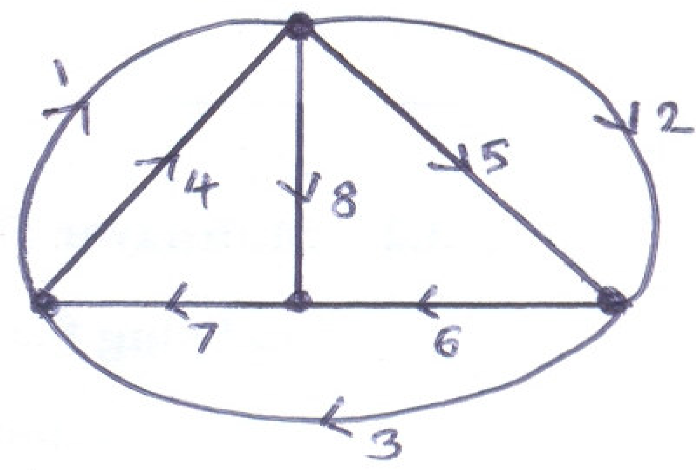
Example: 7) Draw the dual of the network shown below



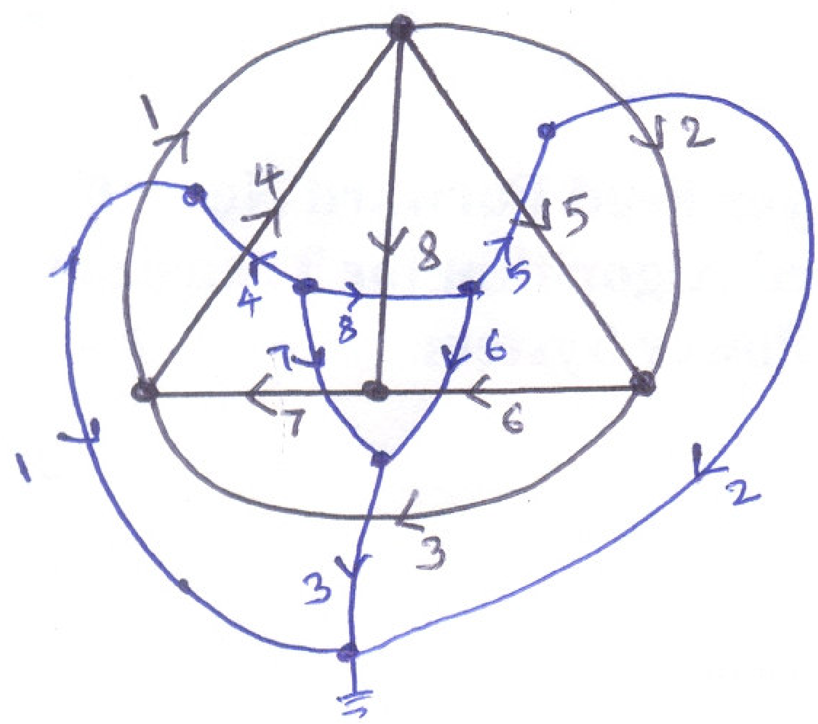
Solution:



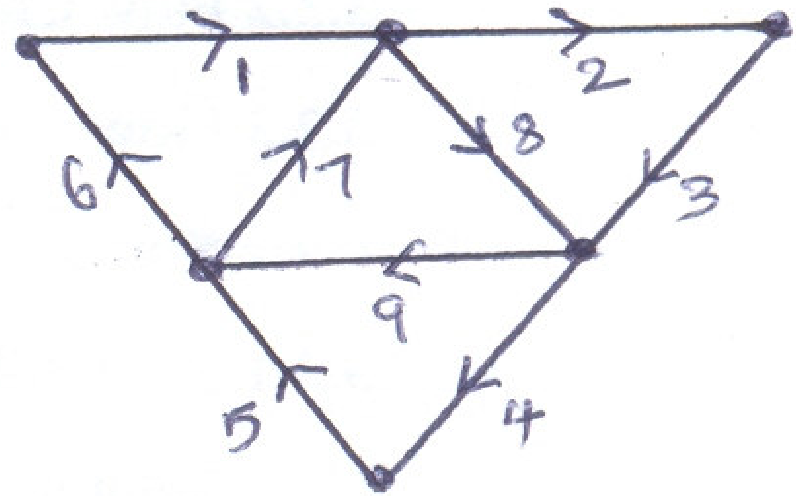
Example: 8) Draw the dual of the given oriented graph



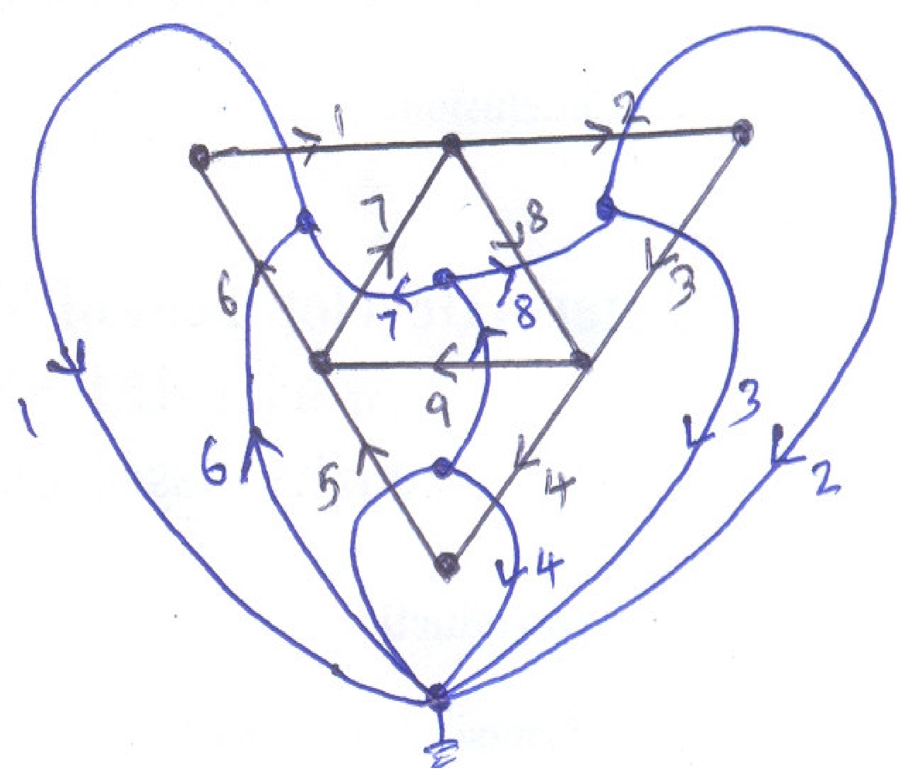
Solution:



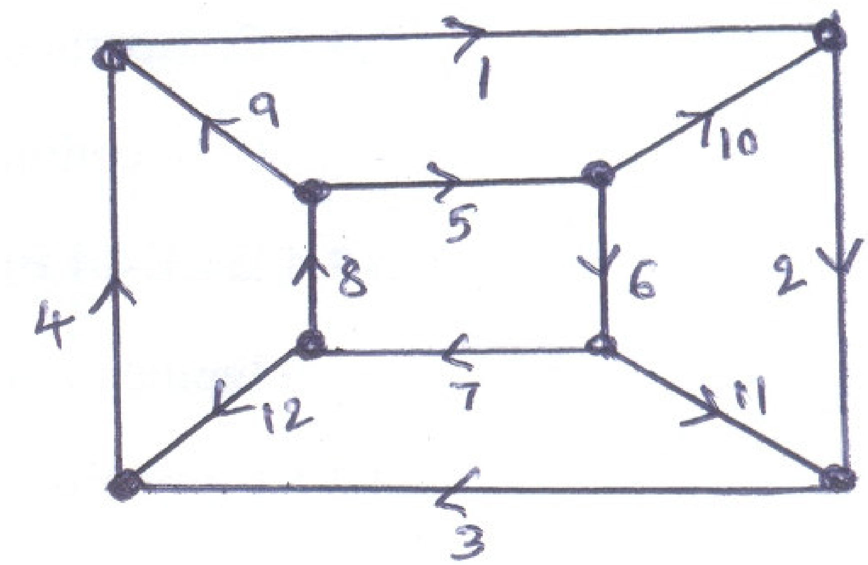
Example: 9) Draw the dual of the given oriented graph



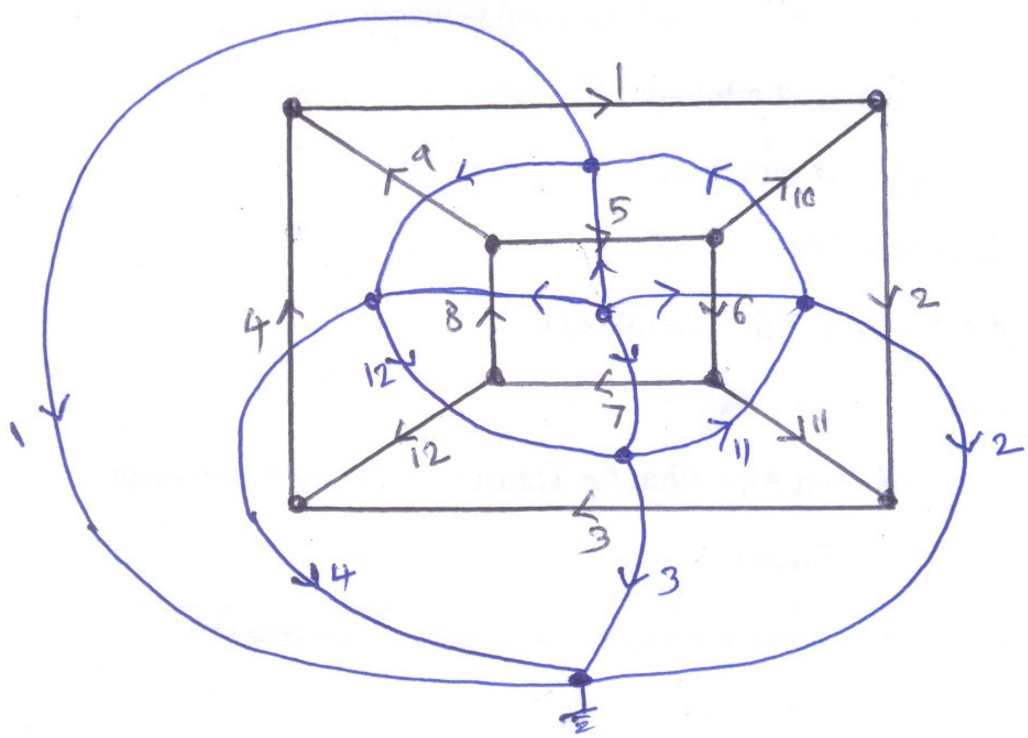
Solution:



Example: 10) Draw the dual of the given oriented graph



Solution:



UNIT - II

LAPLACE AND FREQUENCY DOMAIN ANALYSIS

S - Domain Analysis :-

* The soln. of differential eqns. for network is of form

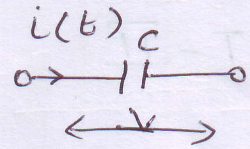
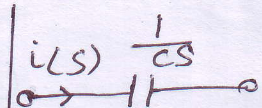
$$i(t) = k_n e^{s_n t}$$

$$s_n = \sigma_n + j\omega_n$$

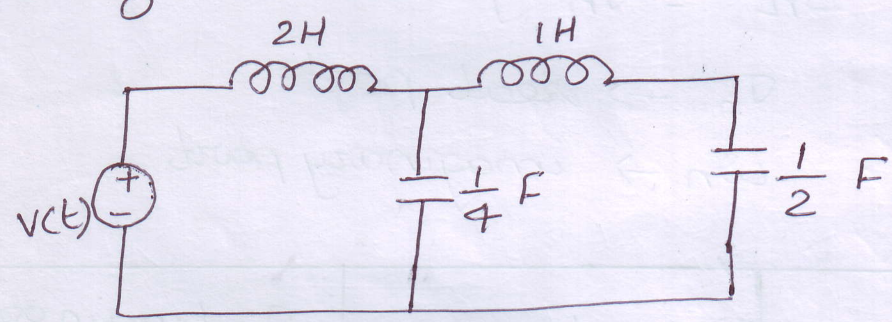
$\sigma_n \rightarrow$ real part

$\omega_n \rightarrow$ imaginary part

Element	Time-domain Representation	S-domain eqvt. circuit	
		With zero Initial Condition	With non-zero intial Condition
Resistor	i $V = Ri$	$I(S)$ R $V(S) = RI(S)$	$I(S)$ R $V(S) = RI(S)$
Inductance	i L V $V = L\frac{di}{dt}$ $i =$ [illegible] $i(0^-)$	$I(s)$ LS $V(S)$ $V(S) = LSI(S)$ $I(S) = \frac{V(S)}{[illegible]}$	L $V(S) = LSI(S) - Li(0^-)$ $I(S) = \frac{V(S)}{LS} +$ $i(0^-)$

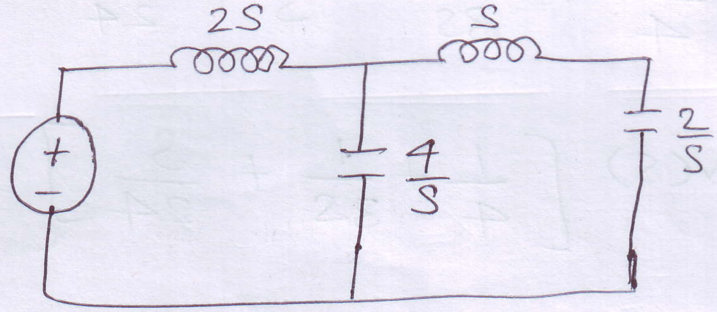
Capacitor	 $i = C\frac{dv}{dt}$ $V = \frac{1}{C}\int i\,dt + v(0^-)$	 $V(S) = \frac{I(S)}{CS}$	$V(S) = \frac{I(S)}{CS} + \frac{V(0^-)}{S}$ $I(S) = V(S)\,CS - CV(0^-)$

1. Draw the s-domain circuit for the circuit shown in fig. Assume initial conditions are zero.

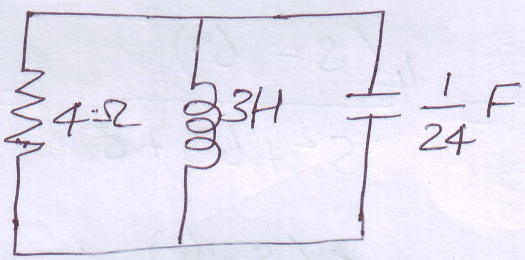


Element	time domain representation	S-domain representation
Voltage Source	$v(t)$	$V(S)$
Inductor 1	2H	2S
2	1H	1S
Capacitor 1	$\frac{1}{4}$ F	$\frac{1}{\frac{1}{4}S} = \frac{4}{S}$
2	$\frac{1}{2}$ F	$\frac{1}{\ldots} = \frac{2}{\ldots}$

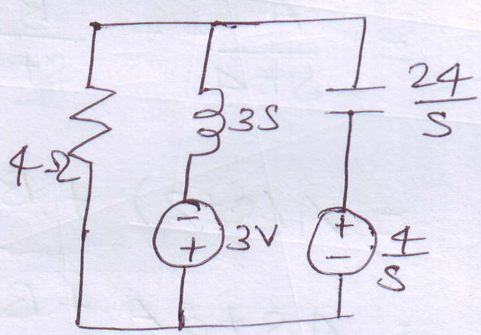
S- Domain Circuit :-



2. Find v(t), $t > 0$ for $i(0^-) = 1A$ and $v(0^-) = 4H$



Soln
eqt. S-domain circuit,



$$\frac{V(S)}{4} + \frac{V(S)+3}{3S} + \frac{V(S) - \frac{4}{S}}{\frac{24}{S}} = 0$$

$$\frac{V(S)}{4} + \frac{V(S)}{3S} + \frac{3}{3S} + \frac{V(S) \cdot S}{24} - \frac{4}{24} = 0$$

$$\frac{V(S)}{4} + \frac{V(S)}{3S} + \frac{1}{S} + \frac{S}{24}V(S) + \frac{4}{24} = 0$$

$$V(S)\left[\frac{1}{4} + \frac{1}{3S} + \frac{S}{24}\right] = \frac{1}{6} - \frac{1}{S}$$

$$V(S)\left[\frac{6S+8+S^2}{24S}\right] = \frac{S-6}{6S}$$

$$V(S) = \frac{(S-6)(24\cancel{S})}{6\cancel{S}(S^2+6S+8)}$$

$$= \frac{4(S-6)}{S^2+6S+8} = \frac{4(S-6)}{(S+4)(S+2)}$$

$$= \frac{4(S-6)}{(S+4)(S+2)}$$

$$\Rightarrow \frac{4(S-6)}{(S+4)(S+2)} = \frac{A}{S+4} + \frac{B}{S+2} \rightarrow (A)$$

$$4(S-6) = A(S+2) + B(S+4)$$

$$4S - 24 = AS + 2A + BS + 4B$$

equating s coefficients & constants

$$4 = A + B \longrightarrow (1)$$

$$-24 = 2A + 4B$$

Solving 1 & 2

$$1 - 2 \Rightarrow \begin{array}{r} \cancel{A} + B = 4 \\ \cancel{A} + 2B = -12 \\ - \quad - \qquad + \\ \hline -B = 16 \end{array}$$

$$\boxed{B = -16}$$

put B in 1

$$A - 16 = 4$$

$$\boxed{A = 20}$$

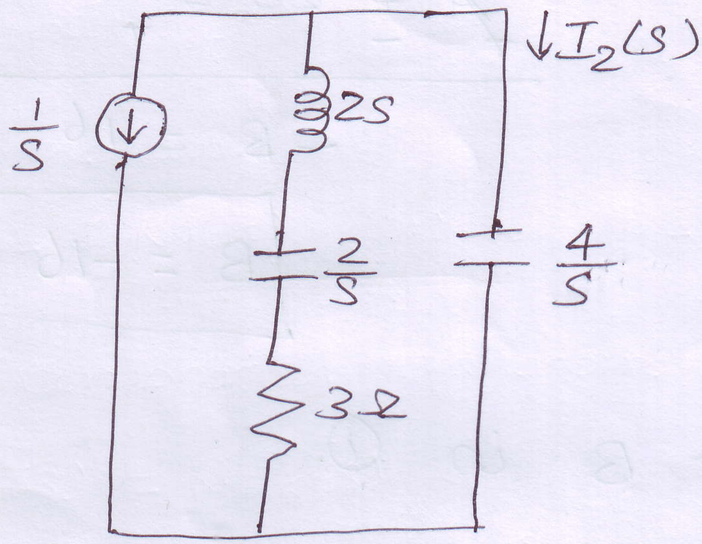
Sub A & B in A

$$A \Rightarrow \frac{20}{s+4} - \frac{16}{s+2} = V(s)$$

Taking inverse laplace,

$$V(t) = 20e^{-4t} - 16e^{-2t}.$$

2. Find $I_2(s)$ by current division. Assume all intial conditions to zero.



Current division,

$$I_2 = \frac{\text{Total current} \times \text{Oppasite branch Resistance}}{\text{Sum of all resistance}}$$

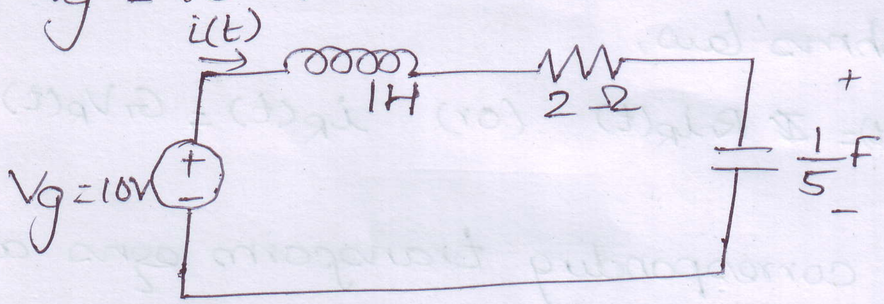
$$= \frac{-\frac{1}{s} \times \left(2s + \frac{2}{s} + 3\right)}{2s + \frac{2}{s} + 3 + \frac{4}{s}}$$

$$= \frac{-\frac{1}{s}\left(\frac{2s^2+3s+2}{s}\right)}{\frac{2s^2+3s+6}{s}}$$

$$= -\frac{1}{s} \times \frac{(2s^2+3s+2)}{\cancel{s}} \times \frac{\cancel{s}}{2s^2+3s+6}$$

$$I_2 = -\frac{1}{\ldots}\left[\frac{2s^2+3s+2}{\ldots}\right]$$

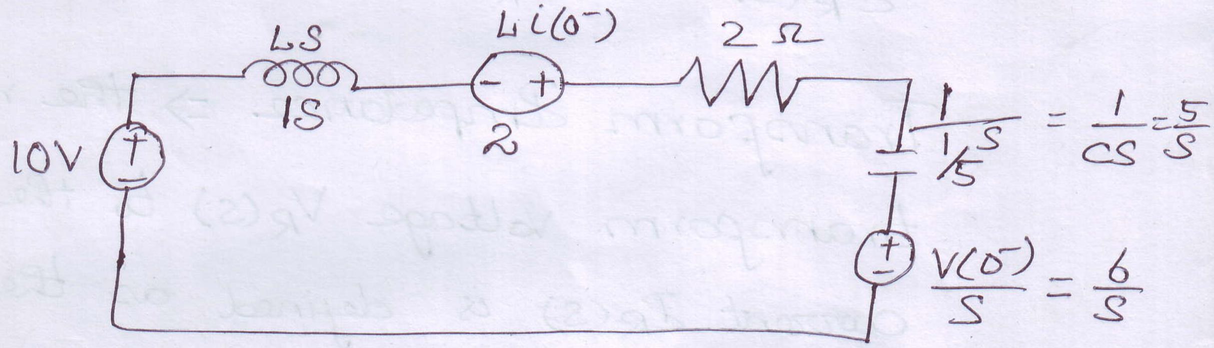
3. Use laplace transform to find $i(t)$ for $t > 0$, $V(0^-) = 6V$, $i(0^-) = 2A$ and $V_g = 10V$.



$L[1] = \frac{1}{s}$

$L[10] = \frac{10}{s}$

S-domain circuit,



Applying KVL,

$$\frac{10}{s} - \frac{6}{s} + 2 = I(s)\left[s + 2 + \frac{5}{s}\right]$$

$$2s + 4 = I(s)\,(s^2 + 2s + 5)$$

$$I(s) = \frac{2(s+2)}{(s+1)^2 + 2^2}$$

$$= \frac{2(s+1)}{(s+1)^2 + 2^2} + \frac{2}{(s+1)^2 + 2^2}$$

I.L.T

$$i(t) = 2e^{-t}\cos 2t + e^{-t}\sin 2t$$

NOTE:-

$s^2 + 2s + 5$

$\rightarrow 2ab = 2;\ a = 1$

TRANSFORM IMPEDANCE :-

(i) Resistor :-

By ohms' law,

$$V_R(t) = \cancel{I}\, R\, i_R(t) \quad (\text{or}) \quad i_R(t) = G\, V_R(t) \;;\; G = \frac{1}{R}$$

The corresponding transform eqns are,

$$V_R(s) = R\, I_R(s)$$

$$I_R(s) = G\, V_R(s)$$

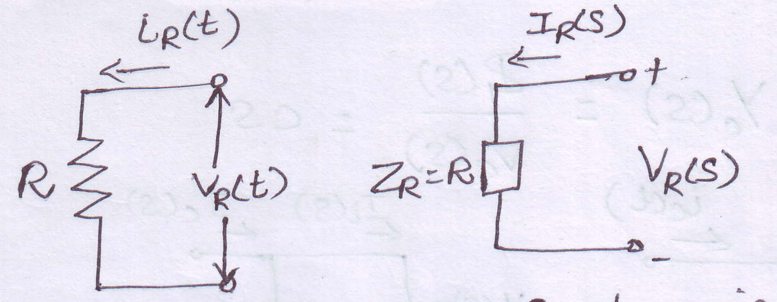
Transform Impedance $\Rightarrow$ the ratio of transform transform voltage $V_R(s)$ to the transform current $I_R(s)$ is defined as the transform impedance of Resistor.

$$Z_R(s) = \frac{V_R(s)}{I_R(s)} = R.$$

Transform admittance, $Y_R(s) = \dfrac{I_R(s)}{V_R(s)} = G$

$\therefore$ Resistor is frequency insensitive. to

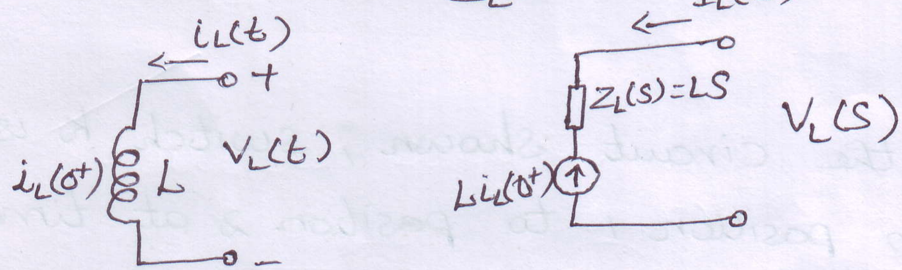
P



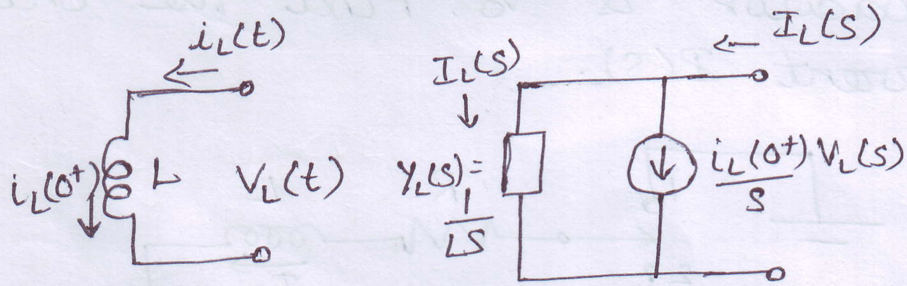
Time domain   S-domain

Inductance :

$$Z_L(S) = \frac{V_L(S)}{I_L(S)} = SL$$

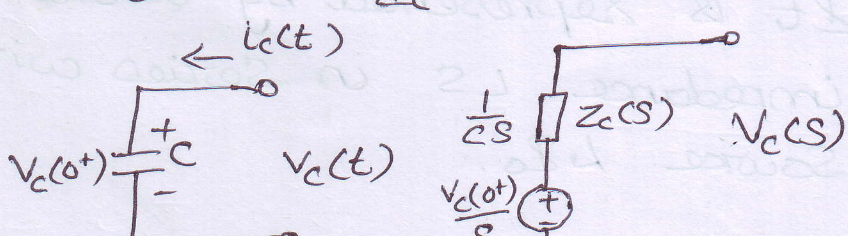


$$Y_L(S) = \frac{I_L(S)}{V_L(S)} = \frac{1}{LS}$$

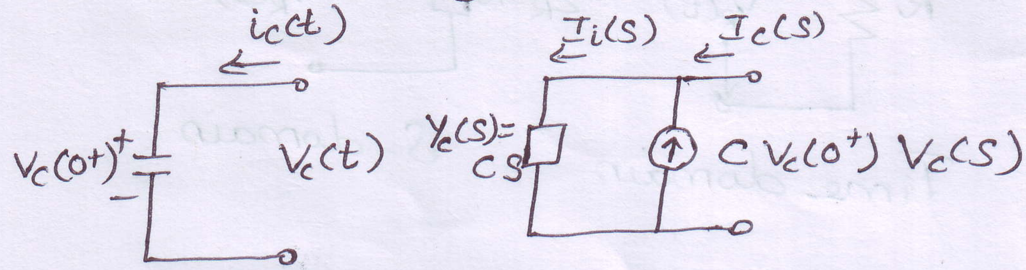


Capacitance :

$$Z_C(S) = \frac{V_C(S)}{I_C(S)} = \frac{1}{CS}$$

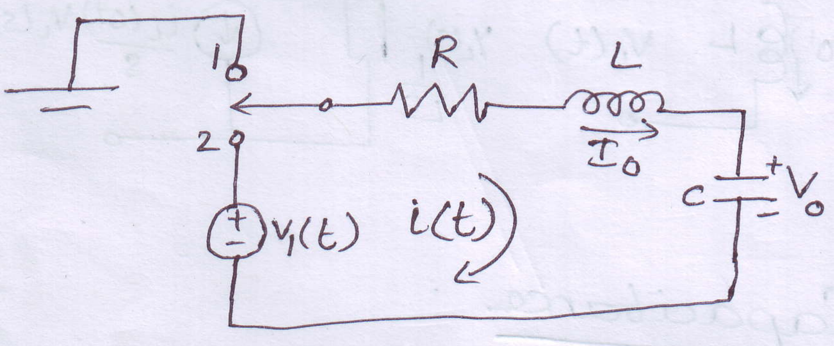


$$Y_c(s) = \frac{I_1(s)}{V_c(s)} = Cs$$



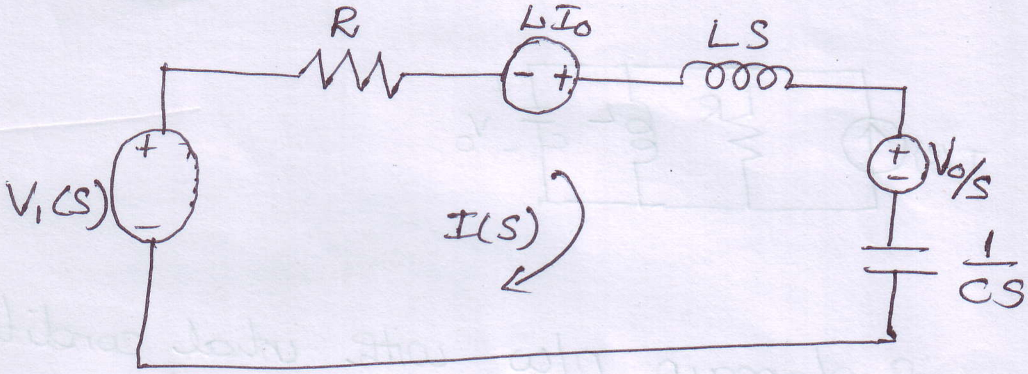
---

1. In the circuit shown, switch k is moved from position 1 to position 2 at time $t=0$. At time $t=0^-$, the current through inductor L is I_0 & the voltage across capacitor is V_0. Find the transform current $I(s)$.



* Inductor has an intial current of I_0. It is represented by a transform impedance LS in series with voltage source LI_0.

Initial voltage of capacitor $\Rightarrow V_0$, represented by transform impedance of $\frac{1}{CS}$ with an initial voltage V_0/s.



Applying KVL,

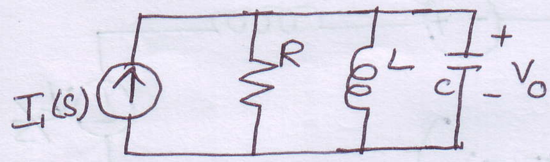
$$V_1(s) + LI_0 - \frac{V_0}{S} = RI(S) + LSI(S) + \frac{1}{CS}I(S)$$

$$V_1(S) + LI_0 - \frac{V_0}{S} = I(S)\left[R + LS + 1/CS\right]$$

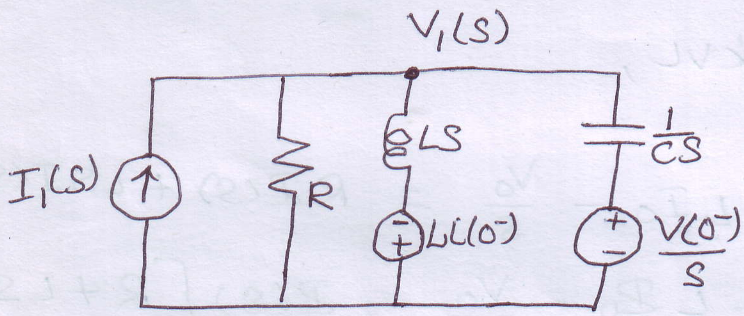
$$\Rightarrow \quad I(S) = \frac{V_1(S) + LI_0 - V_0/S}{R + LS + 1/CS}$$

$$I(S) = \frac{S\,V_1(S) + LI_0 - V_0}{LS^2 + RS + 1/C}$$

2. Calculati $V_1(S)$



S-domain n/w with intial conditions,



$$I_1(S) = \frac{V_1(S)}{R} + \frac{V_1(S) + LI_0}{LS} + \frac{V_1(S) - \frac{V_0}{S}}{1/CS}$$

$$I_1(S) = V_1(S)\left[\frac{1}{R} + \frac{1}{LS} + CS\right] + \frac{LI_0}{LS} - V_0C$$

$$I_1(S) + CV_0 - \frac{I_0}{S} = V_1(S)\left[\frac{1}{R} + \frac{1}{LS} + CS\right]$$

$$V_1(S) = \frac{I_1(S) + CV_0 - I_0/S}{1/R + \frac{1}{LS} + CS}$$

//.

Terminal Pairs or Ports :-

→ Consider a n/ω made of passive elements.

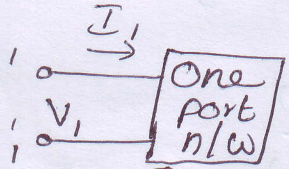


Fig1

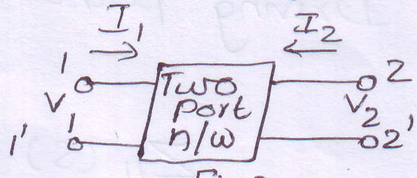


Fig2

Fig1:-

* 1-1' one pair of terminals called port
* Driving source is connected to the pair of terminals.

Fig2:-

* 1-1' & 2-2' called as two port n/ω.
* If driving source connected to 1-1'; load connected across 2-2'.

*

Network Functions For The One-port And Two Port

* For one-port n/ω, the driving point impedance (or) impedance of n/ω is defined as

$$Z(s) = \frac{V(s)}{I(s)}$$

$$Y(s) = \frac{I(s)}{V(s)} \quad \text{(Driving point admittance)}$$

For two port n/w,

* Driving point impedance function at port 1-1'

$$Z_{11}(S) = \frac{V_1(S)}{I_1(S)}$$

ie) ratio of transform voltage at port 1-1' to the transform current at same port.

III^ly

$$Z_{22}(S) = \frac{V_2(S)}{I_2(S)}$$

* Driving point admittance

$$Y_{11}(s) = \frac{I_1(S)}{V_1(S)}$$

$$Y_{22}(S) = \frac{I_2(S)}{V_2(S)}$$

TRASFER FUNCTIONS :-

These functions give the relation b/w vge or current at one port to the vge or current at other ports.

(i) Voltage Transfer Ratio :-

The ratio of voltage transform at one port to voltage transform at the other port, denoted by $G(s)$.

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)}$$

$$G_{12}(s) = \frac{V_1(s)}{V_2(s)}$$

(ii) Current Transfer Ratio :-

Ratio of current transform at one port to current transform at other port, denoted by $\alpha(s)$

$$\alpha_{12}(s) = \frac{I_1(s)}{I_2(s)}$$

$$\alpha_{21}(s) = \frac{I_2(s)}{I_1(s)}$$

(iii) Transfer Impedance :-

Ratio of voltage transform at one port to the current transform at other port.

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)} \qquad Z_{12}(s) = \frac{V_1(s)}{I_2(s)}$$

(iv) Transfer Admittance :-

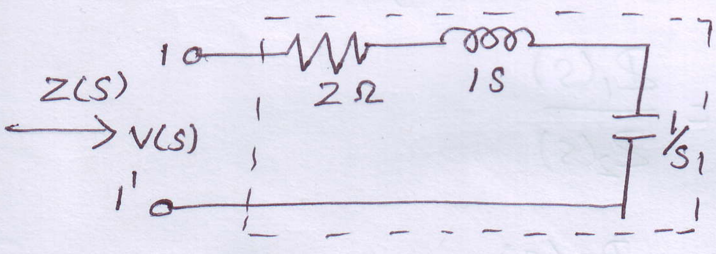
defined as the ratio of current transform at one port to the current transform at the other port.

$$Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$$

$$Y_{12}(s) = \frac{I_1(s)}{V_2(s)}$$

---

1. For the network shown, obtain the driving point impedance.



Soln :- Applying kirchoffs' law at port 1-1'.

$$Z(s) = \frac{V(s)}{I(s)}$$

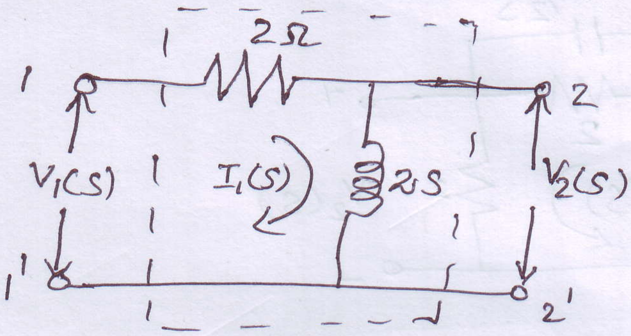
V(S) → applied at port 1-1'.

I(S) → current flowing through the n/w.

$$Z(s) = \frac{V(s)}{I(s)} = 2 + s + 1/s$$

$$Z(s) = s^2 + 2s + 1$$

2. For the n/w shown, obtain the transfer functions $G_{21}(S)$ & $Z_{21}(S)$ & driving point impedance $Z_{11}(S)$



Applying kirchoff's law,

$$V_1(S) = 2I_1(S) + 2SI_1(S) = 2I_1(S)[1+S]$$

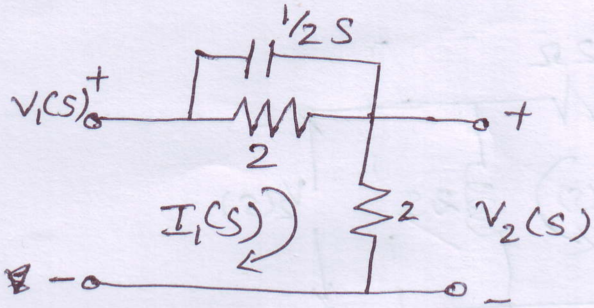
$$V_2(S) = 2SI_1(S)$$

$$G_{21}(S) = \frac{V_2(S)}{V_1(S)} = \frac{2SI_1(S)}{2I_1(S)(1+S)} = \frac{S}{1+S}$$

$$Z_{21}(S) = \frac{V_2(S)}{I_1(S)} = 2S$$

$$Z_{11}(S) = \frac{V_1(S)}{I_1(S)} = 2(S+1)$$

3. For the n/w shown, obtain the transfer functions $G_{21}(s)$, $Z_{21}(s)$ & driving point impedance $Z_{11}(s)$.



soln :-

$$Z_{eq} = 2 \parallel \tfrac{1}{2s}$$

$$= \frac{2 \times \frac{1}{2s}}{2 + \frac{1}{2s}} = \frac{\frac{1}{s}}{\frac{4s+1}{2s}} = \frac{1}{s} \times \frac{2s}{4s+1}$$

$$Z_{eq} = \frac{2}{4s+1}$$

Applying kirchoff's law,

$$V_2(s) = 2 I_1(s)$$

&

$$V_1(s) = I_1(s)\left(\frac{2}{4s+1} + 2\right)$$

$$= I_1(s)\left(\frac{2+8s+2}{4s+1}\right)$$

$$= I_1(s)\left(\frac{8s+4}{4s+1}\right)$$

Transfer fn,

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{2 I_1(s)}{I_1(s)\left(8s+4 \right.}$$

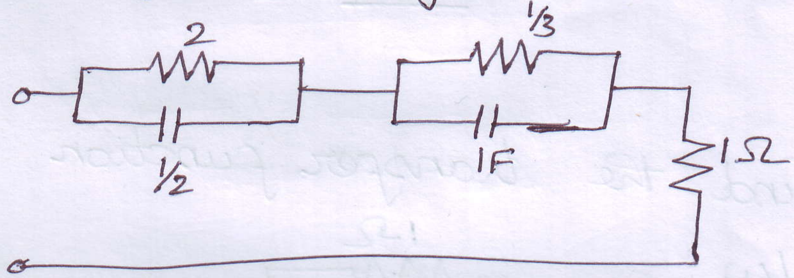
$$G_{21}(S) = \frac{2(4S+1)}{8S+4}$$

$$G_{21}(S) = \frac{8S+2}{8S+4}$$

$$Z_{21}(S) = \frac{V_2(S)}{I_1(S)} = 2$$

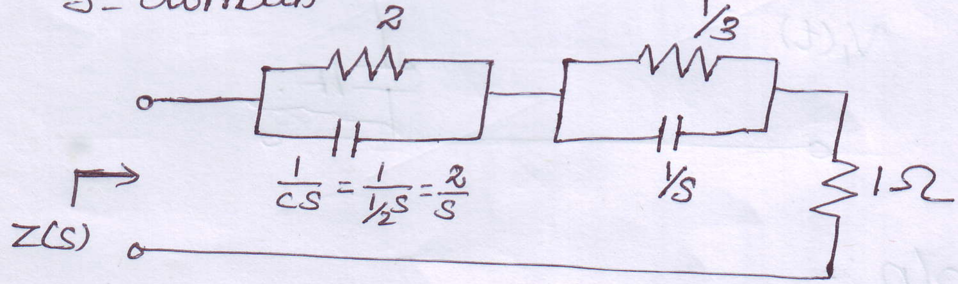
$$Z_{11}(S) = \frac{V_1(S)}{I_1(S)} = \frac{8S+4}{4S+1}$$

4. Find z(s) for the given circuit.



Soln:-

S_domain



$$Z_{eq} = \left(2 \parallel 2/S\right) + \left(\frac{1}{3} \parallel \frac{1}{S}\right) + 1$$

$$= \frac{2 \times 2/S}{2 + 2/S} + \frac{1/3 \times 1/S}{\frac{1}{3} + \frac{1}{S}} + 1$$

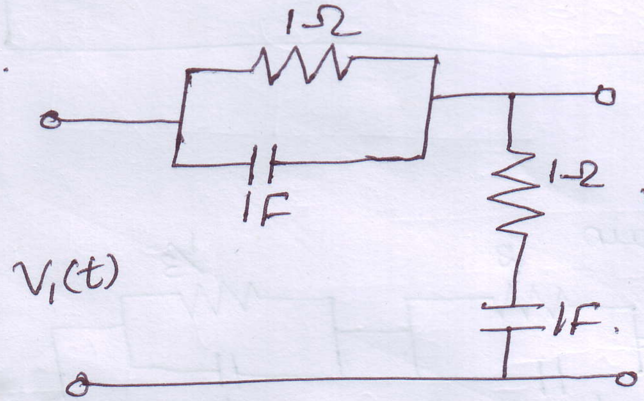
$$= \left(\frac{4}{S} \times \frac{S}{2(S+1)}\right) + \left(\frac{1}{3S} \times \frac{3S}{S+3}\right) + 1$$

$$= \frac{2}{S+1} + \frac{1}{S+3} + 1$$

$$= \frac{2(S+3) + (S+1) + (S+1)(S+3)}{(S+1)(S+3)}$$

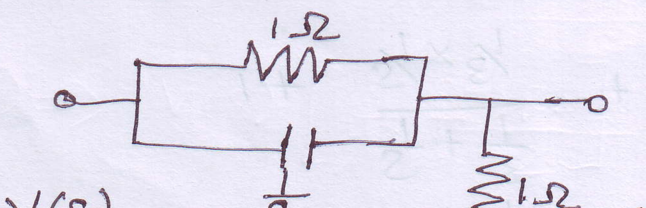
$$= \frac{S^2 + 7S + 10}{S^2 + 4S + 3}$$

5. Find the transfer function $G_{21}(s)$ for the n/w.



Soln :-

S. domain n/w :-



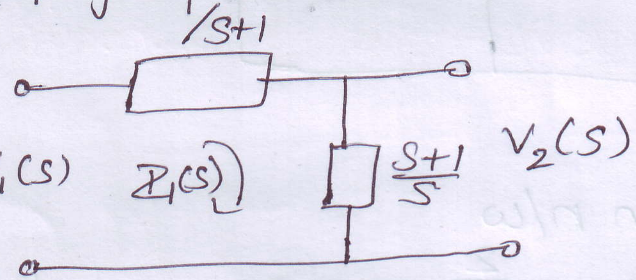
$\Rightarrow 1\Omega$ & $1/s$ are in parallel

$$= \frac{1 \times 1/s}{1 + 1/s} = \frac{1/s}{\frac{s+1}{s}} = \frac{1}{s+1}.$$

$\Rightarrow 1\Omega$ & $1/s$ are in Series

$$= 1 + 1/s = \frac{s+1}{s}.$$

Simplified S- domain n/w



$$V_1(s) = \left[\frac{1}{s+1} + \frac{s+1}{s}\right] I_1(s)$$

$$V_2(s) = \left(\frac{s+1}{s}\right) I_1(s)$$

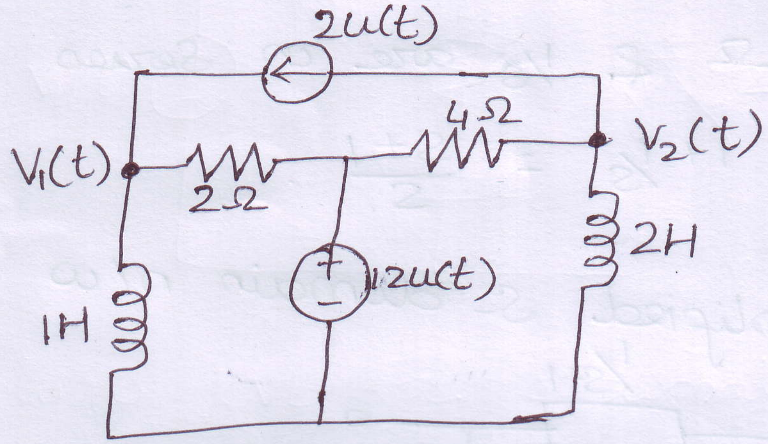
$$G_{21}(s) = \frac{V_2(s)}{V_1(s)}$$

$$= \frac{\left(\frac{s+1}{s}\right) I_1(s)}{\left(\frac{1}{s+1} + \frac{s+1}{s}\right) I_1(s)}$$

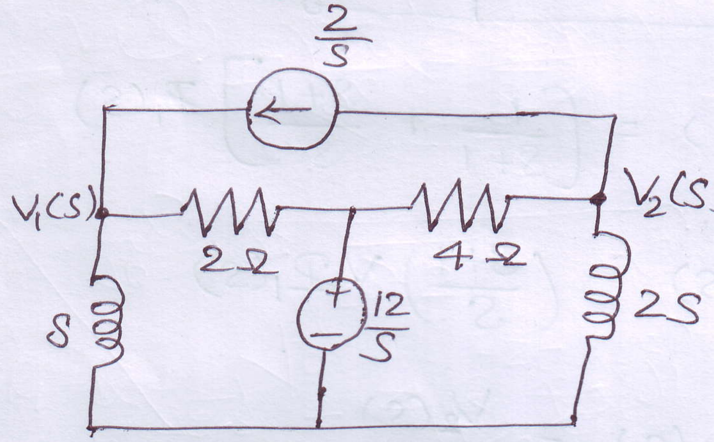
$$= \frac{\frac{s+1}{s}}{\frac{s^2+3s+1}{s(s+1)}} = \frac{s+1}{s} \times \frac{s(s+1)}{s^2+3s+1}$$

$$= \frac{s^2+2s+1}{s^2+3s+1}$$

6. For the s-domain circuit, find $V_1(s)$, $V_2(s)$, $V_1(t)$ & $V_2(t)$.



S-domain n/w



NOTE

$L[u(t)] = 1/s$

Using KCL,

$$\frac{V_1(S)}{S} + \frac{V_1(S) - 12/S}{2} = \frac{2}{S} \longrightarrow ①$$

$$\frac{V_2(S) - 12/S}{4} + \frac{V_2(S)}{2S} + \frac{2}{S} = 0 \longrightarrow ②$$

$$① \Rightarrow V_1(S)\left[\frac{1}{S} + \frac{1}{2}\right] - \left[\frac{12}{2S}\right] = \frac{2}{S}$$

$$V_1(s)\left[\frac{2+s}{2s}\right] = \frac{2}{s} + \frac{6}{s}$$

$$V_1(s) = \frac{8}{s} \times \frac{2s}{s+2}$$

$$V_1(s) = \frac{16}{s+2}$$

$$\Longrightarrow v_1(t) = 16e^{-2t}u(t).$$

$$\Rightarrow V_2(s)\left[\frac{1}{4} + \frac{1}{2s}\right] - \frac{12}{4s} + \frac{2}{s} = 0.$$

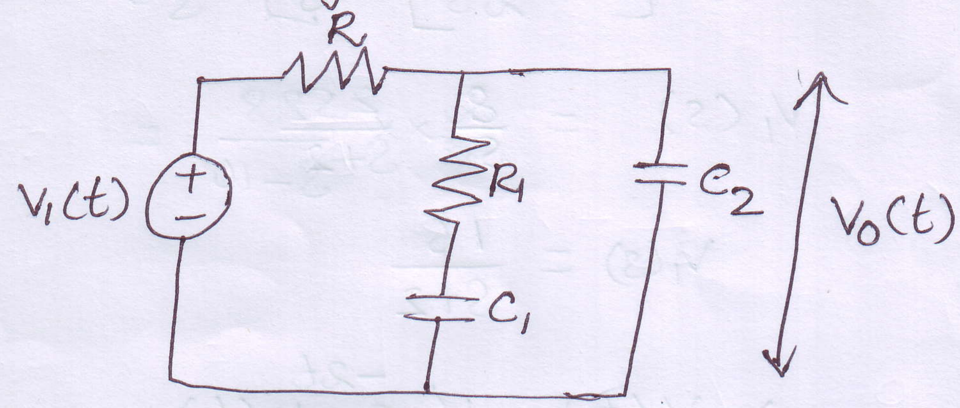
$$V_2(s) = \left(\frac{1}{s} \times \frac{4s}{s+2}\right)$$

$$V_2(s) = \frac{4}{s+2}$$

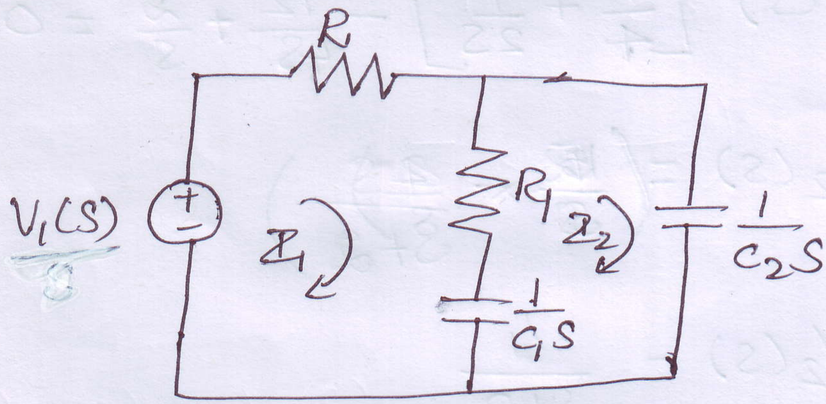
$$v_2(t) = 4e^{-2t}u(t)$$

ll.

7. Obtain $G_{21}(S)$ for the circuit



Applying KVL for S-domain n/w



$$\begin{bmatrix} R_1 + R + \frac{1}{C_1 S} & -\left(R_1 + \frac{1}{C_1 S}\right) \\ -\left(R_1 + \frac{1}{C_1 S}\right) & \frac{1}{C_2 S} + \frac{1}{C_1 S} + R_1 \end{bmatrix} \begin{bmatrix} I_1(S) \\ I_2(S) \end{bmatrix} = \begin{bmatrix} V_1(S) \\ 0 \end{bmatrix}$$

$$\Rightarrow \left(R_1 + R + \frac{1}{C_1 S}\right) I_1(S) - \left(R_1 + \frac{1}{C_1 S}\right) I_2(S) = V_1(S)$$

$$-\left(R_1 + \frac{1}{C_1 S}\right) I_1(S) + \left(\frac{1}{C_2 S} + \frac{1}{C_1 S} + R_1\right) I_2(S) = 0$$

$$V_0(S) = \frac{1}{C_2 S} I_2(S)$$

To find $I_2(S)$:-

$$I_2(s) = \frac{\Delta_2}{\Delta}$$

$$= \frac{\begin{vmatrix} R_1 + R + \frac{1}{C_1 S} & V_1(S) \\ -\left(R_1 + \frac{1}{C_1 S}\right) & 0 \end{vmatrix}}{\begin{vmatrix} R_1 + R + \frac{1}{C_1 S} & -\left(R_1 + \frac{1}{C_1 S}\right) \\ -\left(R_1 + \frac{1}{C_1 S}\right) & R_1 + \frac{1}{C_1 S} + \frac{1}{C_2 S} \end{vmatrix}}$$

$V_0(S) = V_2(S)$

$$\frac{V_0(S)}{V_1(S)} = \frac{\frac{1}{C_2 S}\left(R_1 + \frac{1}{C_1 S}\right)}{\left(R_1 + R + \frac{1}{C_1 S}\right)\left(R_1 + \frac{1}{C_1 S} + \frac{1}{C_2 S}\right) - \left(R_1 + \frac{1}{C_1 S}\right)^2}$$

$$= \frac{R_1 C_1 S + 1}{R_1 R C_1 C_2 S^2 + (R_1 C_1 + R C_1 + R C_2) S + 1}$$ //.

Poles & Zeros Of Network Functions :-

N/w Functions,

$$N(S) = \frac{P(S)}{Q(S)} = \frac{a_0 s^n + a_1 s^{n-1} \ldots + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + \ldots + b_{m-1} s + b_m}$$

$a_0, a_1, \ldots a_n \rightarrow$ Coeff. of polynomial $P(S)$

$b_0, b_1, \ldots b_n \rightarrow$ " " $Q(S)$

* They are real & (+)ve for passive n/ws.

After factorization,

$$N(S) = \frac{P(S)}{Q(S)} = \frac{a_0 (S-Z_1)(S-Z_2)\ldots(S-Z_n)}{b_0 (S-P_1)(S-P_2)\ldots(S-P_m)}$$

$Z_1, Z_2, \ldots Z_n$ are 'n' roots for $P(S)=0$

$P_1, P_2, \ldots P_m$ are 'm' roots for $Q(S)=0$

& $\frac{a_0}{b_0} = H$ (constant) called as Scale factor

$Z_1, Z_2, \ldots Z_n$ are called as zeros (o).

$P_1, P_2, \ldots P_m$ are called Poles (x).

* <u>Simple poles</u> (or) <u>Simple Zeros</u> :-

If the poles (or) Zeros are not repeated, then the function is said to be having simple poles (or) Simple zeros.

* If the poles or zeros are repeated then the fn. is said to be having multiple poles (or) multiple zeros.

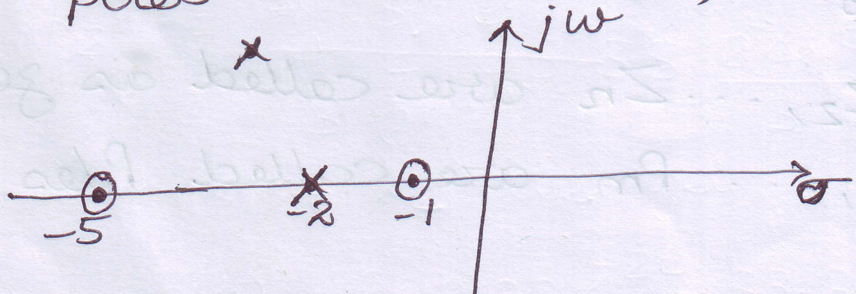
When $n > m$, then $(n-m)$ zeros are at $s = \infty$

$m > n$; then $(m-n)$ poles are at $s = \infty$

eg:- $$N(s) = \frac{(s+1)^2 (s+5)}{(s+2)(s+3+j2)(s+3-j2)}$$

Zeros are, $s = -1$ (twice) & $s = -5$

poles are $s = -2$; $s = -3-2j$; $s = -3+2j$



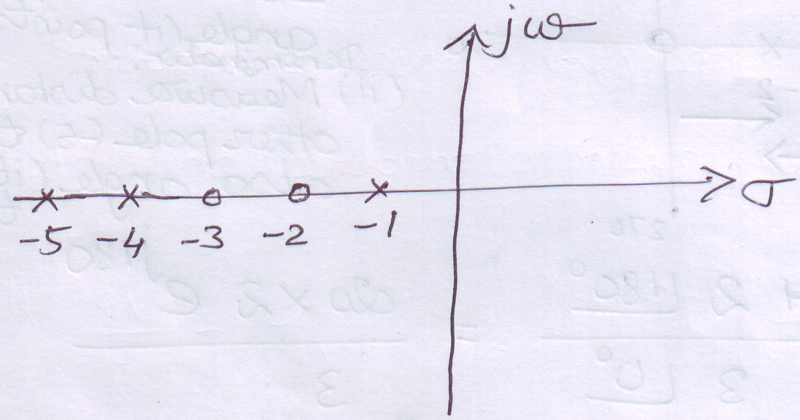
S/m is stable

* when real parts of poles & zeros are negative.

(or)

1. For the given transfer function, plot the pole-zero plot.

$$H(s) = \frac{(s+3)(s+2)}{(s+1)(s+4)(s+5)}$$

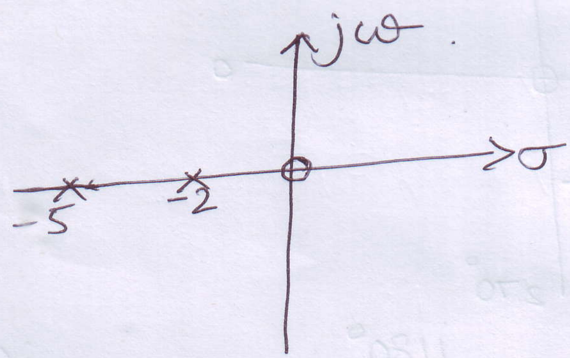


Zeros : $s = -3, -2$

poles : $s = -1, -4, -5$

2. Draw the pole zero diagram for the given n/w function $I(s)$ & hence obtain $i(t)$

$$I(s) = \frac{20s}{(s+5)(s+2)}$$



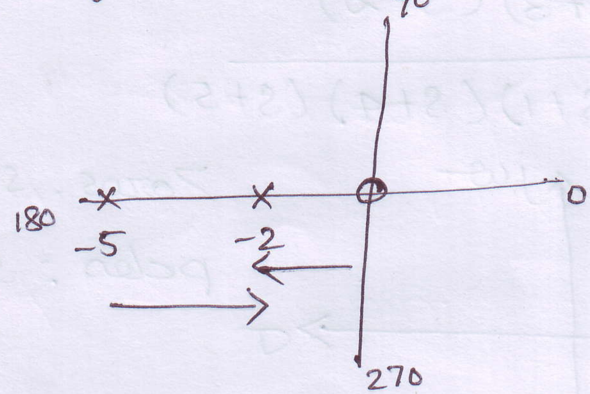
Zeros : $s = 0$

poles : $s = -5, -2$

$$I(s) = \frac{k_1}{(s+2)} + \frac{k_2}{(s+5)}$$

$$i(t) = k_1 e^{-2t} + k_2 e^{-5t}$$

To find k_1 :



NOTE

Numerator :-

(i) Measure distance from zero to (-2) pole & also angle (it points towards 180°)

Denominator :-

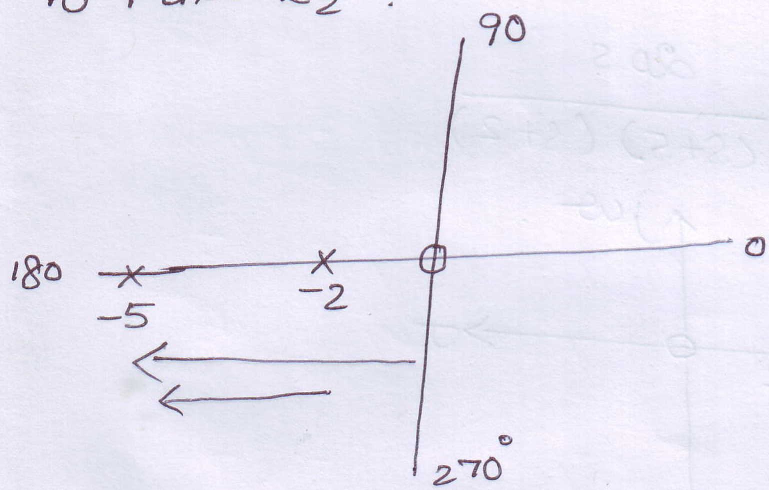
(ii) Measure distance from other pole (-5) to (-2) pole also angle (it point towards 0)

$$k_1 = \frac{4 \; 2 \angle 180^\circ}{3 \angle 0^\circ} = \frac{20 \times 2 \, e^{j180}}{3}$$

$$= 13.33 \, e^{j180} = 13.33 \, (\cos 180 + j \sin 180)$$

$$= -13.33$$

To Find k_2 :



$$k_2 = \frac{4 \, (5) \, e^{j180^\circ}}{3 \, e^{j180^\circ}} = \frac{20 \times 5}{3} e^{j(180-180)} = \frac{100}{3} = 33.3$$

Sub k_1 & k_2 we get,

$$i(t) = (-13.33 e^{-2t} + 33.3 e^{-5t}) \text{ Amps.}$$

1. Obtain the time domain response. Given

$$I(s) = \frac{5s}{(s+1)(s^2+4s+8)}$$

Soln :-

Zeros, $s = 0$

poles, $s = -1, -2-2j, -2+2j$

NOTE :-

$$s^2+4s+8 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16-4\times 8}}{2}$$

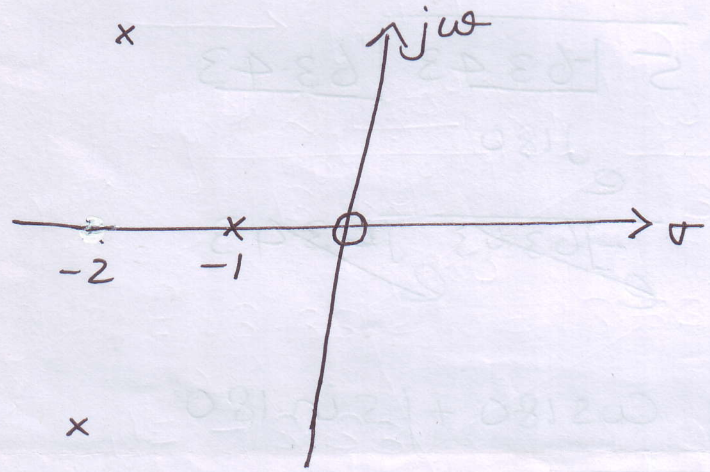
$$= \frac{-4 \pm \sqrt{-16}}{2}$$

$$= \frac{-4 \pm \sqrt{4i^2}}{2}$$

$$= \frac{-4 \pm 4i}{2}$$

$$= -2+2j, -2-2j$$

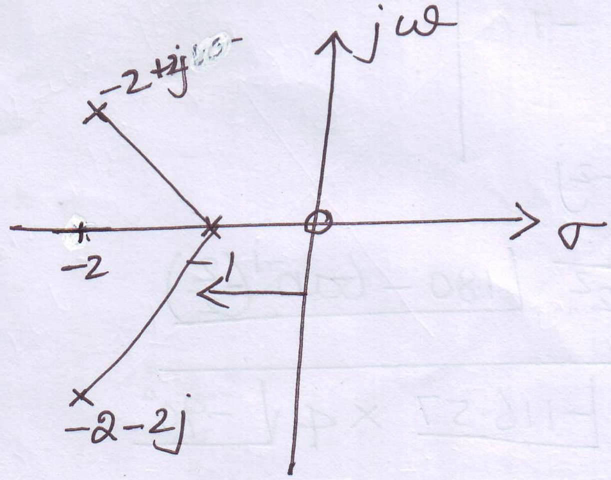
$$= -2 \pm 2j$$



To find residue at $s=-1$

$$I(s) = \frac{k_0}{s+1} + \frac{k_1}{s+2+2j} + \frac{k_2}{s+2-2j}$$

To find k_0 :-



$$k_0 = \frac{5 \times 1 \angle 180}{\sqrt{1^2+2^2} \angle \tan^{-1}(-2/1) \sqrt{5} \angle \tan^{-1}(2)}$$

distance from -1 to $-2+2j$

$\Rightarrow -1-(-2+2j)$

$-1+2-2j$

$1-2j$

$$= \frac{5 \times 1\, e^{j180}}{\sqrt{5} \angle \tan^{-1}(-2) \sqrt{5} \angle \tan^{-1}(2)}$$

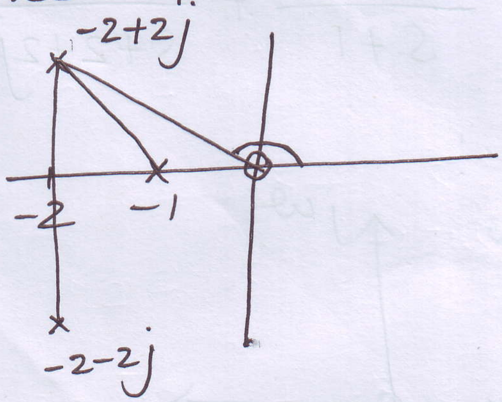
$$= \frac{5 e^{j180}}{5 \angle -63.43 \angle 63.43}$$

$$= \frac{e^{j180}}{\cancel{e^{-j63.43}}\ \cancel{e^{j63.43}}}$$

$$= \cos 180 + j \sin 180$$

$$\boxed{k_0 = -1}$$

To find k_1 :-



Distance from -1 to $-2+2j$

$-1-(-2+2j) = -1+2-2j$

$= 1-2j$

$\Rightarrow \sqrt{1^2+2^2} \tan^{-1}\left(\frac{-2}{1}\right)$

$\sqrt{5} \tan^{-1}(-2)$

$\sqrt{5}\ (\angle -63.43)$

$180-(-63.43)$

$= -(116.57)$

$$k_1 = \frac{5 \times \sqrt{2^2+2^2} \angle 180 - \tan^{-1}\left(\frac{2}{2}\right)}{\sqrt{5} \angle -116.57 \times 4 \angle -90^\circ}$$

$$k_1 = \frac{5\sqrt{8}\,\underline{\left|-(180-45)\right.}}{\sqrt{5}\,\underline{\left|-116.57\right.} \times 4\,\underline{\left|-90^\circ\right.}}$$

$$= \frac{5\sqrt{8}\,\underline{\left|-135^\circ\right.}}{\sqrt{5}\,\underline{\left|-116.57\right.}\; 4\,\underline{\left|-90^\circ\right.}}$$

$$= \cancel{5\sqrt{8}\; -(\cos 135^\circ + j\sin 13}$$

$$= \frac{5\sqrt{8}\; e^{+j(-135+116.57+90)}}{\sqrt{5}\times 4}$$

$$= 1.58\, e^{j71.56}$$

$$= 0.5 + j1.5$$

P_2 & P_3 are complex conjugate, the value of k_2 is equal to the complex conjugate of k_1.

$$k_2 = k_1^*$$

$$= 0.5 - j1.5$$

$$k_2 = 1.58\, e^{-j71.56}$$

$$I(s) = \frac{-1}{s+1} + \frac{1.58e^{+j71.56}}{s+2+2j} + \frac{1.58e^{-j71.56}}{s+2-2j}$$

Taking ILT,

$$i(t) = -e^{-t} + 1.58e^{j71.56}\, e^{-(2+2j)t} + 1.58\, e^{-j71.56}\, e^{-(2+2j)}$$

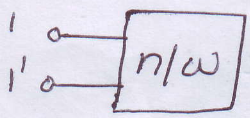
1

UNIT - III

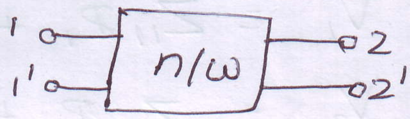
TWO PORT NETWORKS

Port :-

A pair of terminals at which a signal may enter or leave a terminal is called a port.



one port network



Two port n/w.

Driving point of network (or) Input Port :-

The driving force connected to the pair of terminals is energy source & the port at which energy source is connected is called driving point of the n/w or input port.

Output port :-

The port at which load is connected is called o/p port.

Z - Parameters :-

→ Also called as impedance parameters.

→ Also called as open circuit impedance parameters.

$$V_1 = f_1(I_1, I_2)$$

$$V_2 = f_2(I_1, I_2)$$

V_1 & V_2 are dependent variables

I_1 & I_2 are independent variables.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

In matrix,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

* Z_{11} & Z_{22} driving point impedance

* Z_{12} & Z_{21} transfer impedance.

Z_{11} → open circuit input driving point impedance

Z_{22} → Open circuit output driving point impedance

Y- Parameters :-

→ admittance parameters

→ short circuit admittance parameters.

$$I_1 = f_1(V_1, V_2)$$

$$I_2 = f_2(V_1, V_2)$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

In matrix form,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} \quad ; \quad Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0}$$

$$Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} \quad ; \quad Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$

Y_{21} → forward transfer admittance

Y_{11} → i/p driving point admittance

Y_{12} → reverse transfer admittance

Y_{22} → output driving point admittance.

h-parameters :-

→ hybrid parameters

→ Used for constructing models of transistors.

$$V_1 = f_1(I_1, V_2)$$

$$I_2 = f_2(I_1, V_2).$$

$I_1, V_2 \rightarrow$ independent variables

$I_2, V_1 \rightarrow$ dependent variables

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

In matrix form,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

When $V_2 = 0$ port 2-2' is short circuited

$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0}$$

$h_{11} \rightarrow$ Short circuit input impedance $\left(\frac{1}{Y_{11}}\right)$

$$h_{21} = \frac{I_2}{I_1} \Big|_{V_2=0}$$

→ Short circuit forward current $\left(\frac{Y_{21}}{\ldots}\right)$

$I_1 = 0$ (open circuit)

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0}$$

$h_{12} \rightarrow$ open circuit reverse voltage gain $\left(\frac{Z_{12}}{Z_{22}}\right)$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0}$$

$h_{22} \rightarrow$ open circuit output admittance $\left(\frac{1}{Z_{22}}\right)$

ABCD Parameters :-

→ known as transmission parameters

→ Used for the analysis of power transmission in which the input port is referred as the sending end & while the o/p port is referred as receiving end.

$$V_1 = f_1(V_2, -I_2)$$

$$I_1 = f_2(V_2, -I_2)$$

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \rightarrow$ transmission matrix.

$I_2 = 0$ (open circuit)

$$A = \frac{V_1}{V_2}\Big|_{I_2=0}$$

$\frac{1}{A} \rightarrow$ open circuit voltage gain dimensionless parameter.

$A \rightarrow$ reverse voltage ratio with o/p open circuit.

$$C = \frac{I_1}{V_2}\Big|_{I_2=0}$$

$$\frac{1}{C} = \frac{V_2}{I_1}\Big|_{I_2=0} = Z_{21}$$

$\frac{1}{C} \rightarrow$ open circuit transfer impedance.

$V_2 = 0$ (Short circuited)

$$B = -\frac{V_1}{I_2}\Big|_{V_2=0}$$

$$-\frac{1}{B} = \frac{I_2}{V_1}\Big|_{V_2=0}$$

$1/B = Y_{21}$ → Short circuit transfer admittance.

B → Reverse transfer impedance with o/p short circuited.

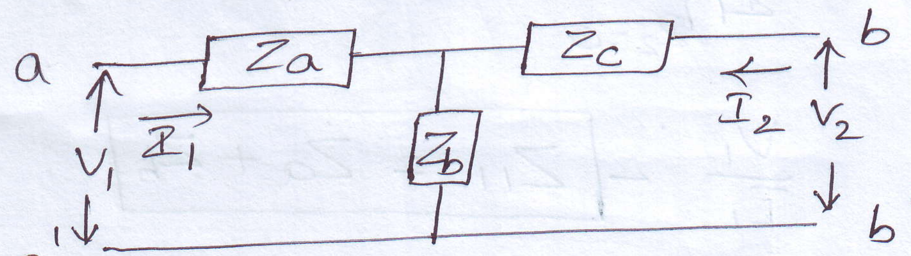
$$D = -\frac{I_1}{I_2}\Big|_{V_2=0}$$

$-\frac{1}{D} = \frac{I_2}{I_1}$ (Short circuit current gain)

$-D$ → Reverse current ratio with o/p short circuited.

Problems

1. Find the Z parameters for the circuit.



Z-parameter eqns,

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Eqn. Obtained from the circuit,

$$V_1 = Z_a I_1 + Z_b (I_1 + I_2)$$

$$V_1 = Z_a I_1 + Z_b I_1 + Z_b I_2 \longrightarrow ①$$

$$V_2 = Z_c I_2 + Z_b (I_2 + I_1)$$

$$V_2 = Z_c I_2 + Z_b I_2 + Z_b I_1 \longrightarrow ②$$

We know,

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0}.$$

Sub $I_2 = 0$ in eqn ①

$$V_1 = (Z_a + Z_b) I_1$$

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_a + Z_b$$

$$\frac{V_1}{I_1} = \boxed{Z_{11} = Z_a + Z_b}$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

Sub $I_1 = 0$ in eqn 1

$$V_1 = Z_b I_2$$

$$Z_{12} = \frac{V_1}{I_2} = Z_b$$

$$\boxed{Z_{12} = Z_b}$$

Sub $I_2 = 0$ in eqn 2

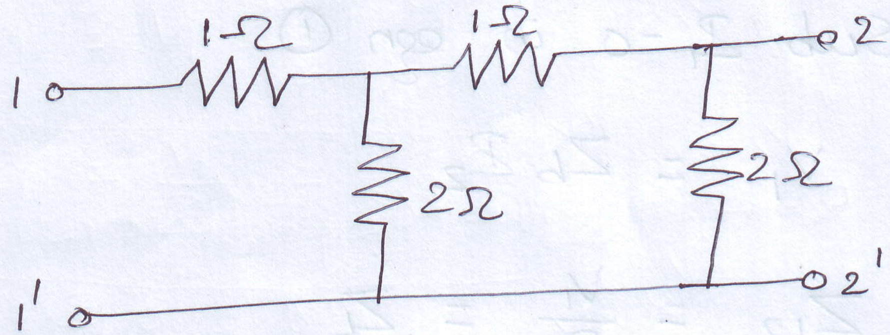
$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = Z_b$$

$$\boxed{Z_{21} = Z_b}$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

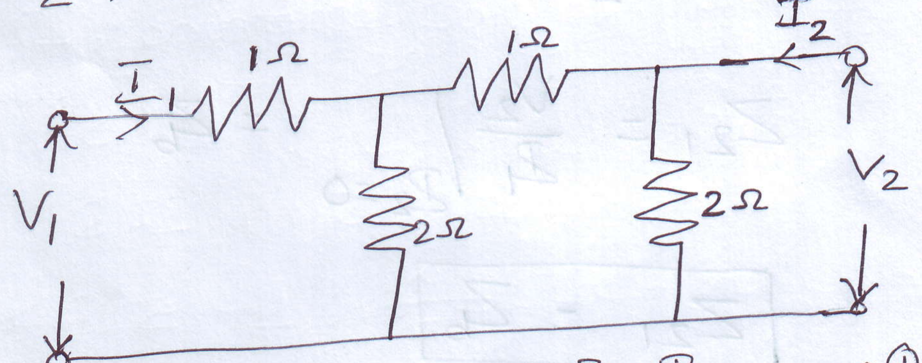
$$\boxed{Z_{22} = Z_b + Z_c}$$

2. Find the Z-parameters for the given network. (Direct method)



Soln :-

* Redraw the circuit with I_1, V_1 & I_2, V_2 parameters.



$V_1 = Z_{11} I_1 + Z_{12} I_2$ ⟶ (1)

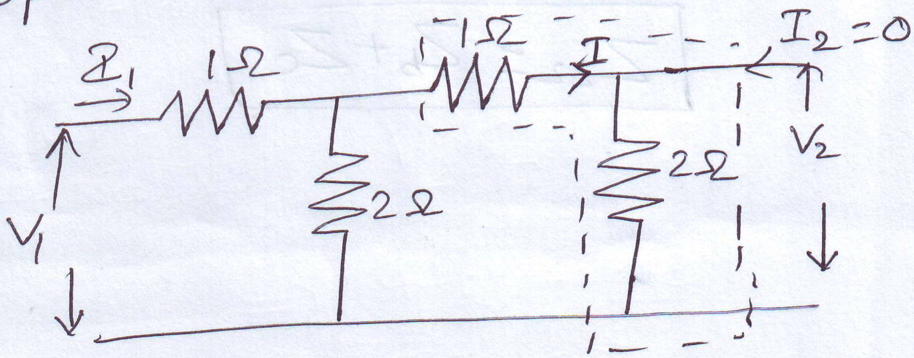
$V_2 = Z_{21} I_1 + Z_{22} I_2$ ⟶ (2)

} General eqn. of Z-parameters

Step I :-

* Open circuit the o/p terminal $\boxed{I_2 = 0}$

*



1 Ω & 2 Ω are series 1+2 = 3 Ω

are parallel $\frac{3 \times 2}{}$ = 6 Ω

$\frac{6}{5}\Omega$ & 1Ω are series $\frac{6}{5}+1 = \frac{6+5}{5} = \frac{11}{5}\Omega$

eqn ① $\Rightarrow I_2 = 0 \Rightarrow V_1 = Z_{11} I_1$.

$V_1 = I_1(Z_{eq})$ From ckt,

i) $\boxed{Z_{11} = \frac{V_1}{I_1} = \frac{11}{5}\Omega}$

In Our Circuit Consider a current I flowing to the 2Ω resistor, So V_2 can be written as

$$V_2 = 2I \longrightarrow ③$$

* Value of I can be calculated using current division rule.

$$I = I_1 \times \frac{2}{5} \longrightarrow ④$$

Sub I in eqn ③

$$V_2 = 2\left[\frac{2}{5} \times I_1\right]$$

$$\frac{V_2}{I_1} = \frac{4}{5}$$

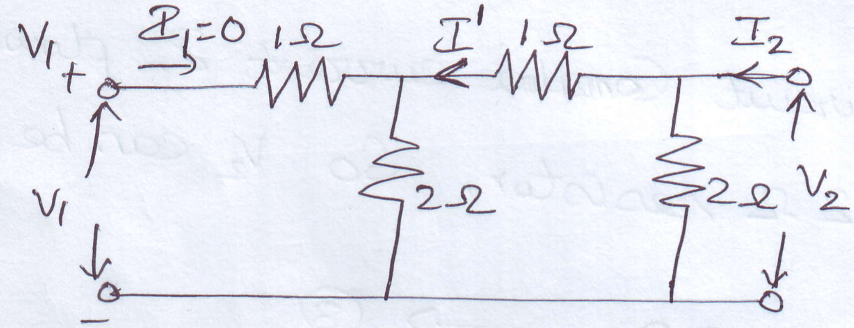
We know,

$$\boxed{Z_{21} = \frac{4}{5}}$$

Step 2 :-

$I_1 = 0$ (means open circuiting the input terminals).

Redrawing the circuit, by $I_1 = 0$



1Ω resistance can be neglected becoz of

$I_1 = 0$.

So, 2Ω & 1Ω are in series $2+1 = 3\Omega$

3Ω & 2Ω are in parallel $\frac{3\times2}{3+2} = 6/5$.

Sub $I_1 = 0$ in ②

$$V_2 = Z_{22} I_2$$

From circuit, $V_2 = 6/5\, I_2$

$$\Rightarrow \frac{V_2}{I_2} = \frac{6}{5}$$

$$\boxed{Z_{22} = 6/5}$$

Since $I_1 = 0$ drop across $2\Omega \equiv V_1$

$$I' = I_2\left(\frac{2}{5}\right)$$

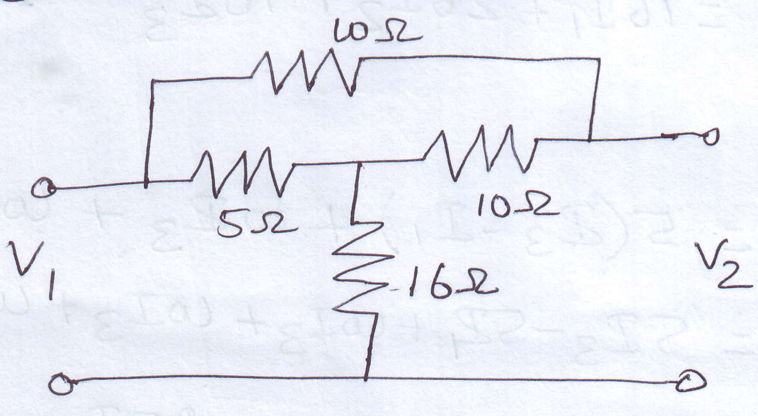
$$V_1 = 2I'$$

$$V_1 = 2\left(\frac{2}{5}\right)I_2$$

$$\frac{V_1}{I_2} = \frac{4}{5}$$

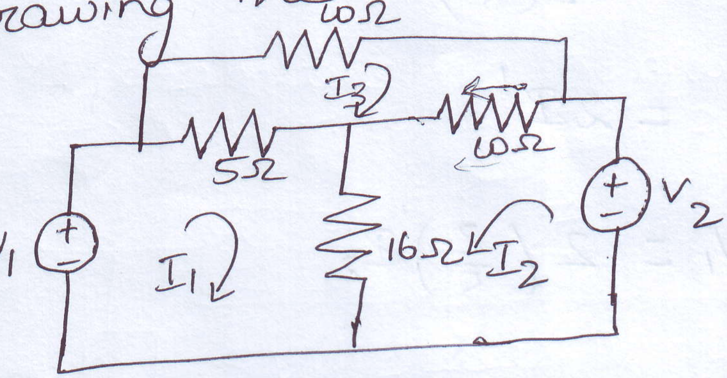
$$\boxed{Z_{12} = \frac{4}{5}}$$

3. Find Z-parameters of the n/w by indirect method.



Soln :-

* Redrawing the circuit,



Writing KVL equations,

loop 1

$$V_1 = 5(I_1 - I_3) + 16(I_1 + I_2)$$

$$V_1 = 5I_1 + 16I_1 + 16I_2 - 5I_3$$

$$\boxed{V_1 = 21I_1 + 16I_2 - 5I_3} \longrightarrow (1)$$

loop 2

$$V_2 = 10(I_2 + I_3) + 16(I_1 + I_2)$$

$$V_2 = 16I_1 + 26I_2 + 10I_3 \longrightarrow (2)$$

loop 3

$$0 = 5(I_3 - I_1) + 10I_3 + 10(I_3 + I_2)$$

$$0 = 5I_3 - 5I_1 + 10I_3 + 10I_3 + 10I_2$$

$$0 = -5I_1 + 10I_2 + 25I_3 \longrightarrow (3)$$

Eliminating I_3

Consider eqn 1 & 3

$$1 \times 5 \Rightarrow 5V_1 = 105I_1 + 80I_2 - 25I_3$$

$$0 = -5I_1 + 10I_2 + 25I_3$$

$$5V_1 = 100I_1 + 90I_2$$

$$V_1 = 20I_1 + 18I_2 \longrightarrow 4$$

Consider eqn 2 & 3

$$2 \times 25 \Rightarrow 25V_2 = 400I_1 + 650I_2 + 250I_3$$

$$3 \times 10 \Rightarrow 0 = \underset{(+)}{-}50I_1 \underset{-}{+} 100I_2 \underset{-}{+} 250I_3$$

$$25V_2 = 450I_1 + 550I_2$$

$$V_2 = 18I_1 + 22I_2 \longrightarrow 5$$

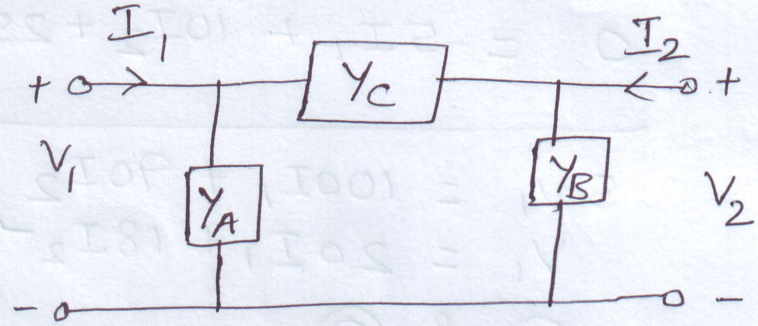
From eqn 4 & 5 we can frame matrix,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 & 18 \\ 18 & 22 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$Z_{11} = 20$; $Z_{12} = 18$; $Z_{21} = 18$; $Z_{22} = 22$.

Problems Based On Y-parameters:-

1. Find Y-parameters of π-n/w.



Soln:-

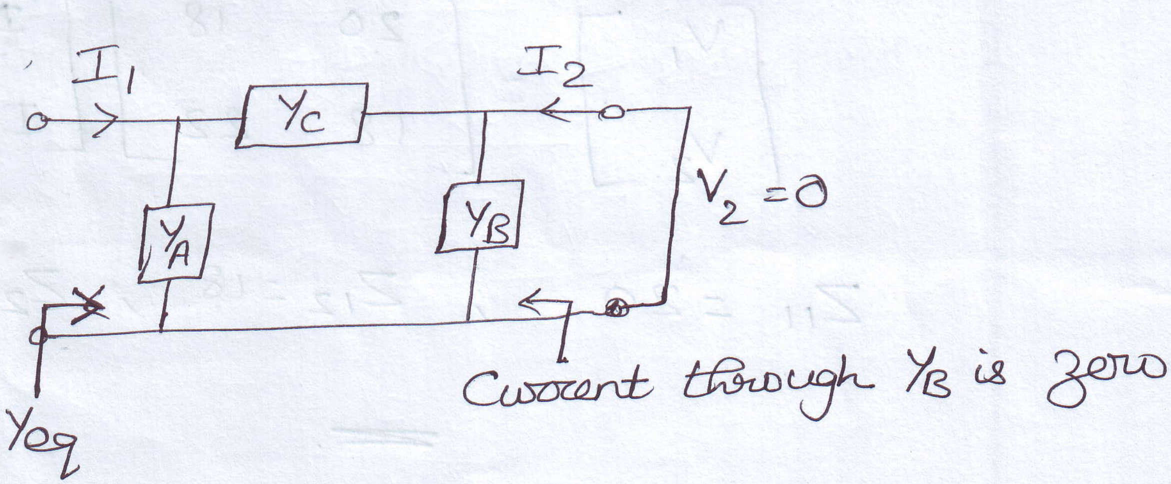
We know,

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \qquad Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \; ; \; Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 . \qquad Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} \; ; \; Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

Step 1:-

$V_2 = 0$ (means short circuit)



Current through $Y_B = 0$

hence,

$$Y_{eq} = Y_A + Y_C$$

$$I_1 = Y_{eq} V_1$$

$$Y_{eq} = Y_{11} = \frac{I_1}{V_1} = Y_A + Y_C.$$

$$\boxed{Y_{11} = Y_A + Y_C} \Rightarrow I_1 = V_1(Y_A + Y_C) \longrightarrow 1$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$I_2 = -I_1 \times \frac{Z_A}{Z_A + Z_C}$$

$$Z_A = 1/Y_A \;,\; Z_C = 1/Y_C.$$

$$I_2 = -I_1 \times \frac{1/Y_A}{\frac{1}{Y_A} + 1/Y_C} = -I_1 \frac{1/Y_A}{\frac{Y_C + Y_A}{Y_A Y_C}}$$

$$I_2 = -I_1 \frac{1}{Y_A} \times \frac{Y_A Y_C}{Y_C + Y_A} = \frac{-I_1 Y_C}{Y_A + Y_C} \longrightarrow 2$$

Sub I_1 in eqn 2

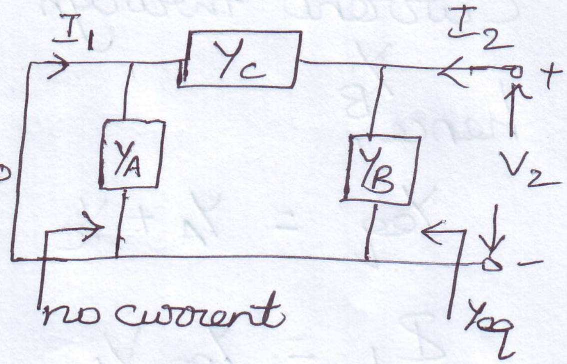
$$I_2 = -\frac{V_1(Y_A + Y_C)Y_C}{Y_A + Y_C}$$

$$Y_{21} = \frac{I_2}{V_1} = -\frac{Y_C(Y_A + Y_C)}{Y_A + Y_C} = -Y_C$$

Step 2 :-

When $V_1 = 0$

Current through $Y_A = 0$.



$$I_2 = V_2 Y_{eq}$$

$$Y_{eq} = Y_B + Y_C$$

$$I_2 = (Y_B + Y_C) V_2$$

$$Y_{22} = \frac{I_2}{V_2} = Y_B + Y_C$$

$$\Rightarrow I_2 = V_2 (Y_B + Y_C) \longrightarrow (3)$$

$$\boxed{Y_{22} = Y_B + Y_C}$$

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$$I_1 = -I_2 \frac{1/Y_B}{\frac{1}{Y_B} + \frac{1}{Y_C}} = \frac{-I_2 \frac{1}{Y_B}}{\frac{Y_C + Y_B}{Y_B Y_C}}$$

$$= -I_2 \times \frac{1}{Y_B} \times \frac{Y_B Y_C}{Y_C + Y_B}$$

$$I_1 = -I_2 \frac{Y_C}{Y_B + Y_C} \longrightarrow (4)$$

Sub I_2 in eqn (4)

$$I_1 = - \frac{Y_C}{Y_B + Y_C} V_2 (Y_B + Y_C)$$

$$I_1 = -Y_C V_2$$

Ans:-

$Y_{11} = Y_A + Y_C$

$Y_{12} = Y_{21} = -Y_C$

$$\boxed{Y_{12} = \frac{I_1}{\ldots} \ldots Y \ldots}$$

Note :-

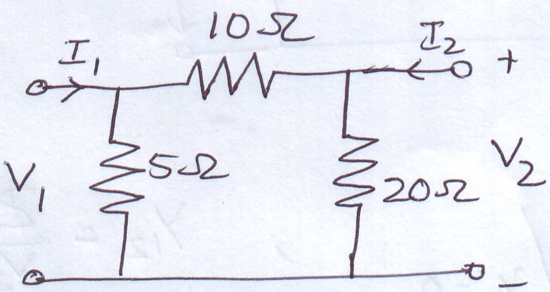
For π – n/w

$Y_{11} = Y_A + Y_C$

$Y_{12} = Y_{21} = -Y_C$

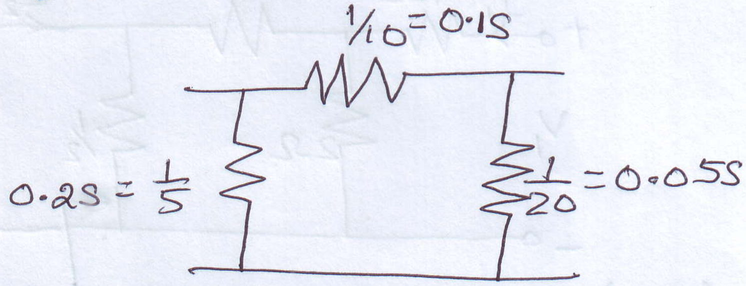
$Y_{22} = Y_B + Y_C$.

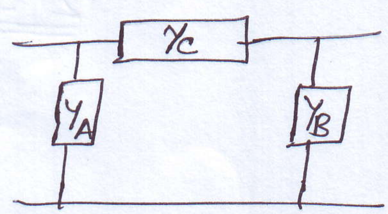
2. Find the Y- parameters for the given n/w



Soln :-

Redrawing the above resistive n/w in terms of admittance,





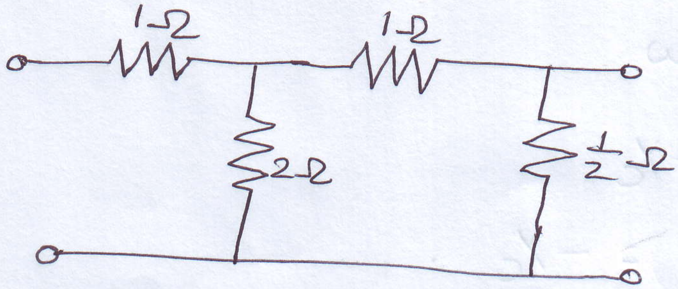
By comparing with standard π n/w

$Y_A = 0.2S$; $Y_B = 0.05S$; $Y_C = 0.1S$

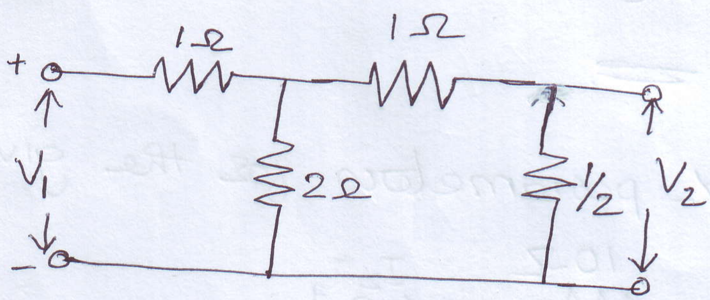
$Y_{11} = Y_A + Y_C = 0.2S + 0.1S = 0.3S$

$Y_{12} = Y_{21} = -Y_C = -0.1S$

3. Find Y- parameters for the given n/w



Soln:-

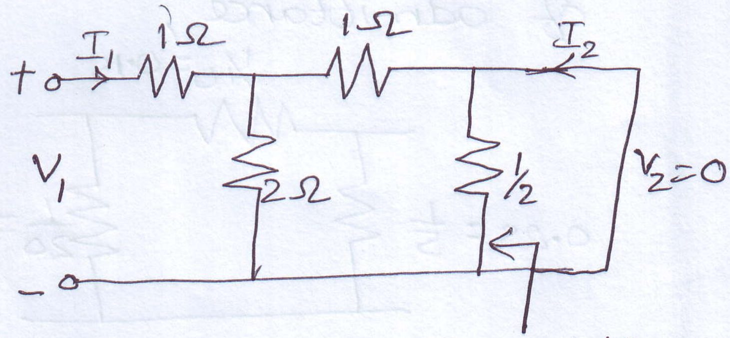


Step 1:-

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \quad ; \quad Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} \quad ; \quad Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

$V_2 = 0$



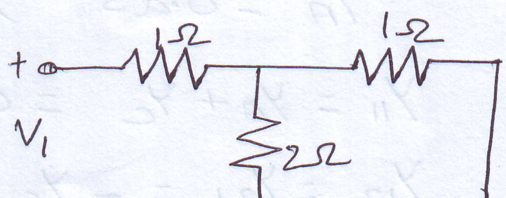
$V_1 = I_1(Z_{eq})$

$\Downarrow$ Current through $\frac{1}{2}$Ω is zero.

$Z_{eq} = 1 + \frac{1 \times 2}{1+2}$

Circuit can be redrawn as,

$= 1 + \frac{2}{3} = \frac{3+2}{3}$



$Z_{eq} = 5/3$

$$\frac{1}{Z_{eq}} = Y_{eq} = 3/5$$

$$\Rightarrow I_1 = \frac{V_1}{Z_{eq}} = V_1 Y_{eq}$$

$$I_1 = V_1 Y_{11} = V_1(3/5)$$

$$Y_{11} = \frac{I_1}{V_1} = 3/5$$

$$\boxed{Y_{11} = 3/5}$$

By Current division rule,

$$I_2 = -I_1 \times \frac{2}{3} \longrightarrow ①$$

Sub I_1 in eqn ①

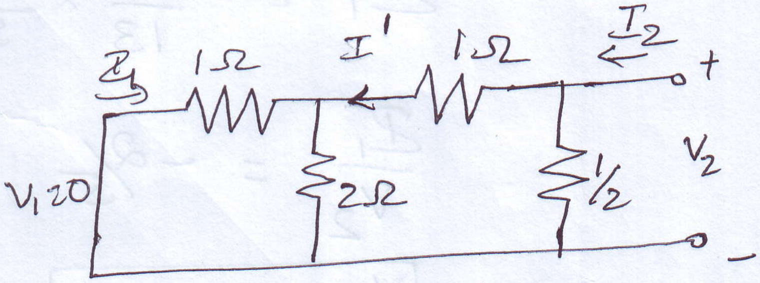
$$I_2 = -V_1\left(\frac{3}{5}\right)\left(\frac{2}{3}\right)$$

$$Y_{21} = \frac{I_2}{V_1} = -\frac{2}{5}$$

$$\boxed{Y_{21} = -2/5}$$

<u>Step 2</u> :-

$V_1 = 0$



Fig(1)

$$\left[(1\Omega \parallel^{el} 2\Omega) + 1\right] \parallel 1/2$$

$$\left(\frac{1\times 2}{1+2}\right) + 1 = \frac{2}{3} + 1 = \frac{2+3}{3} = \frac{5}{3}$$

$$\frac{5}{3} \parallel 1/2 = \frac{\frac{5}{3}\times\frac{1}{2}}{\frac{5}{3}+\frac{1}{2}} = \frac{5/6}{\frac{10+3}{6}} = \frac{5/6}{13/6} = \frac{5}{6}\times\frac{6}{13}$$

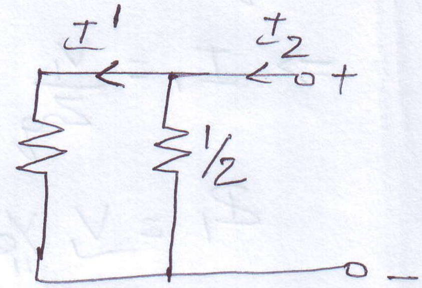
(11)

$$V_2 = \frac{5}{13} I_2$$

$$Y_{22} = \frac{I_2}{V_2} = 13/5$$

$$\Rightarrow \boxed{I_2 = \frac{13}{5} V_2}$$

$$\boxed{Y_{22} = 13/5}$$



Fig(2)

$$Y_{12} = \frac{I_1}{V_2}\Big/_{V_1=0}$$

From Fig(2)

$$I' = \frac{I_2(1/2)}{\frac{1}{2}+5/3} = \frac{I_2/2}{\frac{3+10}{6}} = \frac{I_2/2}{13/6} = \frac{I_2}{2} \times \frac{6}{13}$$

$$I' = \frac{3 I_2}{13}$$

From Fig(1)

$$I_1 = -I'\left(\frac{2}{2+1}\right) = -\frac{3 I_2}{13}\left(\frac{2}{3}\right) = -\frac{2 I_2}{13} \rightarrow (1)$$

Sub $I_2 = \frac{13}{5} V_2$ in (1)

$$I_1 = -\frac{2}{13} \times \frac{13}{5} V_2 = -\frac{2}{5} V_2$$

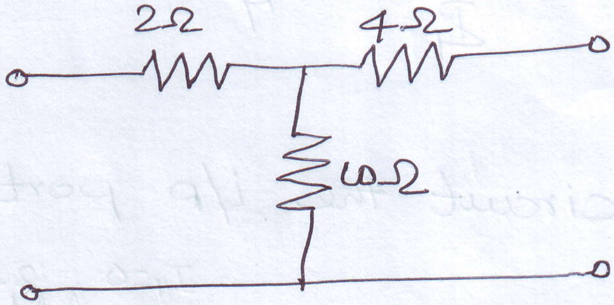
$$\frac{I_1}{V_2} = -2/5$$

$$\boxed{Y_{12} = -2/5}$$

$Y_{11} = 3/5$; $Y_{12} = Y_{21} = -2/5$; $Y_{22} = 13/5$

Hybrid Parameters Problems :-

1. Find the h-parameters for the network.

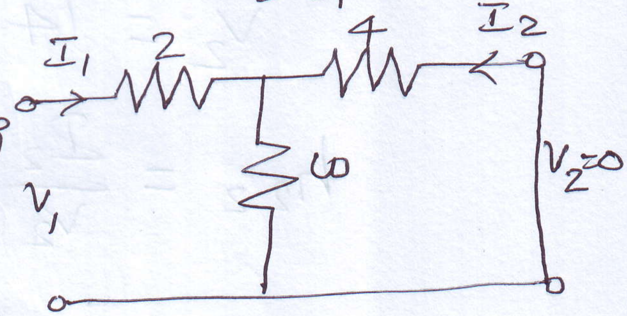


Soln :-

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad ; \quad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \quad ; \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

$\underline{\underline{V_2 = 0}}$ i.e.) Short circuiting



$$R_{eq} = \left(\frac{4 \times 10}{4+10}\right) + 2 = \frac{40}{14} + 2$$

$$= \frac{20}{7} + 2 = \frac{20+14}{7} = \frac{34}{7}$$

$$V_1 = I_1\left(\frac{34}{7}\right)$$

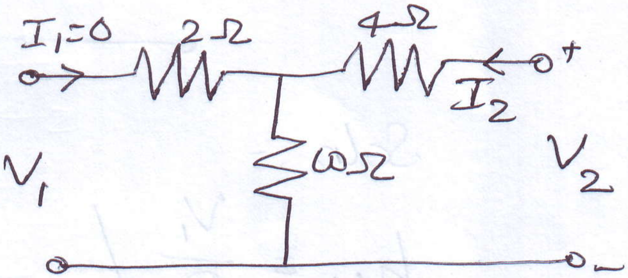
We know,

$$h_{11} = \frac{V_1}{I_1} = \frac{34}{7}\ \Omega \ 4.$$

$$I_2 = -I_1 \frac{10}{10+4} = -I_1\left(\frac{10}{14}\right) = -I_1(5/7)$$

$$h_{21} = \frac{I_2}{I_1} = -\frac{5}{7}$$

Open circuit the i/p port $I_1 = 0$.



$$V_2 = 4I_2 + 10(I_2 + I_1)$$
$$I_1 = 0.$$

hence

$$V_2 = 4I_2 + 10I_2$$

$$V_2 = 14I_2$$

$$h_{22} = \frac{I_2}{V_2} = \frac{1}{14}$$

$$V_1 = 10(I_1 + I_2)$$

We know $I_1 = 0$

$$V_1 = 10I_2$$

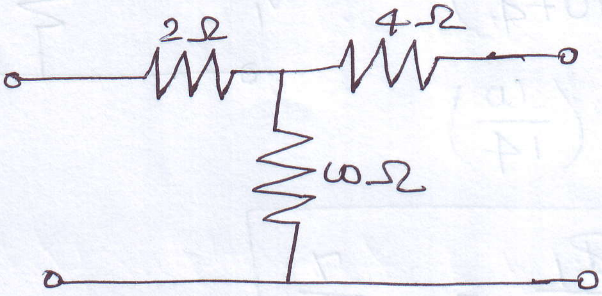
Sub I_2 in V_1

$$V_1 = \frac{\cancel{10}^{5} \times V_2}{\cancel{14}_{7}} = \frac{5V_2}{7}$$

$$h_{12} = \frac{V_1}{V_2} = 5/7$$

Problems based on Transmission Parameters :- (13)

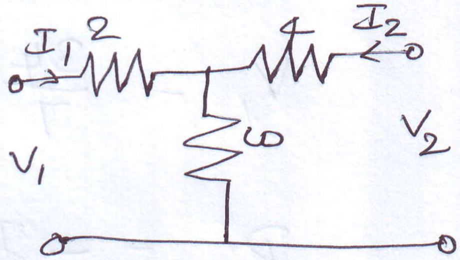
2. Find the transmission parameters for the n/w.



$A = \frac{V_1}{V_2}\Big|_{I_2=0}$; $B = \frac{-V_1}{I_2}\Big|_{V_2=0}$

$C = \frac{I_1}{V_2}\Big|_{I_2=0}$; $D = \frac{-I_1}{I_2}\Big|_{V_2=0}$

Open Circuit the output so that $I_2 = 0$



$V_1 = 2I_1 + 6(I_1 + I_2)$

we know $I_2 = 0$

$V_1 = 2I_1 + 6I_1$

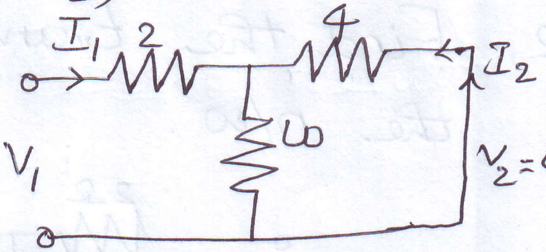
$\boxed{V_1 = 12 I_1}$

$V_2 = 6I_1 \Rightarrow \frac{I_1}{V_2} = \frac{1}{6} = 0.1$

$A = \frac{V_1}{V_2} = \frac{12I_1}{10I_1} = 1.2$ //

$C = \frac{I_1}{V_2} = 0.1$ //.

$V_2 = 0$ (ie Short Circuit V_2)



$$I_2 = -I_1\left(\frac{10}{10+4}\right)$$

$$I_2 = -I_1\left(\frac{10}{14}\right)$$

$$\boxed{D = -\frac{I_1}{I_2} = -\frac{7}{5}}$$

$$B = -\frac{V_1}{I_2}\Big|_{V_2=0}$$

$$V_1 = R_{eq} I_1 = \left\{\left(\frac{4\times 10}{4+10}\right)+2\right\} I_1$$

$$V_1 = \frac{34}{7} I_1 \Rightarrow \boxed{I_1 = \frac{7V_1}{34}}$$

$$I_2 = -I_1\left(\frac{10}{10+4}\right) = -\left(\frac{10}{14}\right) I_1 = -\frac{5}{7} I_1$$

Sub I_1

$$I_2 = -\frac{5}{7}\left(\frac{7V_1}{34}\right) = -\frac{5}{34} V_1$$

$$B = \frac{-V_1}{I_2} = \frac{34}{5}$$ //

$A = 1.2$; $B = \frac{34}{5}$; $C = 0.1$; $D = -1.4$

Problems based on the Interrelationships between Parameters :-

1. Determine Y-Parameters if the Z-parameters are $Z = \begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}$

<u>Soln</u>:-

$Z_{1} = 3$; $Z_{12} = 2$; $Z_{21} = 2$; $Z_{22} = 6$

$\Delta Z = Z_{11} Z_{22} - Z_{21} Z_{12}$

$\Delta Z = 3(6) - 2(2)$

$\boxed{\Delta Z = 14}$

$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{6}{14} = \frac{3}{7}$

$Y_{12} = \frac{-Z_{12}}{\Delta Z} = \frac{-2}{14} = -1/7$

$Y_{21} = \frac{-Z_{21}}{\Delta Z} = \frac{-2}{14} = -1/7$

$Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{3}{14} = 3/14$

2. Determine h-parameters if the Y-parameters are $Y = \begin{bmatrix} 0.1 & 0.1 \\ 0.4 & 0.5 \end{bmatrix}$

Soln:-

$Y_{11} = 0.1$; $Y_{12} = 0.1$; $Y_{21} = 0.4$; $Y_{22} = 0.5$

$$h_{11} = \frac{1}{Y_{11}} = \frac{1}{0.1} = 10$$

$$h_{12} = \frac{-Y_{12}}{Y_{11}} = \frac{-0.1}{0.1} = -1$$

$$h_{21} = \frac{Y_{21}}{Y_{11}} = \frac{0.4}{0.1} = 4$$

$$h_{22} = \frac{Y_{11}Y_{22} - Y_{21}Y_{12}}{Y_{11}} = \frac{(0.1)(0.5) - (0.4)(0.1)}{0.1}$$

$$h_{22} = 0.1$$

---

3. A two port n/w has the following Z-parameters. Find h-parameters and transmission parameters. $Z_{11} = 1$; $Z_{12} = Z_{21} = -0.2$
$Z_{22} = 0.6$

Soln:-

$$h_{11} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}} = \frac{1(0.6) - (-0.2)(-0.2)}{0.6}$$

$$h_{12} = \frac{Z_{12}}{Z_{22}} = -\frac{0.2}{0.6} = -1/3$$

$$h_{22} = \frac{1}{Z_{22}} = \frac{1}{0.6} = 5/3$$

$$h_{21} = \frac{-Z_{21}}{Z_{22}} = \frac{0.2}{0.6} = 1/3$$

$$A = \frac{Z_{11}}{Z_{21}} = -\frac{1}{0.2}$$

$$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} = -2.8$$

$$C = \frac{1}{Z_{21}} = -\frac{1}{0.2} = -5$$

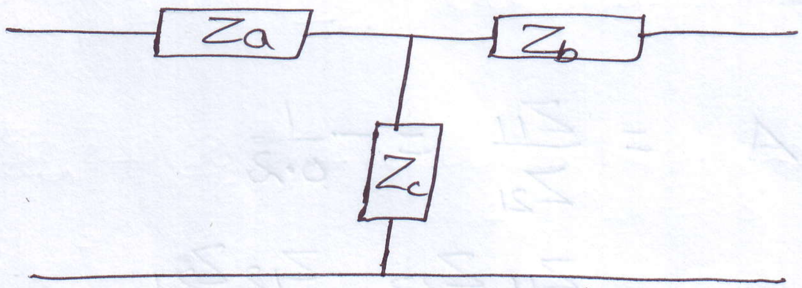
$$D = \frac{Z_{22}}{Z_{21}} = -\frac{0.6}{0.2} = -3$$

3. The Z parameters of a two port network are

$Z_{11} = 10\,\Omega$; $Z_{22} = 15\,\Omega$; $Z_{12} = Z_{21} = 5\,\Omega$.

Find the equivalent T n/w & ABCD parameters.

Soln :-



equivalent T_n/w

Z-parameters:-

$$Z_a = Z_{11} - Z_{21} = 5\,\Omega$$

$$Z_b = Z_{22} - Z_{12} = 10\,\Omega$$

$$Z_c = 5\,\Omega$$

ABCD parameters :-

$$A = \frac{Z_a}{Z_c} + 1 = 2$$

$$B = (Z_a + Z_b) + \frac{Z_a Z_b}{Z_c} = 25$$

$$C = \frac{1}{Z_c} = 0.2\,\mho$$

$$D = 1 + \frac{Z_b}{Z_c} = 3 \,/\!/.$$

Lattice Networks :-

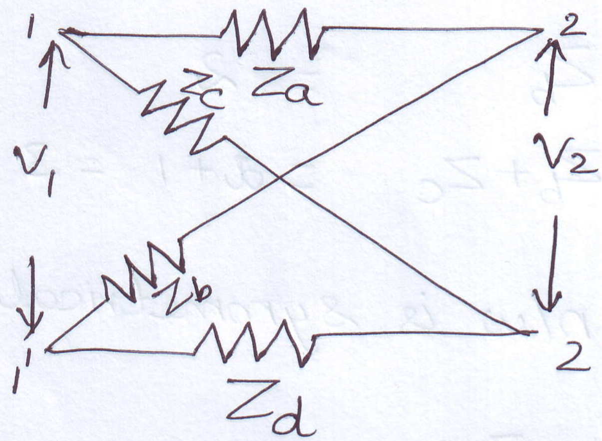
→ 4 terminal two port n/w

→ Used in filter sections.

→ Z_a , Z_d series arms

Z_b , Z_c diagonal arms.

* $Z_d = 0$ (the lattice becomes π-section)



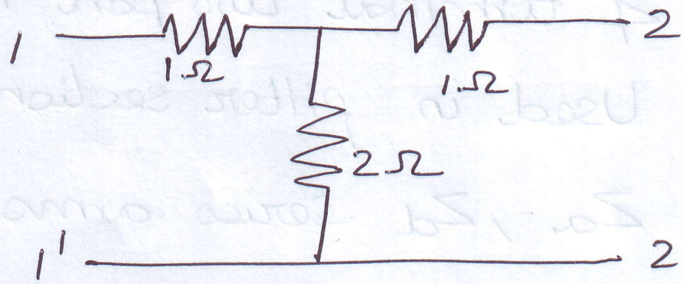
If the network is symmetric,

$Z_a = Z_d$; $Z_b = Z_c$

$Z_a = Z_{11} - Z_{12}$

$Z_b = Z_{11} + Z_{12}$.

1. Obtain the lattice equivalent of a symmetrical T-n/w.



We know,

$$Z_{11} = Z_a + Z_b = 1+2 = 3$$

$$Z_{12} = Z_b = 2$$

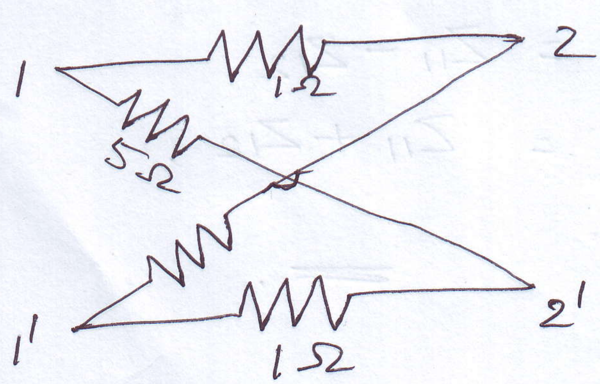
$$Z_{21} = Z_b = 2$$

$$Z_{22} = Z_b + Z_c = 2+1 = 3$$

The given n/w is symmetrical & reciprocal

$$Z_a = Z_{11} - Z_{12} = 1\Omega$$

$$Z_b = Z_{11} + Z_{12} = 5\Omega$$



UNIT-IV

ELEMENTS OF NETWORK SYNTHESIS

Hurwitz Polynomials :-

* The poles of the stable s/m must lie on the left half of s-plane.

* Any n/w fn. can be written as the ratio of two polynomials,

$$Z(S) = \frac{P(S)}{Q(S)}$$

Properties Of Hurwitz Polynomials :-

a) Z(S) must be a real function of S

$$Z(S) = \frac{P(S)}{Q(S)} = \frac{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}$$

where all quotients a_i, b_j are real, hence Z(S) is real if s is real.

b) All roots of P(S) must have zero real parts (or) negative real parts i.e. the roots lie in negative half of S-plane.

Properties :-

1. All quotients in the polynomial are positive.
$P(S) = a_0 S^n + a_1 S^{n-1} + \dots a_{n-1} S + a_n$

2. The roots of the odd and even parts of a hurwitz polynomial P(S) lie on the jω axis.

$$P(S) = \underbrace{O(S)}_{\text{odd}} + \underbrace{e(S)}_{\text{even}}$$

$\Rightarrow$ Both have roots on jω axis.

3. If the polynomial P(S) is either even or odd, the roots of P(S) lie on the jω axis.

4. All the quotient terms are positive in the continued fraction expansion of the ratio of odd to even (or) even to odd parts of the polynomial P(S).

5. If the polynomial satisfies the Condition of Hurwitz, then the polynomial must be Hurwitz to within an even multiplicative factor $w(S)$ i.e) if

$P_1(S) = w(S)\, P(S)$, then $P(S)$ & $w(S)$ are hurwitz, $P_1(s)$ must be hurwitz.

6. If the ratio of the polynomial $P(S)$ & its derivative $P(S)$ gives a continued fraction expansion with all positive coefficients, then the polynomial $P(S)$ is Hurwitz.

Positive Real Function :-

A rational function $F(S)$ is positive real function if it satisfies

1. $F(S)$ is real for real value of S.
2. The poles & zeros of $F(S)$ lie in the left half of the s-plane or on the imaginary axis.
3. The real part of $F(S)$ is greater than ... zero when the real part of 's'

Properties of Positive real Functions :-

1. If $F(S) = \frac{P(S)}{q(S)}$ where $P(S)$ & $q(S)$ are polynomials in 's' then the coefficients of $P(S)$ & $q(S)$ are real.

2. The sum of positive real function is positive real.

3. $P(S)$ & $q(S)$ are Hurwitz polynomial.

4. If $F(S)$ is real, then $\frac{1}{F(S)}$ is also positive real.

5. The degree of of P & q can differ atmost by unity. This condition prohibits multiple poles & zeros at $s=\infty$.

6. The degree of the lowest powers of P & q differ atmost by one. This condition prohibits the possibility of multiple poles or zeros at $s=0$.

1. Determine whether the given function is positive real. $F(s) = \dfrac{s^2+1}{s^3+4s}$

(i) The function $F(s)$ has all the real quotient terms hence $F(s)$ is real for real s.

(ii) poles & zeros

$$Z_{1,2} = \pm j$$

$$P_1 = 0 \; ; \; P_{2,3} = \pm j2 .$$

Poles & zeros lie on imagniary axis.

(iii) $$Re\left[F(j\omega)\right] = Re\left[\frac{(j\omega)^2+1}{(j\omega)^3+4j\omega}\right]$$

$$= Re\left[\frac{-\omega^2+1}{j\omega(-\omega^2+4)}\right]$$

$$= Re\left[\frac{j(-\omega^2+1)}{-\omega(-\omega^2+4)}\right]$$

$$\simeq 0$$

$Re\left[F(s)\right] = 0$ for all values of $s \geq 0$.

$F(s)$ is positive real.

Test the following polynomials for hurwitz property.

(i) $s^5 + 5s^3 + 11s^2 + 25s + 6$.

s^4 term is missing, hence $P(s)$ is not hurwitz.

(ii) $s^5 + 2s^4 + 5s^3 + 10s^2 + 9s + 18$.

$$n(s) = s^5 + 5s^3 + 9s$$

$$m(s) = 2s^4 + 10s^2 + 18$$

$$
\begin{array}{r}
2s^4+10s^2+18 \,\big)\, s^5 + 5s^3 + 9s \,\big(\, s/2 \\
s^5 + 5s^2 + 9s \\
\hline
0
\end{array}
$$

Hurwitz polynomial.

(iii) $s^7 + s^5 + s^3 + s$

$$P(s) = s^7 + s^5 + s^3 + s$$

$$P'(s) = 7s^6 + 5s^4 + 3s^2 + 1$$

$$\Longrightarrow \frac{P(s)}{P'(s)}$$

$$7s^6+5s^4+3s^2+1\,)\; s^7+s^5+s^3+s\;(\,\frac{s}{7}$$
$$s^7+\frac{5}{7}s^5+\frac{3}{7}s^3+\frac{s}{7}$$
$$\frac{2}{7}s^5+\frac{4}{7}s^3+\frac{6}{7}s\,)\;7s^6+5s^4+3s^2+1\;(\,\frac{49s}{2}$$
$$7s^6+14s^4+21s^2$$

$$7s^6+5s^4+3s^2+1\,)\; s^7+s^5+s^3+s\;(\,\frac{s}{7}$$
$$s^7+\frac{5}{7}s^5+\frac{3}{7}s^3+\frac{s}{7}$$
$$\frac{2}{7}s^5+\frac{4}{7}s^3+\frac{6}{7}s\,)\;7s^6+5s^4+3s^2+1\;(\,\frac{49s}{2}$$
$$7s^6+14s^4+21s^2$$
$$-9s^4-18s^2+1\,)\;\frac{2}{7}s^5+\frac{4}{7}s^3+\frac{6}{7}s\;(\,-\frac{2s}{63}$$
$$\frac{2}{7}s^5+\frac{4}{7}s^3-\frac{2}{63}s$$

Not an hurwitz Polynomial, becoz of negative quotient.

(A)

Synthesis Of Driving Point Immitance Functions:

* immitance (Impedance or admittance)

* The immitance function must be positive real function so that its synthesis can be done to obtain an electrical n/w, using passive elements.

Passive Elements → Inductor (L), Capacitor (C) Resistor.

* A n/w using any two types of passive elements can be synthesized generally in two forms called,

1. Foster Form 2. Cauer Form.

Canonical (or) Simple Form Of Realization :-

Foster Form & Cauer forms are used for the network realization because the n/w is realized using minimum number of passive elements using these basic forms. Hence these forms are called as Canonical

Foster Form

- Foster Form I
- Foster Form II

Cauer Form

- Cauer Form I
- Cauer Form II

Elements	Z(S)	Y(S)
Resistance R	R	$1/R = G$
Inductance L	SL	$\frac{1}{SL}$
Capacitance C	$\frac{1}{SC}$	SC

FOSTER I FORM :-

Foster I form uses the partial fraction of the driving point impedance function Z(S).

$$Z(S) = Z_1(S) + Z_2(S) + \dots + Z_n(S)$$

<u>NOTE</u>

Degree Of Numerator < Degree of Denominator

FOSTER II FORM :-

Foster II form uses the partial fractions of the driving point function $Y(S) = \frac{1}{Z(S)}$.

$$Y(S) = Y_1(S) + Y_2(S) + \dots + Y_n(S)$$

Thus a n/w can be realized by connecting the admittances $Y_1, Y_2, \dots Y_n$ in parallel.

Cauer I Form :-

* This form uses continued fraction expansion of the driving point impedance function $Z(S)$.

* Numerator & denominator are arranged in descending powers of S, starting from highest to lowest power of S.

$$Z(S) = \underset{(\text{series})}{Z_1(S)} + \cfrac{1}{\underset{(\text{Shunt})}{Y_2(S)} + \cfrac{1}{Z_3(S)} + \cdots}$$

Quotient of first division ⇒ Impedance (Series arm)

Quotient of second division ⇒ $Y(S)$ [shunt arm]

$Y(S)$ & $Z(S)$ represents shunt & series arms respectively.

__NOTE__ :-

If the degree of Nr is same or less than the denominator, there is possibility of negative coefficients in the continued fraction expansion. In such case restart with inversion & first quotient in such case represents admittance $Y(S)$ which is a shunt arm. Series arm is absent in such a case.

* Cauer Form I is <u>low pass</u> structure of network.

⇒ If driving point admittance function Y(S) is expressed in continued fraction expansion with division at start, first quotient → admittance in shunt arm, then alternatily impedance & admittance in series & shunt arms.
Starting with inversion in such case gives impedance in series arm as the first quotient.

Cauer II Form :-

* Also uses continued fraction expansion of driving point immitance function either Z(S) or Y(S).

* Numerator & denominator are arranged in ascending powers of S, starting from lowest to highest power of S.

$$Z(S) = \underset{(series)}{Z_1(S)} + \cfrac{1}{\underset{(shunt)}{Y_2(S)} + \cfrac{1}{Z_3(S) + \cdots}}$$

* In this form there is a possibility of getting negative coefficients is expansion procedure. In such a case, restart with inversion, which gives $Y(S)$ as the first quotient which indicates shunt arm, without a series arm in the circuit.

Cauer II Form is basically high pass.

⟹ If $Y(S)$ is to be synthesized & expansion starts with the division, first quotient gives $Y(S)$ in shunt arm and if starts with inversion, first quotient gives $Z(S)$ in series arm & vice versa for $Z(S)$.

Synthesis Of Driving Point Immitance Function Of LC Network :-

LC n/w driving point functions can be LC impedance functions denoted as Z_{LC} (or) LC admittance functions denoted as $Y_{LC}(S)$.

A LC n/w doesnot contain power dissipative components i.e. resistance & only consists of reactive elements L & C components. Hence such n/w is also called a reactance n/w or lossless n/w.

Properties Of LC Driving Point Immitance Function :-

1. The immitance function is always a ratio of odd to even or even to odd polynomials
2. The poles & zeros are simple. There are no multiple poles or zeros either at origin or infinity or at any point.
3. The poles & zeros are located ~~at~~ on $j\omega$ axis only.
4. The poles & zeros interlace (alternate) each other on $j\omega$ axis. There are no consecutive poles or zeros on $j\omega$ axis.
5. The imaginary poles & zeros occur in the form of complex conjugate pairs.

6. The highest powers of numerator & denominator must differ by unity.

7. The lowest powers of numerator & denominator must differ by unity.

9. Residues at imaginary axis poles are real & positive.

10. The slope of the graph of reactance against frequency is always positive.

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1. Determine Foster & Cauer forms of realization of driving point impedance function,

$$Z(s) = \frac{4(s^2+1)(s^2+9)}{s(s^2+4)}.$$

<u>Soln</u>:-

Foster I Form :-

Degree of $N(s) > D(s)$ degree

* So perform division & then obtain partial fraction of remainder.

$$Z(S) = \frac{4S^4 + 40S^2 + 36}{S^3 + 4S}$$

$$S^3 + 4S \overline{\big)\, 4S^4 + 40S^2 + 36 \,(\, 4S}$$

$$\underline{4S^4 + 16S^2}$$

$$24S^2 + 36$$

$$Z(S) = 4S + \frac{24S^2 + 36}{S^3 + 4S}$$

$$= 4S + \frac{24S^2 + 36}{S(S^2 + 4)}$$

$$Z(S) = 4S + \frac{A}{S} + \frac{BS}{S^2 + 4}$$

$$\frac{\cancel{A}\; 24S^2 + 36}{S\,(S^2 + 4)} = \frac{A}{S} + \frac{BS}{S^2 + 4}$$

$$\frac{24S^2 + 36}{S(S^2 + 4)} = \frac{A(S^2 + 4) + BS^2}{S(S^2 + 4)}$$

$$24S^2 + 36 = AS^2 + 4A + BS^2$$

equating S^2

$$A + B = 24$$

equating constant

$B = 36$

$$Z(s) = 4s + \frac{9}{s} + \frac{15s}{s^2+4}$$

$$Z(s) = Z_1(s) + Z_2(s) + Z_3(s)$$

$Z_1(s)$, $Z_2(s)$ & $Z_3(s)$ are in series Combination.

We know,

$Z = R$ $\quad Y = 1/R$

$Z = LS$ $\quad Y = \frac{1}{LS}$

$Z = \frac{1}{C}$ $\quad Y = CS$

$$Z_1(s) = LS = 4S$$

i.e.) $\boxed{L = 4H}$

$$Z_2(s) = \frac{9}{s} = \frac{1}{(\frac{1}{9})s}$$

i.e.) $\boxed{C = 1/9 F}$

$$Z_3(s) = \frac{15s}{s^2+4} = \frac{1}{\frac{1}{15s}(s^2+4)}$$

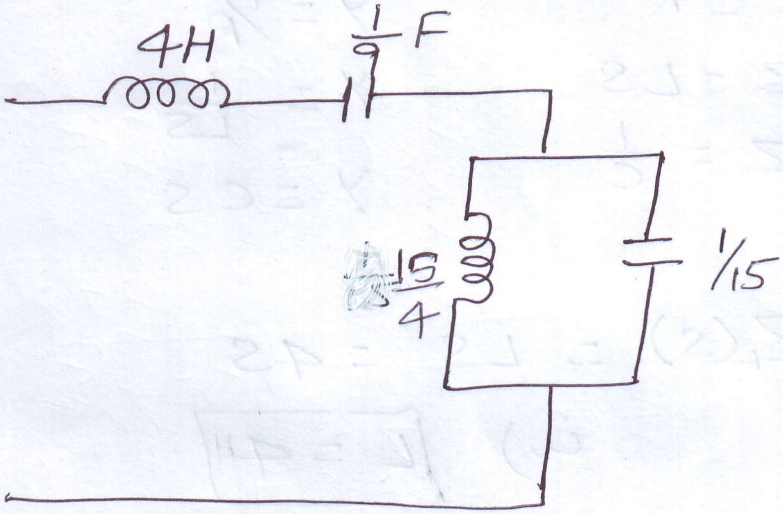
$$Z_3(s) = \frac{1}{\frac{s}{15} + \frac{4}{15s}} = \frac{1}{\frac{s}{15} + \frac{1}{(\frac{15}{4})s}}$$

$$Z_3(S) = \frac{1}{Y_3(S)}$$

$$Y_3(S) = \frac{S}{15} + \frac{1}{\left(\frac{15}{4}\right)S}$$

iii) $\frac{S}{15}$ iii) $C = 1/15\ F$

$\frac{1}{LS}$ iii) $L = 15/4\ H$



Foster II Form :-

$$Y(S) = \frac{1}{Z(S)}$$

$$= \frac{1}{\frac{4(S^2+1)(S^2+9)}{S(S^2+4)}}$$

$$= \frac{S(S^2+4)}{}$$

$$= \frac{s^3+4s}{4(s^2+1)(s^2+9)}$$

$$Y(s) = \frac{4\left[\frac{s^3}{4}+s\right]}{4(s^2+1)(s^2+9)}$$

$$\frac{\frac{s^3}{4}+s}{4(s^2+1)(s^2+9)} = \frac{As}{s^2+1} + \frac{Bs}{s^2+9}$$

$$\frac{s^3}{4}+s = As(s^2+9) + Bs(s^2+1)$$

$$\Rightarrow \quad A = 3/32 \quad ; \quad B = 5/32$$

$$Y(s) = \frac{3/32\, s}{s^2+1} + \frac{5/32\, s}{s^2+9}$$

$$Y_1(s) = \frac{3/32\, s}{s^2+1} = \frac{1}{\frac{32}{3s}(s^2+1)}$$

$$Y_1(s) = \frac{1}{\frac{32}{3\cancel{s}}\left[\frac{s^2}{s}+\frac{1}{s}\right]} = \frac{1}{\frac{32}{3}\left(s+1/s\right)}$$

$$Y_1(s) = \frac{1}{Z_1(s)} = \frac{1}{\frac{32s}{3}+\frac{32}{3s}}$$

$$Z_1(s) = \frac{32s}{3} + \frac{32}{3s}$$

$$Ls = \frac{32s}{3} \quad \text{w)} \quad L = \frac{32}{3}$$

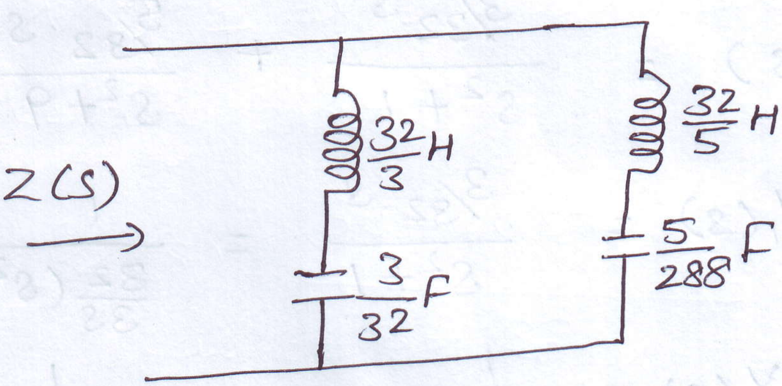
$$Y_2(s) = \frac{\frac{5s}{32}}{s^2+9} = \frac{1}{\frac{32}{5s}(s^2+9)}$$

$$Y_2(s) = \frac{1}{\frac{32s^2}{5s} + \frac{288}{5s}} = \frac{1}{Z_2(s)}$$

$$Z_2(s) = \frac{32}{5}s + \frac{288}{5s}$$

$$Ls = \frac{32}{5}s \qquad \text{i)}\ L = \frac{32}{5}$$

$$\frac{1}{Cs} = \frac{1}{\left(\frac{5}{288}\right)s} \qquad \text{ii)}\ C = \frac{5}{288}$$



Cauer I Form :-

* N(s) & D(s) arranged in descending power

$$Z(s) = \frac{4(s^2+1)(s^2+9)}{s(s^2+4)}$$

$$Z(s) = \frac{4s^4+40s^2+36}{s^3+4s}$$

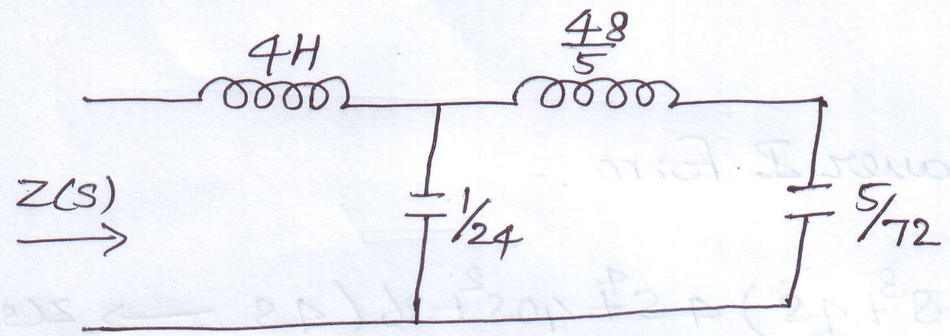
Cauer I Form :-

$$s^3+4s\,)\,4s^4+40s^2+36\,(\,4s \longrightarrow Z(s)$$
$$\underline{4s^4+16s^2}$$
$$24s^2+36\,)\,s^3+4s\,(\,\frac{s}{24} \longrightarrow Y(s)$$
$$\underline{s^3+\tfrac{3}{2}s}$$
$$\tfrac{5}{2}s\,)\,24s^2+36\,(\,\tfrac{48}{5}s \longrightarrow Z(s)$$
$$\underline{24s^2}$$
$$36\,)\,\tfrac{5}{2}s\,(\,\tfrac{5}{72}s \longrightarrow Y(s)$$
$$\underline{\tfrac{5}{2}s}$$
$$0$$

This can be written as,

$$Z(s) = \underbrace{4s}_{Z(s)} + \cfrac{1}{\underbrace{\frac{s}{24}}_{Y(s)} + \cfrac{1}{\underbrace{\frac{48}{5}s}_{Z(s)} + \cfrac{1}{\underbrace{\frac{5}{72}s}_{Y(s)}}}}$$

As fraction starts with division, first quotient gives impedance Z(S) which is series arm,



Cauer II Form :-

* N(S) & D(S) are arranged in ascending order.

$$Z(S) = \frac{36+40S^2+4S^4}{4S+S^3}$$

$4s+s^3 \,)\, 36+40s^2+4s^4 \,(\, \frac{9}{s} \rightarrow Z(s)$

$\quad 36+9s^2$

$\quad \overline{\qquad\qquad}$

$\qquad 31s^2+4s^4 \,)\, 4s+s^3 \,(\, \frac{4}{31s} \rightarrow Y(s)$

$\qquad\qquad 4s+\frac{16}{31}s^3$

$\qquad\qquad \overline{\qquad\qquad}$

$\qquad\qquad\quad \frac{15}{31}s^3 \,)\, 31s^2+4s^4 \,(\, \frac{961}{15s} \rightarrow Z(s)$

$\qquad\qquad\qquad 31s^2$

$\qquad\qquad\qquad \overline{\qquad\qquad}$

$\qquad\qquad\qquad\quad 4s^4 \,)\, \frac{15}{31}s^3 \,(\, \frac{15}{124s}$

$\qquad\qquad\qquad\qquad \frac{15}{31}s^3$

$\qquad\qquad\qquad\qquad \overline{\qquad}$

$\qquad\qquad\qquad\qquad 0$

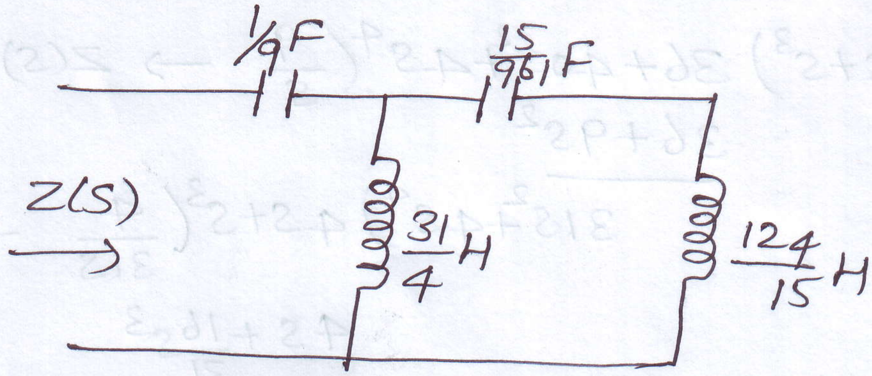
* As started with division first quotient gives Z(s)
* Second quotient gives Y(s)

$Z_1(s) = \frac{9}{s} = \frac{1}{(\frac{1}{9})s}$ ie) $C = 1/9$

$Y_1(s) = \frac{4}{31s} = \frac{1}{(\frac{31}{4})s}$ ie) $L = \frac{31}{4}$

$Z_2(s) = \frac{961}{15s} = \frac{1}{(\frac{15}{961})s}$ ie) $C = \frac{15}{961}$

$Y_2(s) = \frac{15}{124s} = \frac{1}{\ldots}$ ie) $L = \frac{124}{\ldots}$



Note For Cauer Form :-

1. If there is a pole at origin in $Z(s)$, first divide & then invert in continued fraction expansion.

2. If there is no pole at origin in $Z(s)$, first invert & then divide in continued fraction expansion.

Synthesis Of Driving Point Impedance Functions Of RC Networks :-

→ Only R & C components.

Properties :-

1. Poles & Zeros are simple. There are no multiple poles & zeros.
2. The poles & zeros are located on negative real axis.
3. The poles & zeros alternate each other on negative real axis.
4. Poles & zeros are called critical frequencies of the n/w. The critical frequency nearest to origin is always a pole.
5. The critical frequency at a greatest distance away from the origin is always zero, which may be located at ∞ also.

6. The partial fraction expansion of $Z_{RC}(S)$ gives the residues which are always real & positive.

7. No pole located at infinity.

8. The slope is always negative.

9. There no zero at origin.

10. The value of $Z_{RC}(S)$ at $S=0$ is always greater than the value of $Z_{RC}(S)$ at $S=\infty$

=.

1. Realize the given RC n/w impedance function using all four forms.

$$Z(S) = \frac{(S+1)(S+4)}{S(S+2)}$$

<u>Soln</u>:-

Foster I Form :-

$$N(S) \text{ degree} = D(S) \text{ degree}$$

$$Z(S) = \frac{S^2+5S+4}{S^2+2S}$$

$$
\begin{array}{r|l}
s^2+2s \;) & s^2+5s+4 \; (\; 1 \\
 & \underline{s^2+2s} \\
 & \quad 3s+4
\end{array}
$$

$$Z(S) = 1 + \frac{3S+4}{S^2+2S} = 1 + \frac{3S+4}{S(S+2)}$$

$$Z(S) = 1 + \frac{A}{S} + \frac{B}{S+2}$$

By simplification,

ie) $3S+4 = A(S+2) + BS$

$\underline{\underline{S=0}}$

$$4 = 2A$$

$$\boxed{A = 2}$$

$\underline{\underline{S=-2}}$

$$-6+4 = -2B$$

$$\boxed{B = 1}$$

$$Z(S) = 1 + \frac{2}{S} + \frac{1}{S+2}$$

$$Z(S) = Z_1(S) + Z_2(S) + Z_3(S)$$

$$Z_1(S) = R = 1$$

$$Z_2(S) = \frac{2}{S} = \frac{1}{CS} = \frac{1}{(1/2)S} \quad \text{ie)} \; C = 1/2$$

$$Z_3(S) = \frac{1}{S+2}$$

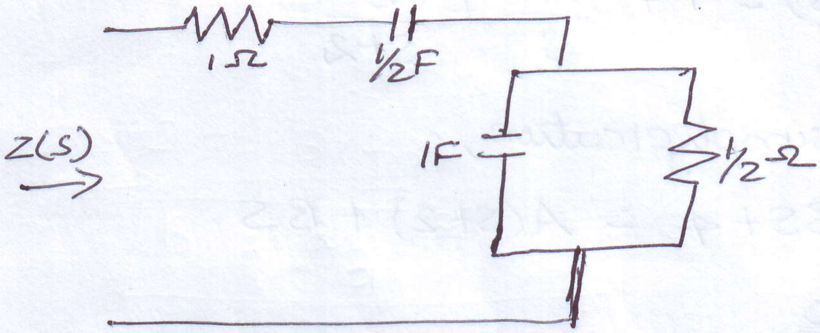
$Z_3(S) = \frac{1}{\;}$ ie) $Y_3(S) = S+2$

We know, $Y_3(s) = sC + 1/R$

$Y_3(s) = s + 2$

ii) $\boxed{C = 1}$

$\boxed{R = 1/2}$



Foster II Form :-

$$Y(s) = \frac{1}{Z(s)} = \frac{1}{\frac{(s+1)(s+4)}{s(s+2)}}$$

$$Y(s) = \frac{s^2+2s}{s^2+5s+4}$$

* degree of N(s) = degree of D(s), we get negative term, which can not be synthesized.

<u>NOTE</u> :- In such case, obtain the partial fractions of $\frac{Y(s)}{s}$ & then multiply both the sides by s.

$$Y(s) = \frac{s(s+2)}{s^2+5s+4}$$

$$\frac{Y(S)}{S} = \frac{(S+2)}{(S+1)(S+4)}$$

$$\frac{Y(S)}{S} = \frac{A}{S+1} + \frac{B}{S+4}$$

$$S+2 = A(S+4) + B(S+1)$$

put $s = -4$

$$-4+2 = B(-4+1)$$

$$-2 = B(-3)$$

$$\boxed{B = 2/3}$$

put $s = -1$

$$-1+2 = A(-1+4)$$

$$1 = A(3)$$

$$\boxed{A = 1/3}$$

$$\frac{Y(S)}{S} = \frac{1/3}{S+1} + \frac{2/3}{S+4}$$

$$Y(S) = \frac{\frac{1}{3}S}{S+1} + \frac{\frac{2}{3}S}{S+2} = Y_1(S) + Y_2(S)$$

So n/w is parallel combination of two branches

$$Y_1(s) = \frac{1}{\frac{3}{S}(S+1)} = \frac{1}{3+3/S} = \frac{1}{Z_1(S)}$$

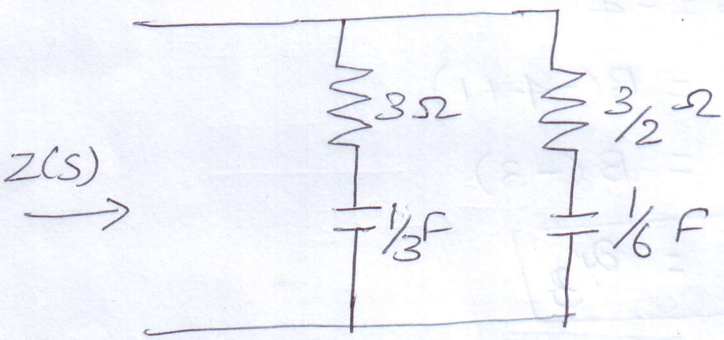
$\Rightarrow Z_1(S) = 3 + 3/S$ [series combination of R & C

R = 3 & C = 1/3

$$Y_2(s) = \frac{\frac{2}{3}s}{s+4} = \frac{1}{\frac{3}{2s}(s+4)} = \frac{1}{\frac{3}{2}+\frac{6}{s}} = \frac{1}{Z_2(s)}$$

$$Z_2(s) = \frac{3}{2} + \frac{6}{s}$$ [series combination of R & C

$R = 3/2$ & $C = 1/6$.



Cauer I Form :-

* It uses continued fraction expansion to give ladder structure.

$$Z(s) = \frac{s^2+5s+4}{s^2+2s}$$

* As there is pole at origin, start with division.

$$s^2+2s\,\big)\, s^2+5s+4 \,\big(1 \longrightarrow Z(s) \qquad \frac{s^{\cancel{2}}}{3\cancel{s}} = s/3$$

$$\underline{s^2+2s} \qquad 2s - \frac{4s}{3} =$$

$$3s+4\,\big)\, s^2+2s\,\big(\frac{s}{3} \longrightarrow Y(s) \qquad \frac{6s-4s}{3} = \frac{2s}{3}$$

$$\underline{s^2+\frac{4s}{3}} \qquad \frac{3\cancel{s}\times 3}{2\cancel{s}} = 9/2$$

$$\frac{2s}{3}\,\big)\, 3s+4\,\big(\frac{9}{2} \longrightarrow Z(s) \qquad \frac{\frac{2s}{3}}{4} = \frac{2s}{3}\times\frac{1}{\cancel{4}\,2}$$

$$\underline{3s}$$

$$4\,\big)\, \frac{2s}{3}\,\big(\frac{s}{6} \longrightarrow Y(s)$$

$$\underline{\frac{2s}{3}}$$

$$0$$

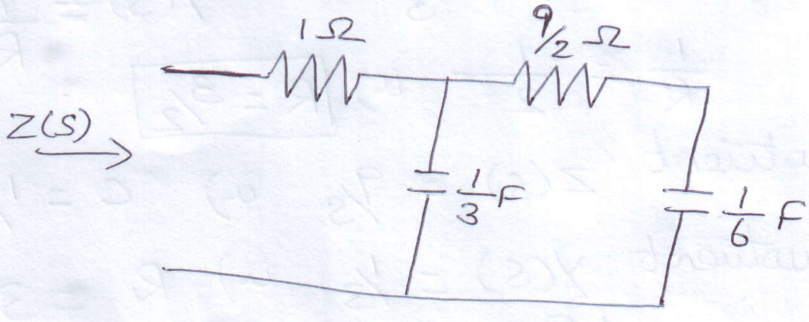
* Starting quotient gives $Z_1(S) = 1$ i.e. resistance 1Ω, series arm.

$Y_1(S) = \frac{S}{3}$ i.e. $Y_1(S) = CS$

$\Rightarrow C = 1/3\,F$

$Z_2(S) = \frac{9}{2} = R_1$

$Y_2(S) = \frac{S}{6} = CS$ i.e. $C = 1/6$.



Cauer II Form :-

Arrange N(s) & D(s) in ascending power of s.

$$Z(s) = \frac{4+5s+s^2}{2s+s^2}$$

$\frac{28}{38}$

$2s+s^2)\ 4+5s+s^2\ (\frac{2}{s} \leftarrow Z(s)$

$\quad 4+2s$

$\quad\quad 3s+s^2)\ 2s+s^2\ (\frac{2}{3} \leftarrow Y(s)$

$\quad\quad\quad 2s+\frac{2}{3}s^2$

$\quad\quad\quad\quad \frac{1}{3}s^2)\ 3s+s^2\ (\frac{9}{s} \leftarrow Z(s)$

$\quad\quad\quad\quad\quad 3s$

$\quad\quad\quad\quad\quad\quad s^2)\ \frac{1}{3}s^2\ (\frac{1}{3} \leftarrow Y(s)$

$\quad\quad\quad\quad\quad\quad\quad \frac{1}{3}s^2$

$\quad\quad\quad\quad\quad\quad\quad 0$

1st quotient $Z(s) = \frac{2}{s}$ (ii) $\frac{1}{Cs}$

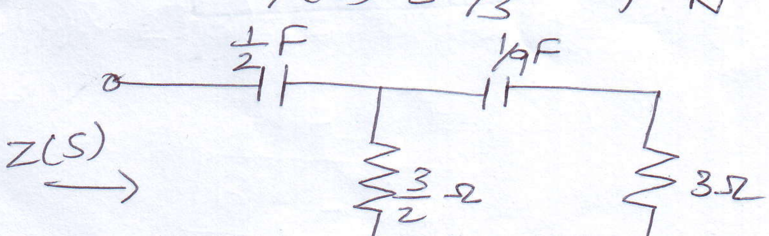
$$\boxed{C = 1/2}$$

2nd quotient $Y(s) = \frac{2}{3}$ (ii) $Y(s) = \frac{1}{R}$

(ii) $\boxed{R = 3/2}$

3rd quotient $Z(s) = 9/s$ (ii) $C = 1/9$

4th Quotient $Y(s) = 1/3$ (ii) $R = 3$



Synthesis Of Driving Point Impedance Functions Of RL networks :-

→ Only R & L components

→ No capacitors

Properties Of RL Driving Point Impedance Functions

1. Poles & zeros are simple. There are no multiple poles & zeros.
2. Poles & zeros are located on negative real axis.
3. Poles & zeros interlace each other on the negative real axis.
4. Poles & zeros are the critical frequencies.
5. The critical frequency at a greatest distance away from origin is always a pole & nearest to origin is always zero.
6. To obtain positive residues, $\frac{Z_{RL}(s)}{s}$
7. There can not be a pole at origin.
8. Slope of graph is always positive.
9. $Z_{RL}(0) < Z_{RL}(\infty)$

1. Realize the given RL n/w impedance function using all the four forms.

$$Z(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}$$

Soln:-

Foster I form :-

NOTE:- degree of N(s) & D(s) is same, If we divide N(s) / D(s) it gives negative terms which are not allowed in the synthesis.

i)

$$\frac{Z(s)}{s} = \frac{(s+1)(s+3)}{s(s+2)(s+4)}$$

$$\frac{Z(s)}{s} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$(s+1)(s+3) = A(s+2)(s+4) + B(s)(s+4) + C(s)(s+2)$$

put $s = 0$

$$(1)(3) = (2)(4)A$$

$$3 = 8A$$

$$\boxed{A = 3/8}$$

put $s = -2$

$$(-2+1)(-2+3) = B(-2)(-2+4)$$

$$(-1) = B(-2)(2)$$

$$\boxed{B = 1/4}$$

put $s=-4$

$$(-4+1)(-4+3) = C(-4)(-4+2)$$
$$(-3)(-1) = C(-4)(-2)$$
$$3 = C(8)$$
$$\boxed{C = 3/8}$$

$$\frac{Z(S)}{S} = \frac{3/8}{S} + \frac{1/4}{S+2} + \frac{3/8}{S+4}$$

$$Z(S) = \frac{\frac{3}{8}S}{S} + \frac{\frac{1}{4}S}{S+2} + \frac{\frac{3}{8}S}{S+4}$$

$$= \frac{3}{8} + \frac{1}{\frac{4(S+2)}{S}} + \frac{1}{\frac{8}{3S}(S+4)}$$

$$Z(S) = Z_1(S) + Z_2(S) + Z_3(S).$$

$$Z_1(S) = 3/8$$

$$Z_2(S) = \frac{1}{4+8/S} = \frac{1}{Y_2(S)} \quad \text{ie)} \; Y_2(S) = 4 + 8/S$$

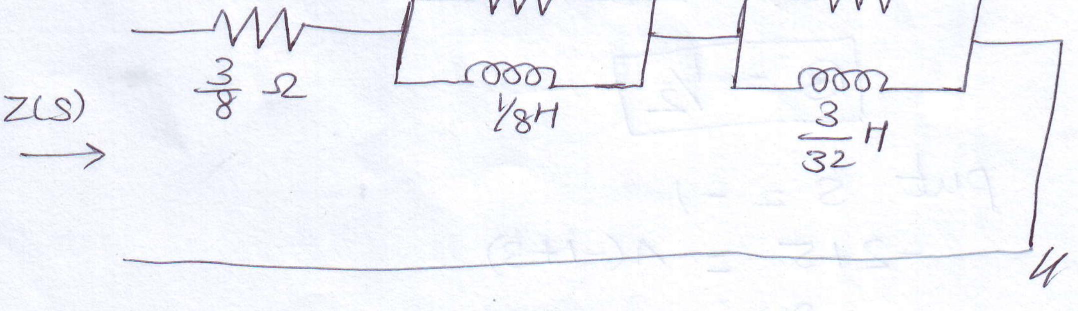
$$Y(S) = \frac{1}{R} \quad \text{ie)} \; R = 1/4$$

$$Y(S) = \frac{1}{LS} = \frac{1}{(\frac{1}{8})S} \quad \text{ie)} \; L = 1/8$$

$$Z_3(S) = \frac{1}{\frac{8}{3S}(S+4)} = \frac{1}{\frac{8}{3} + \frac{32}{3S}}$$

ie) $R = 3/8$

$L = 3/32$



Foster II Form :-

$$Y_{RL}(S) = \frac{1}{Z_{RL}(S)}$$

<u>NOTE</u>

Nr degree = Dr degree, so partial fraction must be obtained by dividing numerator by denominator.

$$Y(S) = \frac{1}{Z(S)} = \frac{1}{\frac{(S+1)(S+3)}{(S+2)(S+4)}}$$

$$Y(S) = \frac{(S+2)(S+4)}{(S+1)(S+3)} = \frac{S^2+6S+8}{S^2+4S+3}$$

$$\begin{array}{r} S^2+4S+3 \,\big)\, S^2+6S+8 \,(\, 1 \\ S^2+4S+3 \\ - \quad - \\ \hline 2S+5 \end{array}$$

$$Y(s) = 1 + \frac{2S+5}{S^2+4S+3} = 1 + \frac{2S+5}{(S+1)(S+3)} = 1 + \frac{A}{S+1} + \frac{B}{S+3}$$

$$2S+5 = A(S+3) + B(S+1)$$

put $S = -3$

$$-6+5 = B(-3+1)$$

$$-1 = -2B$$

$$\boxed{B = 1/2}$$

put $S = -1$

$$-2+5 = A(-1+3)$$

$$Y(s) = 1 + \frac{3/2}{s+1} + \frac{1/2}{s+3}$$

$$Y(s) = Y_1(s) + Y_2(s) + Y_3(s)$$

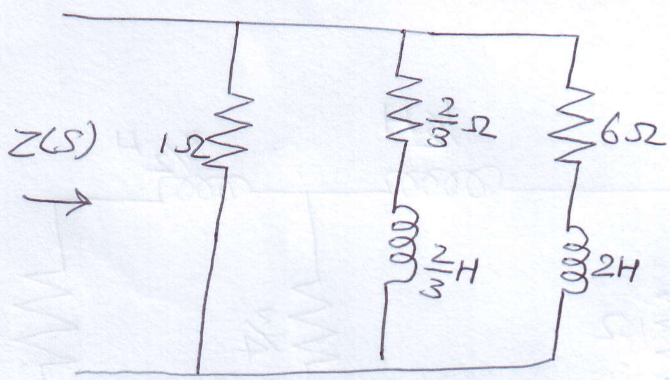
$$Y_1(s) = 1$$

$$Y_2(s) = \frac{1}{\frac{2}{3}(s+1)} = \frac{1}{\frac{2s}{3} + \frac{2}{3}}$$

(i.e) $Z_2(s) = \frac{2s}{3} + 2/3$

$$Y_3(s) = \frac{1}{2(s+3)} = \frac{1}{2s+6}$$

$$Z_3(s) = 2s+6$$



Cauer I Form :-

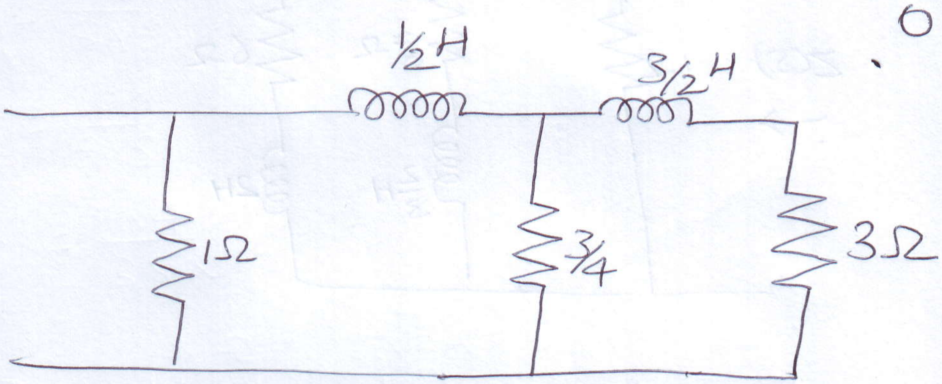
$$Z(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)} = \frac{s^2+4s+3}{s^2+6s+8}$$

$$
\begin{array}{r|l}
s^2+6s+8 \;) & s^2+4s+3 \;(1 \\
 & s^2+6s+8 \\ \hline
 & -2s-5
\end{array}
$$

* Negative quotient ie) there is no zero at origin. So it is necessary to first invert &

The first quotient is y(s) is shunt arm.

$$s^2+4s+3\,\big)\,s^2+6s+8\,\big(\,1 \leftarrow Y$$
$$\underline{s^2+4s+3}$$
$$2s+5\,\big)\,s^2+4s+3\,\big(\,\tfrac{s}{2} \leftarrow Z$$
$$\underline{s^2+\tfrac{5}{2}s}$$
$$\tfrac{3}{2}s+3\,\big)\,2s+5\,\big(\,\tfrac{4}{3} \leftarrow Y$$
$$\underline{2s+4}$$
$$1\,\big)\,\tfrac{3}{2}s+3\,\big(\,\tfrac{3}{2}s \leftarrow Z$$
$$\underline{\tfrac{3}{2}s}$$
$$3\,\big)\,1\,\big(\,\tfrac{1}{3} \leftarrow Y$$
$$\underline{1}$$
$$0$$



Cauer II Form :-

Nr & Dr in ascending power.

$$Z(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)} = \frac{3+4s+s^2}{8+6s+s^2}$$

$$8+6s+s^2 \overline{)\,3+4s+s^2\,}\left(\frac{3}{8} \longrightarrow Z\right.$$

$$\underline{3+\frac{18}{8}s+\frac{3}{8}s^2}$$

$$\frac{14}{8}s+\frac{5}{8}s^2 \overline{)\,8+6s+s^2\,}\left(\frac{64}{14s} \longleftarrow Y\right.$$

$$\underline{8+\frac{320}{112}s}$$

$$\frac{352}{112}s+s^2 \overline{)\,\frac{14}{8}s+\frac{5}{8}s^2\,}\left(\frac{196}{352} \longrightarrow Z\right.$$

$$\underline{\frac{14}{8}s+\frac{196}{352}s^2}$$

$$\frac{24s^2}{352} \overline{)\,\frac{352}{112}s+s^2\,}\left(\frac{968}{21s} \rightarrow Y\right.$$

$$\underline{\frac{352s}{112}}$$

$$s^2 \overline{)\,\frac{24}{352}s^2\,}\left(\frac{24}{352}\right.$$

$$\underline{\frac{24}{352}s^2}$$

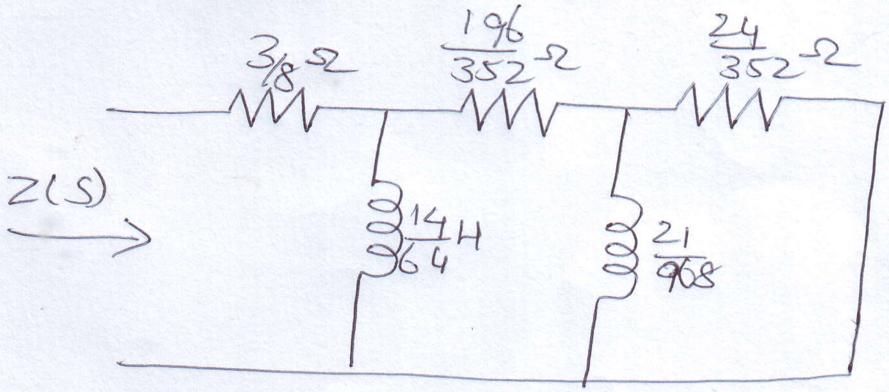
$$0$$

$$Y_1 = \frac{64}{14s}$$

$$Z_1 = \frac{196}{352}$$

$$Y_2 = \frac{968}{21s}$$

$$Z_2 = \frac{24}{352}$$



UNIT - V

DESIGN OF FILTERS

Filters :-

A n/w which freely passes desired band of frequencies while almost supresses other band of frequencies is called filter.

(i) Active Filters

↳ Filters containing active elements such as transistors, op-amps along with resistors, capacitors & inductors are called active filters.

(ii) Passive Filters

↳ Passive elements such as resistors, capacitors & inductors are used.

Pass Band :-

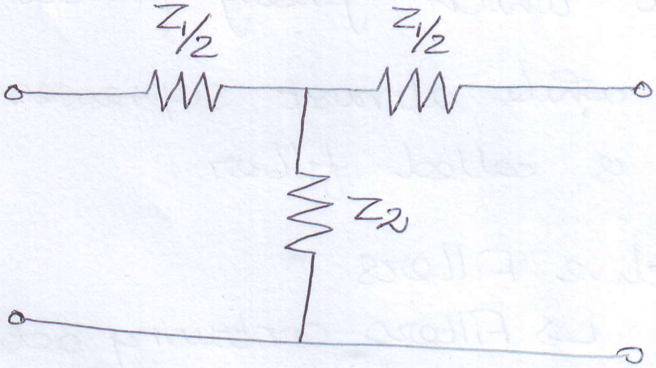
The range of frequencies over which attenuation by filter is zero is called passband.

Stop Band (or) Attenuation Band :-

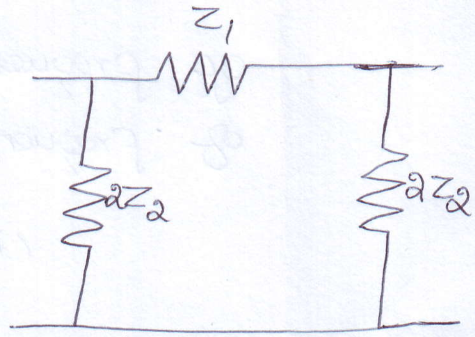
The range of frequencies over which attenuation is infinite is called stop band.

Cutoff frequencies $\rightarrow$ The frequencies which seperate the passband from attenuation or stopband are called cutoff frequencies of the filter represented as f_c.

Symmetrical UnBalanced T-sections :-



(T-Section)



(π-section)

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$\downarrow$

Charac. impedance of T-n/w

$$Z_{0\pi} = \frac{Z_1 Z_2}{\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}}$$

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$$

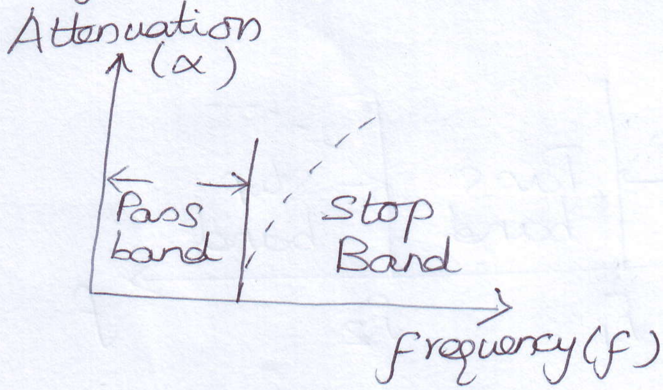
Propagation Constant

$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_{0T}}{Z_2}$$

$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_1}{Z_{0\pi}}$$

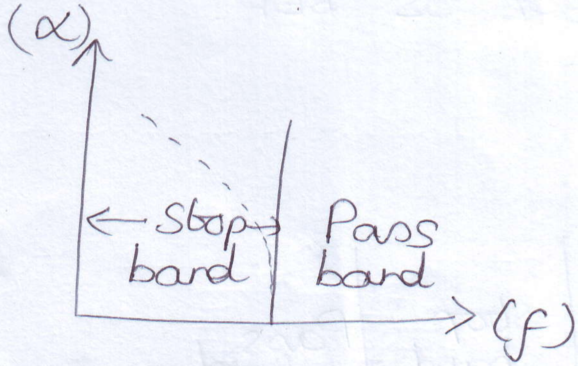
Low Pass Filter :-

If filter passes all frequencies upto cutoff frequency and attenuats all frequency above it, then it is called LPF.



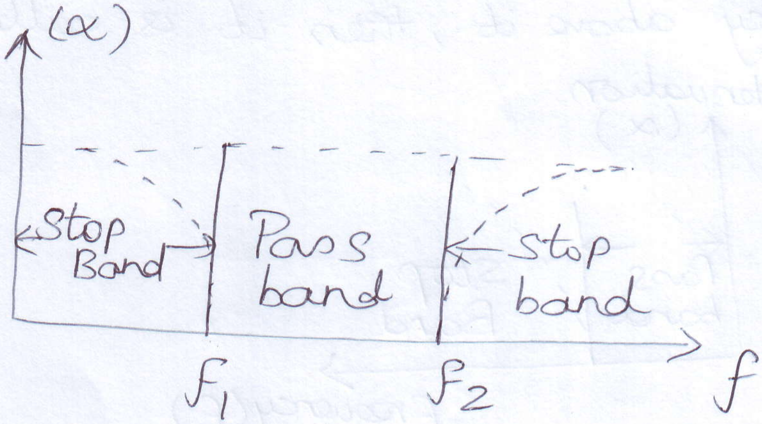
High Pass Filter :-

If filter attenuats all frequencies upto cutoff frequency & passes all frequencies above it, then it is called HPF



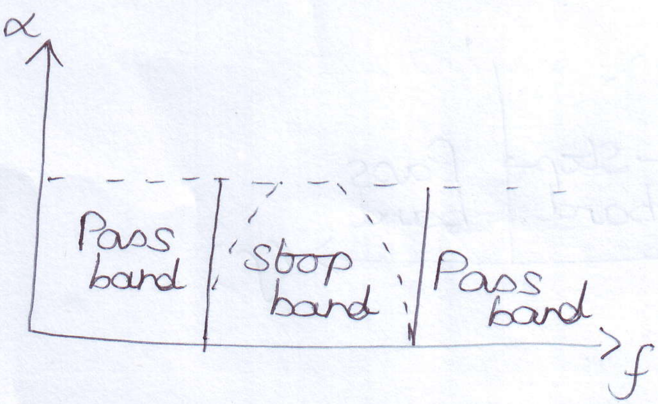
Band Pass Filter :-

If the filter passes all the frequencies b/w the two cut off frequencies and attenuatis all other frequencies, then it is called as BPF.



Band Elimination Filter :-

If the filter attenuatis all the frequencies between the two cutoff frequencies & passes all other frequencies, then it is called as band stop filter or BEF.



Constant-k Sections :-

A T (or) π-section in which series and shunt arm impedances Z_1 & Z_2 satisfy the relationship $\rightarrow Z_1 Z_2 = R_0^2$ where R_0 is a real constant is called constant-k section.

$R_0 \rightarrow$ design impedance

For the same series & shunt arm impedances the charac. impedances of T and π sections can be related,

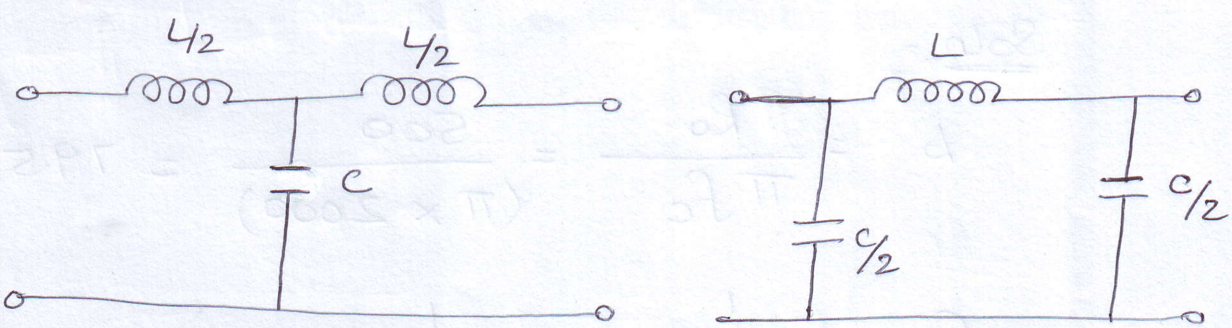
$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$$

Constant-k Section :-

$$Z_{0\pi} = \frac{R_0^2}{Z_{0T}} \qquad Z_1 Z_2 = R_0^2$$

Prototype Filter Sections :-

(i) Low Pass Filters :-



'T' type π-type

Design Equations Of Prototype LPF

$$R_o = \sqrt{\frac{L}{C}}$$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

Dividing equation for R_o by f_c,

$$L = \frac{R_o}{\pi f_c}$$

Multiplying equation for R_o & f_c,

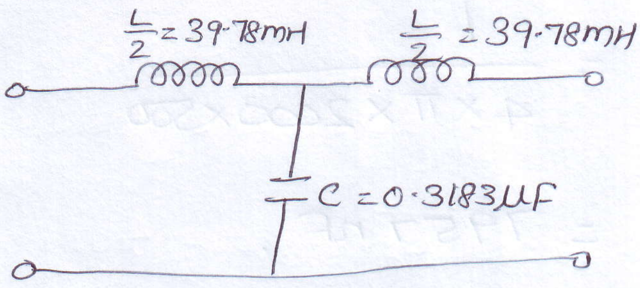
$$C = \frac{1}{(\pi f_c) R_o}$$

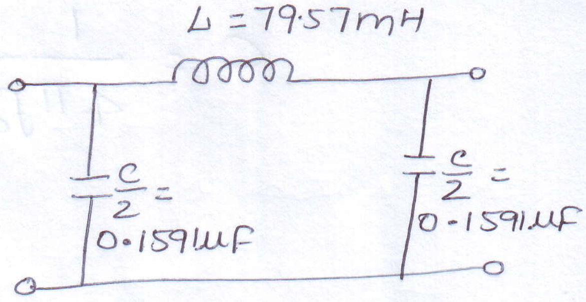
1. Design a prototype lowpass filter sections of design impedance $R_o = 500\Omega$ and cutoff frequency $f_c = 2000HZ$.

<u>Soln</u>:-

$$L = \frac{R_o}{\pi f_c} = \frac{500}{(\pi \times 2000)} = 79.57\,mH$$

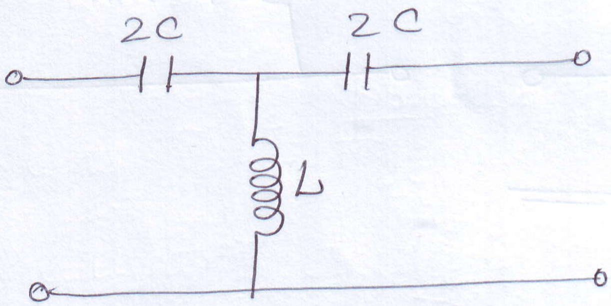
$$C = \frac{1}{(\pi f_c) R_o} = \frac{1}{\pi \times 2000 \times 500} = 0.3183\,\mu F$$

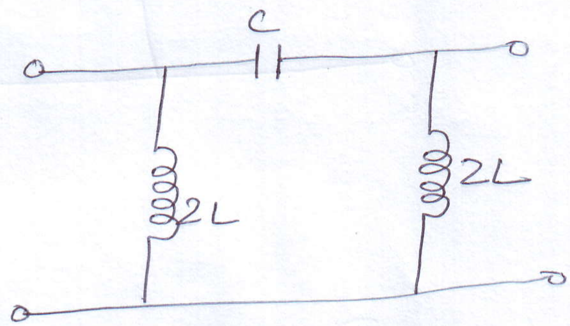




T _ Sec

High Pass Filter :-





T_ Section   Π_ Section

Design equations

$$L = \frac{R_0}{4\pi f_c} \quad ; \quad C = \frac{1}{(4\pi f_c) R_0}$$

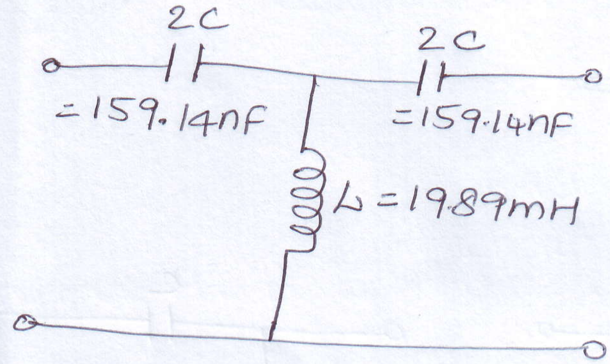
1. Design a prototype highpass filter sections of design impedance $R_0 = 500\Omega$ & cutoff frequency $f_c = 2000$ HZ.

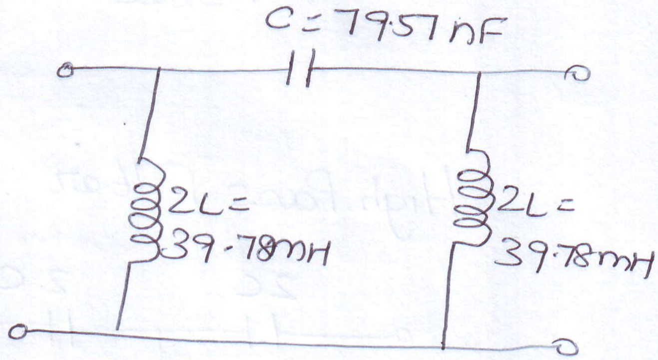
Soln

$$L = \frac{R_0}{4\pi f_c} = \frac{500}{4 \times \pi \times 2000} = 19.89 \text{ mH}$$

$$C = \frac{1}{4\pi f_c R_0} = \frac{1}{4 \times \pi \times 2000 \times 500}$$

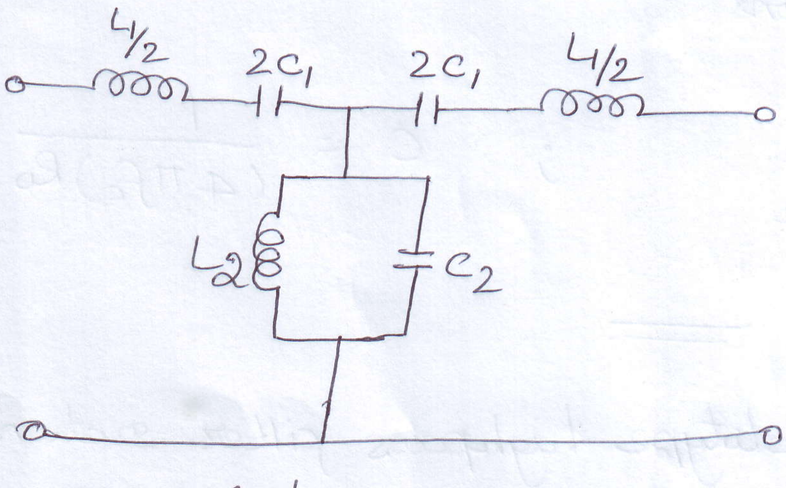
$$= 79.57 \text{ nF}$$



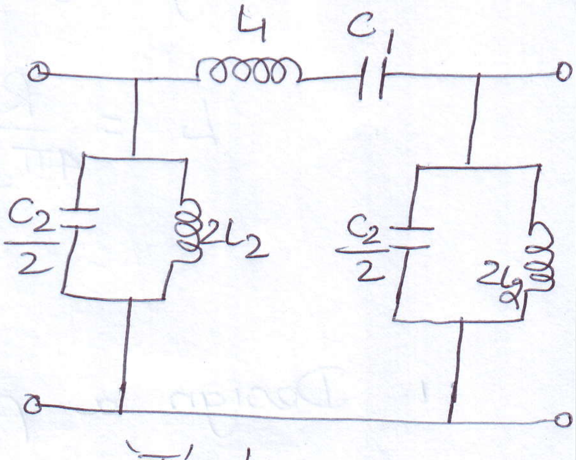


≡

Band Pass Filters :-



'T' type



'π' type

Design Equations :-

$$R_0 = \sqrt{\frac{L_2}{C_1}} = \sqrt{\frac{L_1}{C_2}}$$

$$C_1 = \frac{(f_2 - f_1)}{4\pi R_0 (f_1 f_2)}$$

$$L_1 = \frac{R_0}{\pi (f_2 - f_1)}$$

$$C_2 = \frac{1}{\pi R_0 (f_2 - f_1)}$$

$$L_2 = \frac{R_0 (f_2 - f_1)}{4\pi f_1 f_2}$$

① Design a prototype band pass filter if design impedance $R_0 = 4k\Omega$ and pass band between 1.25kHZ & 2 kHZ.

Soln :-

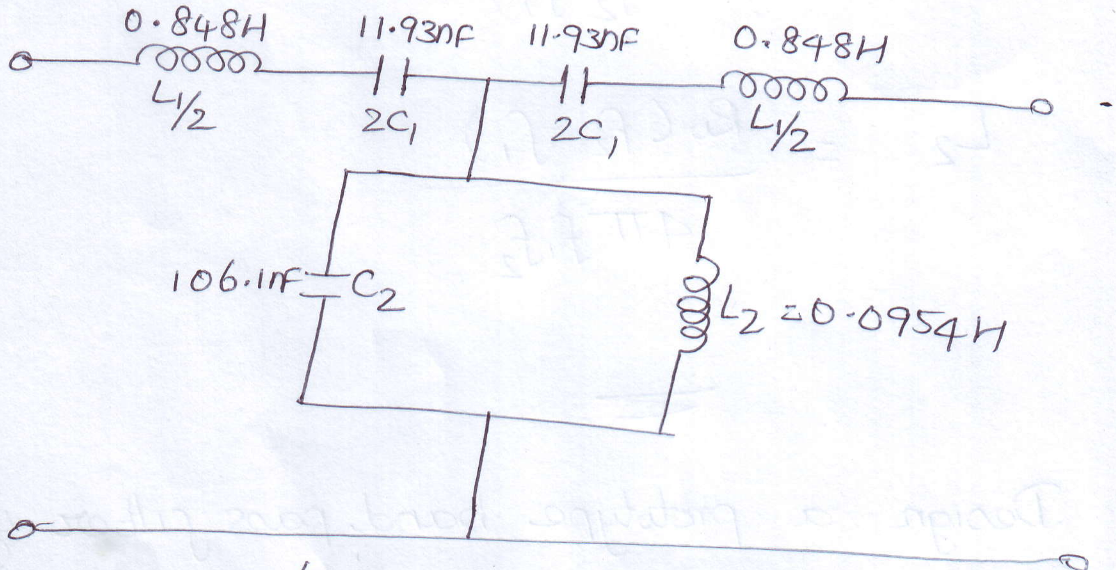
$R_0 = 4k\Omega$; $f_1 = 1.25\,kHZ$; $f_2 = 2\,kHZ$

$$L_1 = \frac{R_0}{\pi(f_2 - f_1)} = \frac{4000}{\pi(2000-1250)} = 1.697\,H$$

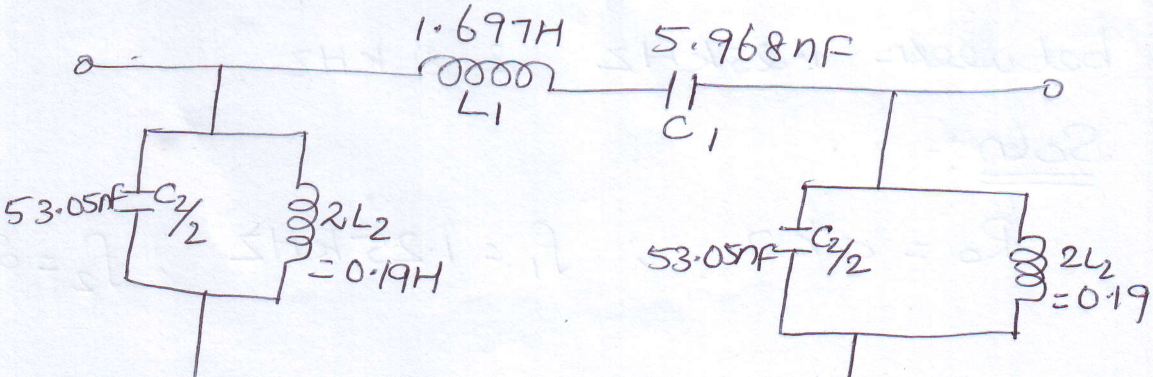
$$C_1 = \frac{(f_2 - f_1)}{4\pi R_0 (f_1 f_2)} = \frac{(2000-1250)}{4\pi \times 4000 \times 1250 \times 2000} = 5.968\,nF$$

$$L_2 = \frac{(f_2 - f_1) R_0}{4\pi f_1 f_2} = \frac{(2000-1250)(4000)}{4\pi \times 1250 \times 2000} = 0.0954\,H$$

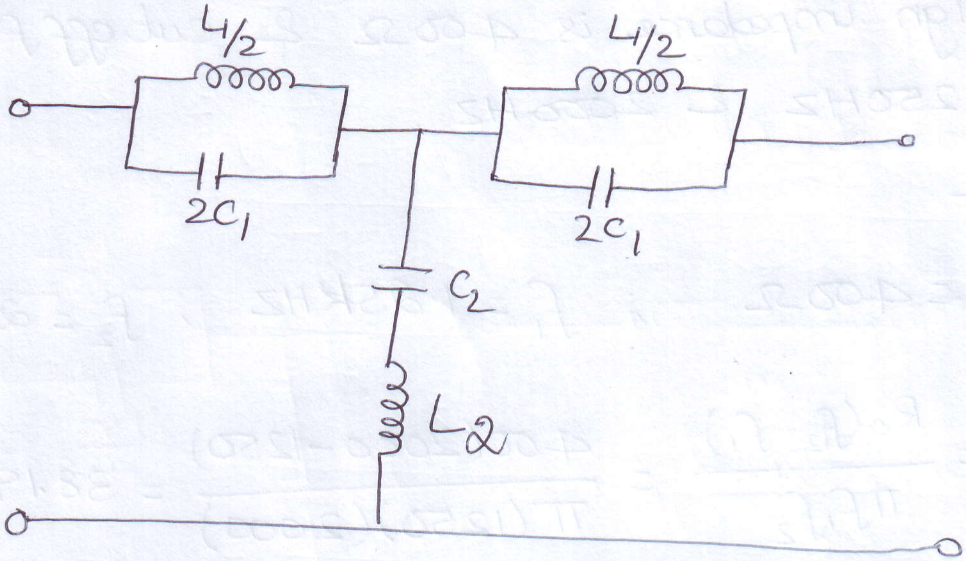
$$C_2 = \frac{1}{\pi R_0 (f_2 - f_1)} = \frac{1}{4000 \times \pi \times (2000-1250)} = 106.1\,nF$$



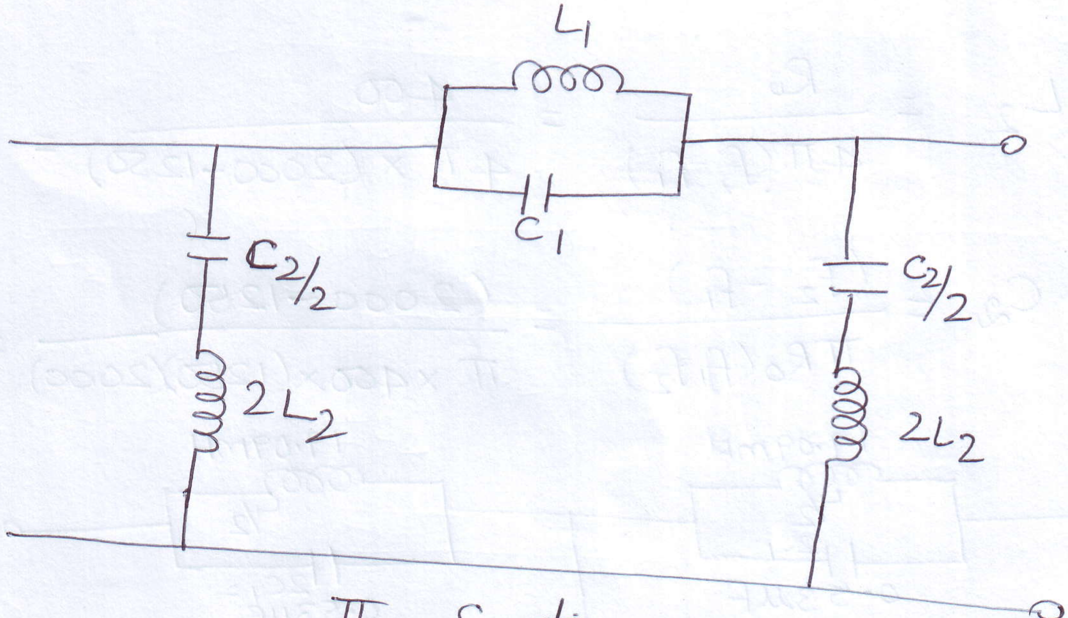
(T-Section)



Band Elimination Filters :-



'T' type



π – Section

$$L_1 = \frac{R_0(f_2 - f_1)}{\pi f_1 f_2} \quad ; \quad C_1 = \frac{1}{4\pi R_0(f_2 - f_1)}$$

$$L_2 = \frac{R_0}{4\pi(f_2 - f_1)} \quad ; \quad C_2 = \frac{(f_2 - f_1)}{\pi R_0 f_1 f_2}$$

1. Design a prototype band elimination filter sections if design impedance is 400 Ω & cut off frequencies are 1250 HZ & 2000 HZ.

Soln :-

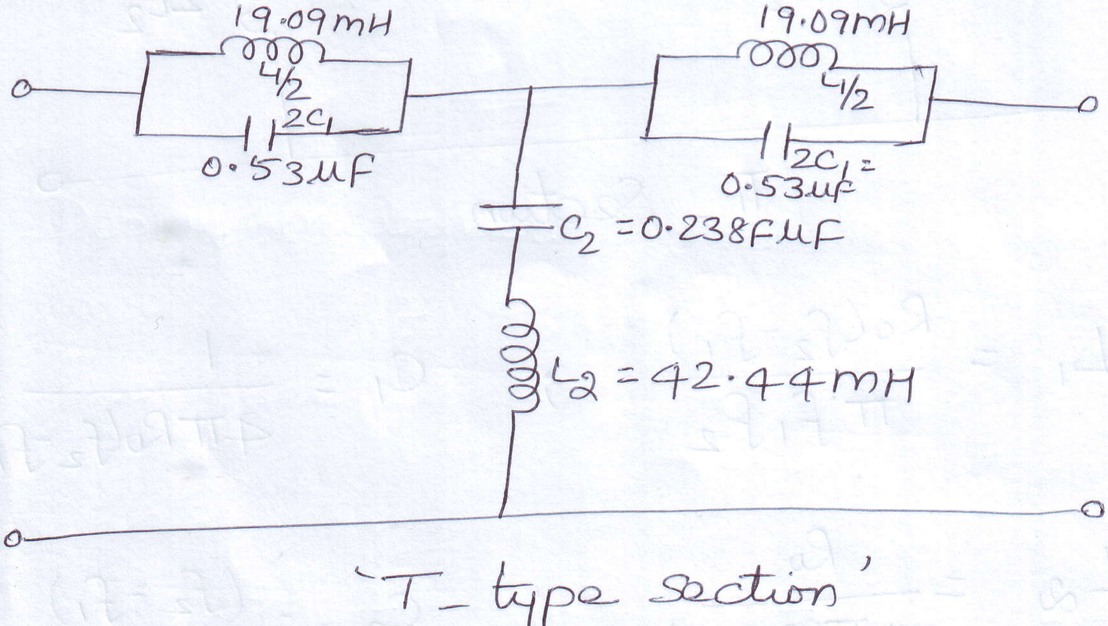
$R_0 = 400\Omega$; $f_1 = 1.25\,kHZ$; $f_2 = 2\,kHZ$

$$L_1 = \frac{R_0(f_2-f_1)}{\pi f_1 f_2} = \frac{400(2000-1250)}{\pi(1250)(2000)} = 38.197\,mH$$

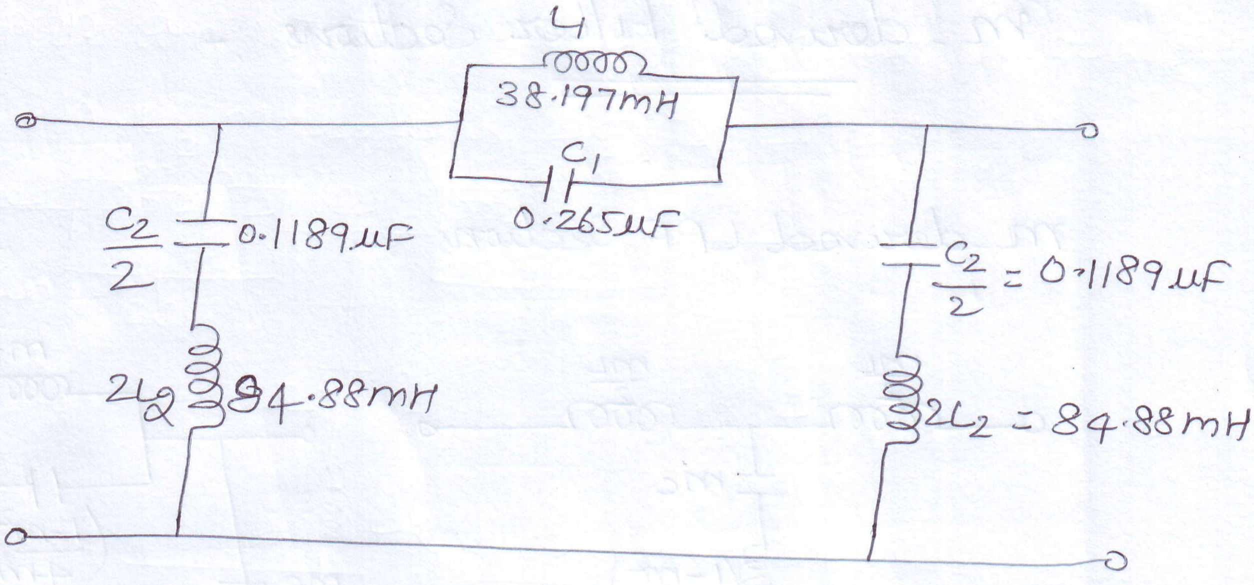
$$C_1 = \frac{1}{4\pi R_0(f_2-f_1)} = \frac{1}{4\pi \times 400 \times (2000-1250)} = 0.265\,\mu F$$

$$L_2 = \frac{R_0}{4\pi(f_2-f_1)} = \frac{400}{4\pi \times (2000-1250)} = 42.44\,mH$$

$$C_2 = \frac{(f_2-f_1)}{\pi R_0(f_1 f_2)} = \frac{(2000-1250)}{\pi \times 400 \times (1250)(2000)} = 0.2387\,\mu F$$



'T - type section'



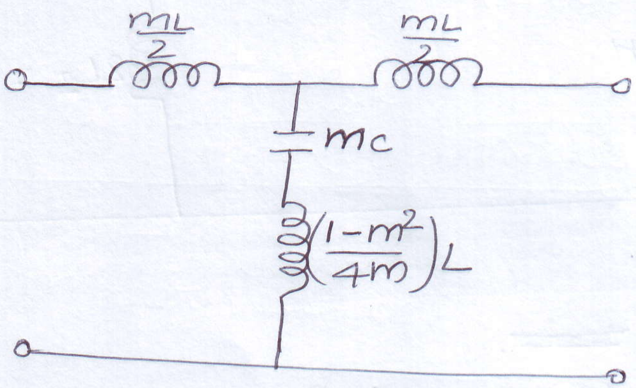
Disadvantages Of Prototype Filter Section :-

(i) Ideally the attenuation should change sharply in the attenuation band. But in all prototype filter section, the attenuation changes gradually in stop band. Hence frequencies near cut-off frequency are passed through the filter.

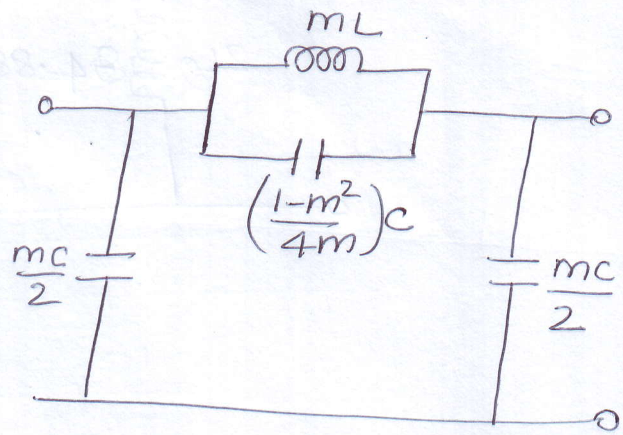
(ii) In the pass band, output of the filter. This indicates that the characteristic impedance should remain constant. But it is observed that, characteristic impedance varies with frequency from value R_0 i.e) design impedance value throughout the pass band. Hence, filter cannot be terminated properly.

m-derived Filter Sections :-

m-derived LPF Sections :-



T-Section



Π-Section

$$f_\infty = \frac{f_c}{\sqrt{1-m^2}}$$

$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2}$$

1. Design m-derived T-type LPF to work into load of 500 Ω & cutoff frequency at 4 kHZ and peak attinuation at 4.5 kHZ.

Soln:-

$R_0 = 500\Omega$; $f_c = 4\,kHZ$; $f_\infty = 4.5\,kHZ$

Before designing m-derived section, we have to design prototype section first,

$$L = \frac{R_0}{\pi f_c} = \frac{500}{\pi \times 4000} = 39.78\, mH$$

$$C = \frac{1}{\pi f_c R_0} = \frac{1}{\pi \times 4000 \times 500} = 0.1591\, \mu F$$

m-derived LPF,

$$m = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2}$$

$$m = \sqrt{1 - \left(\frac{4000}{4500}\right)^2}$$

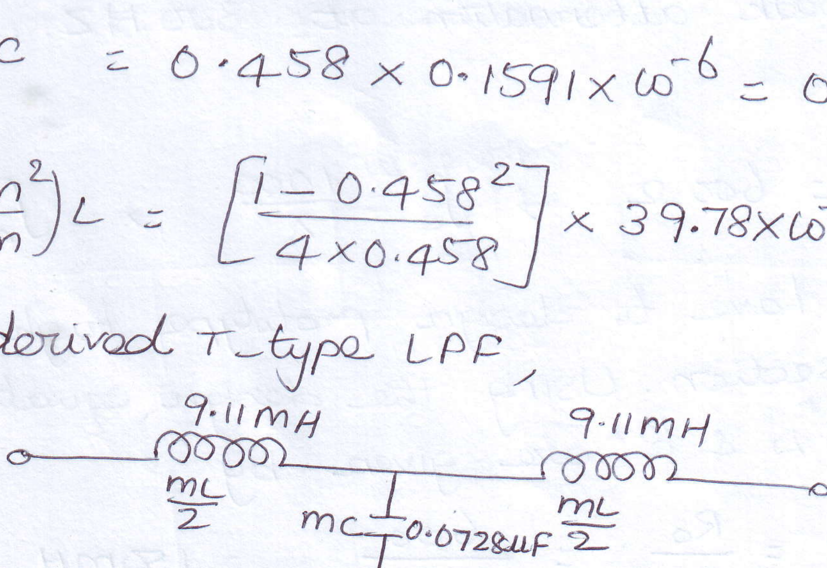
$$m = 0.458 \,//$$

$$\frac{mL}{2} = \frac{0.458 \times 39.78 \times 10^{-3}}{2} = 9.109\, mH$$

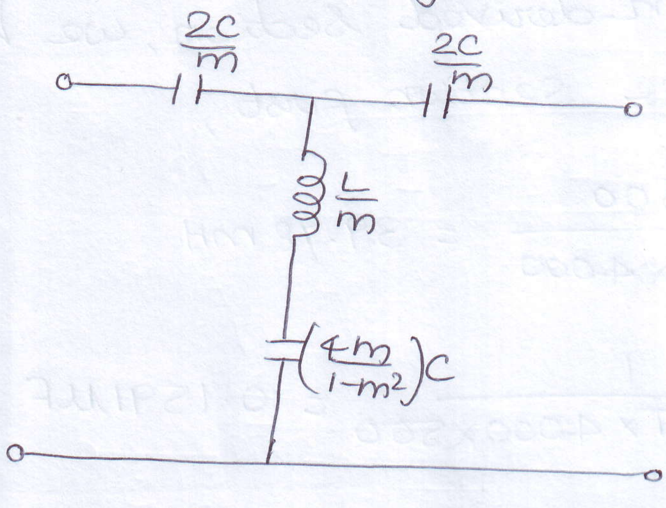
$$mC = 0.458 \times 0.1591 \times 10^{-6} = 0.0728\, \mu F$$

$$\left(\frac{1-m^2}{4m}\right)L = \left[\frac{1-0.458^2}{4 \times 0.458}\right] \times 39.78 \times 10^{-3} = 17.16\, mH$$

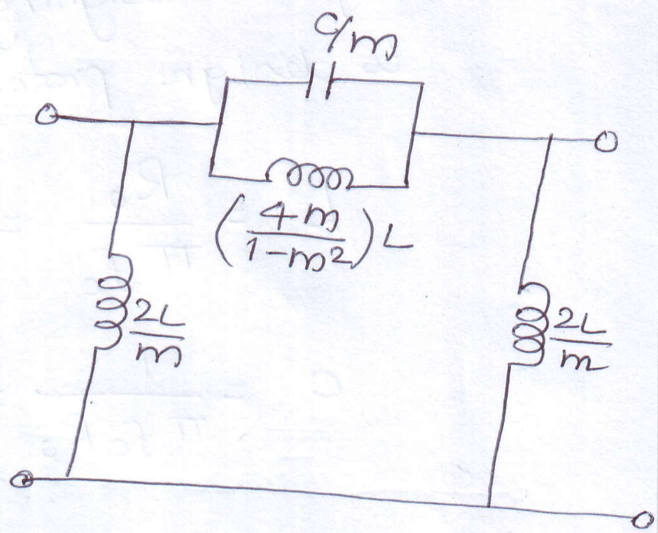
m-derived T-type LPF,



m-derived High Pass Filter :-



T-Section



$$f_\infty = f_c\sqrt{(1-m^2)}$$

$$m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2}$$

---

1. Design m-derived HPF π section to work into load of 600Ω with cutoff frequency of $\left(\frac{1000}{\pi}\right)$Hz and peak attenuation at 300 HZ.

Soln :-

$R_o = 600\Omega$; $f_c = \frac{1000}{\pi}$; $f_\infty = 300\,HZ$

* We have to design prototype highpass filter π-section. Using the design equation, the values of L & C are given by

$L = \frac{R_o}{} = \frac{600}{} = 150mH$

$$c = \frac{1}{4\pi f_c R_0} = \frac{1}{4\pi \times 318.3 \times 600} = 0.416\,\mu F$$

In m-derived HPF, value of m,

$$m = \sqrt{1 - \left(\frac{f_\infty}{f_c}\right)^2}$$

$$m = \sqrt{1 - \left(\frac{300}{318.3}\right)^2}$$

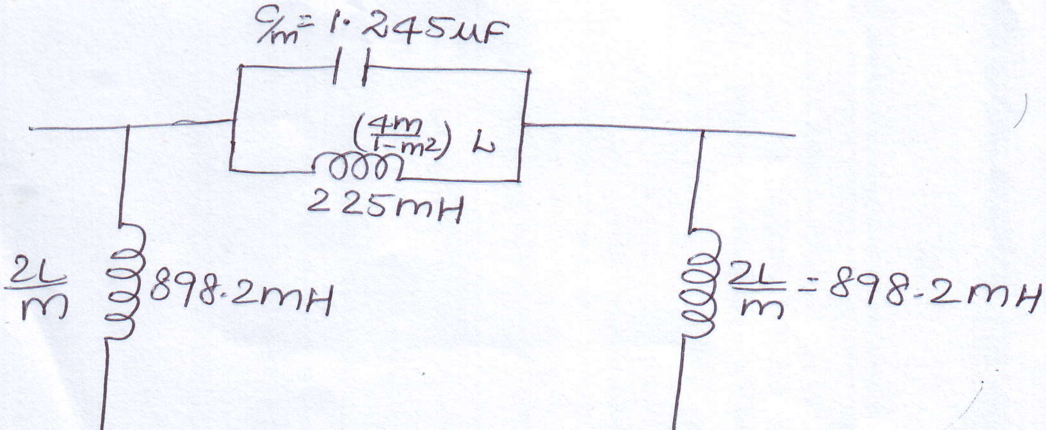
$$m = 0.334$$

For m derived HPF,

$$\frac{c}{m} = \frac{0.416 \times 10^{-6}}{0.334} = 1.245\,\mu F$$

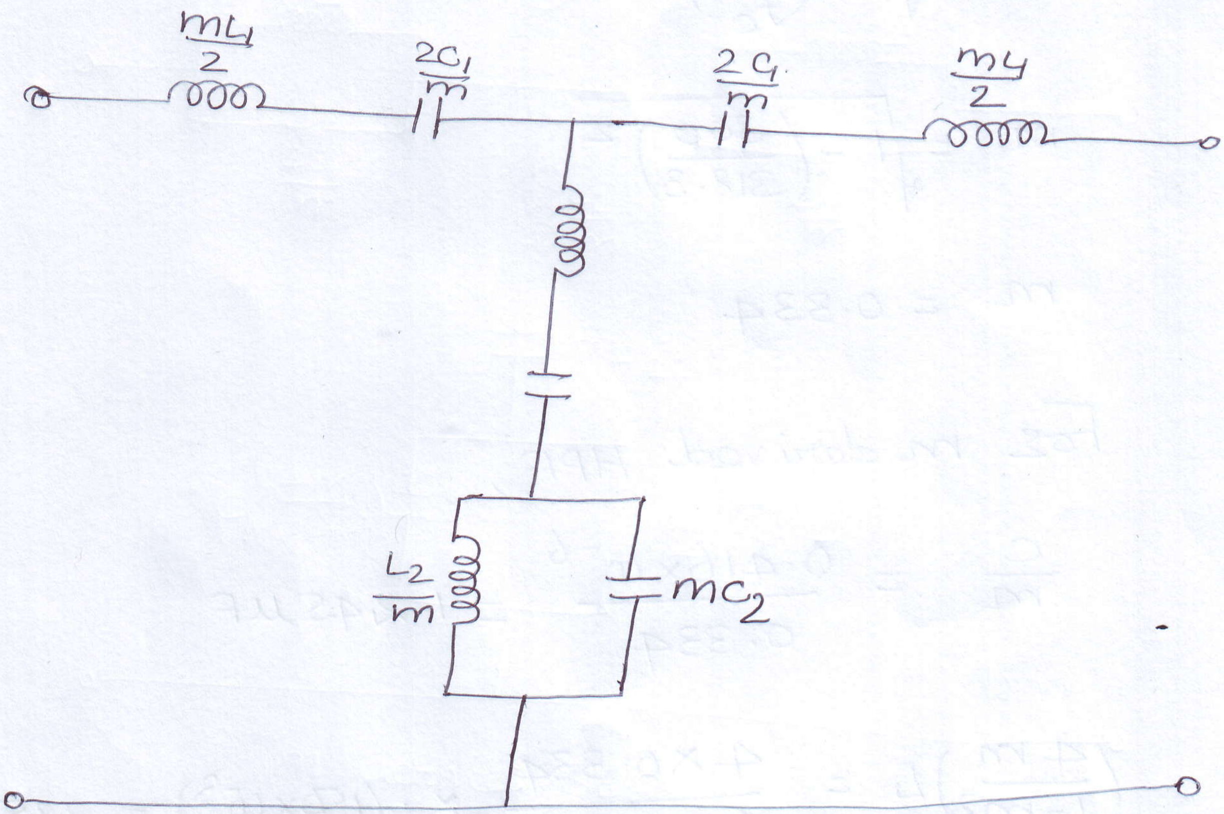
$$\left(\frac{4m}{1-m^2}\right)L = \frac{4 \times 0.334}{\left[1-(0.334)^2\right]} \times (150 \times 10^{-3}) = 225.6\,mH$$

$$\frac{2L}{m} = \frac{2 \times 150 \times 10^{-3}}{0.334} = 898.2\,mH$$



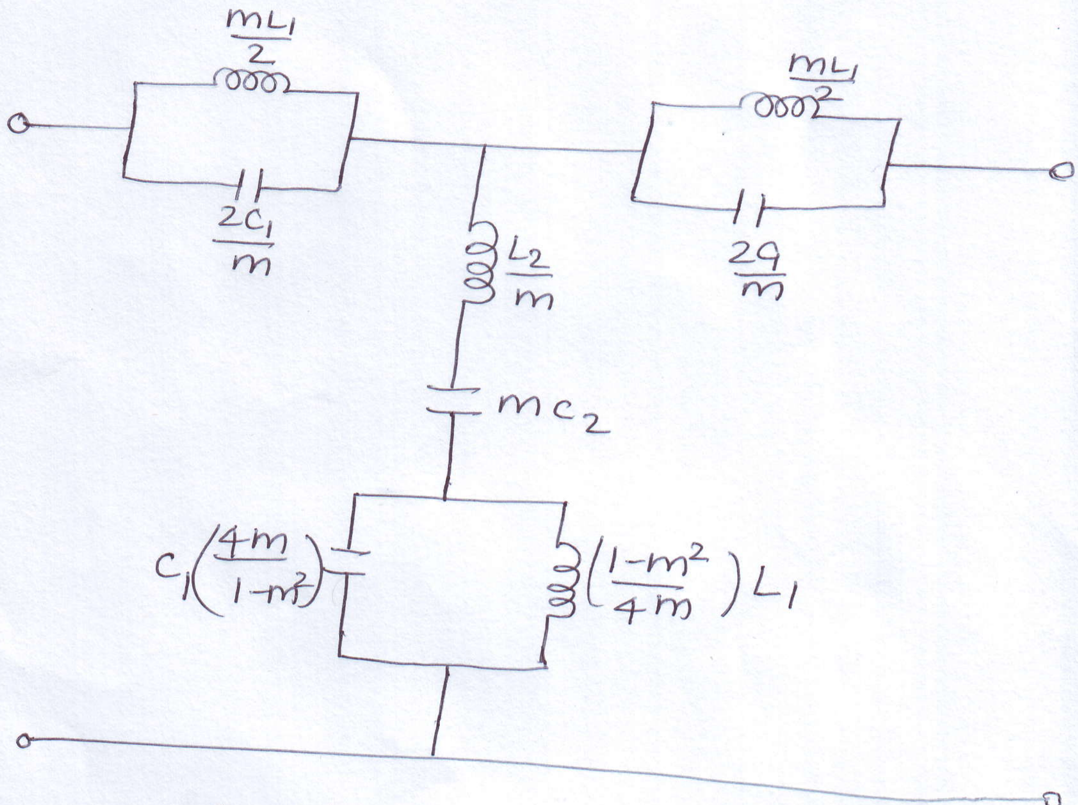
m-derived Band Pass Sections :-

$$m = \sqrt{1 - \left(\frac{f_2 - f_1}{f_{2\infty} - f_{1\infty}}\right)^2}$$



m-derived Band Elimination Filter :-

$$m = \sqrt{1 - \left(\frac{f_{2\infty} - f_{1\infty}}{f_2 - f_1}\right)^2}$$



m-derived BEF

Atthnuators :-

* is a two port resistive network and is used to reduce the signal level by a given amount.

* A fixed attinuator with constant attinuation is called a <u>pad</u>.

* Attenuation is expressed in decibels (dB) or in nepers.

$$\text{Attenuation in dB} = 10\log_{10}\left(\frac{P_1}{P_2}\right)$$

$P_1 \rightarrow$ input power

$P_2 \rightarrow$ Output power

$$\frac{P_1}{P_2} = \frac{I_1^2}{I_2^2} = \frac{V_1^2}{V_2^2}$$

$I_1 \rightarrow$ input current ; $I_2 \rightarrow$ output current

$V_1 \rightarrow$ Voltage at port 1.

$V_2 \rightarrow$ Voltage at port 2.

$$\text{attinuation in dB} = 20\log_{10}\left(\frac{V_1}{V_2}\right)$$

$$= 20\log_{10}\left(\frac{I_1}{I_2}\right)$$

If $\frac{V_1}{V_2} = \frac{I_1}{I_2} = N$; then $\frac{P_1}{P_2} = N^2$

$$dB = 20\log_{10} N$$

(or)

$N = \text{antilog}(dB)$

1. Design a π type attenuator to give 20dB attenuation and to have a characteristic impedance of 100Ω.

Soln:-

$R_o = 100\Omega$, $D = 20dB$

$$N = \text{antilog}\frac{D}{20} = 10$$

$$R_1 = R_o\frac{(N^2-1)}{2N} = \frac{100(10^2-1)}{2\times 10} = 495\Omega$$

$$R_2 = R_o\frac{(N+1)}{(N-1)} = \frac{100(10+1)}{(10-1)} = 122.22\,\Omega$$

2. Design a symmetrical lattice attenuator to have charac. impedance of 800Ω and attenuation of 20dB.

Soln:-

$R_o = 800\Omega$; $D = 20dB$

$$N = \text{antilog}\frac{D}{20} = \text{antilog}\left(\frac{20}{20}\right) = 10$$

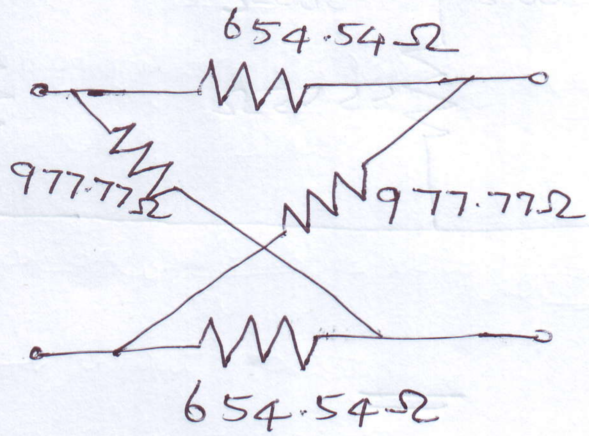
Design eqn. of lattice attenuator

Series arm resistance $R_1 = R_o\frac{(N-1)}{(N+1)} = \frac{800(10-1)}{(10+1)}$

Diagonal arm resistance $R_2 = R_0 \frac{(N+1)}{(N-1)}$

$$= 800\left(\frac{(10+1)}{(10-1)}\right)$$

$$= 977.77\,\Omega$$



3. Design a symmetrical bridged-T attenuator with an attenuation of 20dB and terminated into a load of 500 Ω.

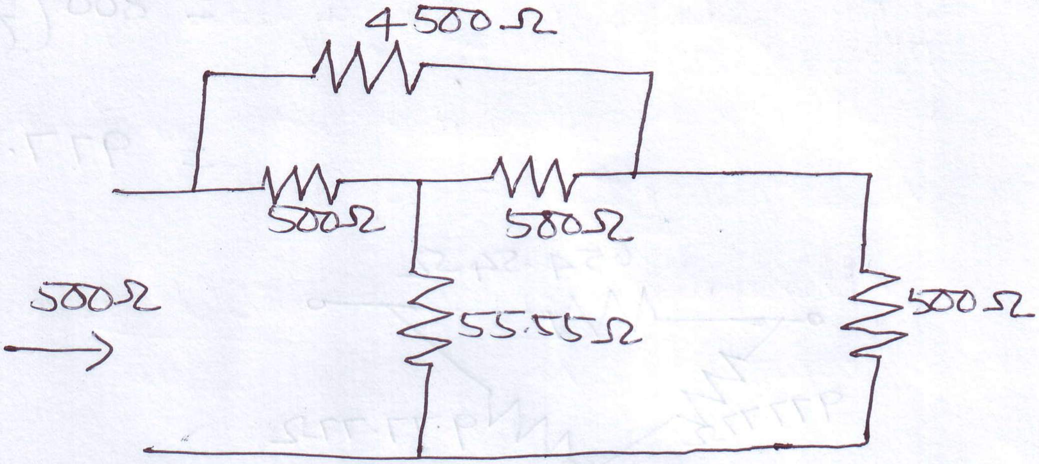
Soln :-

$D = 20\,dB$; $R_0 = 500\,\Omega$

$N = \text{antilog}\,\frac{D}{20} = \text{antilog}\,\frac{20}{20} = 10$

$R_A = R_0(N-1) = 500(10-1) = 4500\,\Omega$

$$R_B = \frac{R_0}{N-1} = \frac{500}{10-1} = 55.55\,\Omega$$



4. Design a L-type attenuator to operate into a load resistance of 600 Ω with an attenuation of 20 dB.

Soln :-

$$N = \text{antilog}\,\frac{dB}{20} = \text{antilog}\,\frac{20}{20} = 10$$

Series arm of attenuator

$$R_1 = R_0\left(\frac{N-1}{N}\right) = 600\left(\frac{10-1}{10}\right) = 540\,\Omega$$

Shunt arm of attenuator

$$R_2 = R_0/(N-1) = \frac{600}{9} = 66.66\,\Omega$$

L-type attenuator,