NOTES

BME202 ENGINEERING MECHANICS

ENGINEERING MECHANICS

UNIT - I - STATICS OF PARTICLES

1. State Newtons laws of Motion?

Ist Law:

A Body Remains in its state of rest (or) Motion unless it is acted by external force to change its state.

II Law:

The Rate of change of Momentum of a body is directly Proportional to the force applied on the body.

III Law:

For every action there is equal and opposite reaction.

2. Define Force? What are the characteristics of force?

Force:

Force is the action of one body on another body.

Charateristics:

It is characterized by its

(i). Point of application
(ii). Magnitude
(iii). Direction.

③. Define the Different System of forces?

when number of forces acting on the body then it is said to be System of forces.

<u>Types of system of forces</u>:

1. collinear forces – Force act along the same line
2. Coplanar forces – Forces acting in the same Plane
3. concurrent forces – Forces Intersect at Common Point.

④. State Lami's theorem.

It states that "If the forces acting at a Point are in Equilibrium, each force will be Proportional to the Sine angle b/w other two forces.

⑤. Define Parallelogram law of forces.

It States that "If two vectors acting at a Point be represented in magnitude and direction by the two adjacent sides of a Parallelogram, then their resultant is represented by the diagonal of the Parallelogram ... that Point.

⑥. If $\vec{A} = 4i + 3j - 5k$ and $\vec{B} = 3i - 2j + 4k$. find $\vec{A} \cdot \vec{B}$ and $\vec{A} \times \vec{B}$.

Ans: (i). $\vec{A} \cdot \vec{B} = (4i + 3j - 5k) \cdot (3i - 2j + 4k)$

$= (4 \times 3) + (3 \times -2) + (-5 \times 4)$

$\boxed{\vec{A} \cdot \vec{B} = -14}$

(ii). $\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 4 & 3 & -5 \\ 3 & -2 & 4 \end{vmatrix}$

$= i(12 - 10) - j(16 + 15) + k(-8 - 9)$

$\boxed{\vec{A} \times \vec{B} = 2i - 31j - 17k}$

⑦. State the Difference b/w Internal and External forces?

Internal force: → Force acts internally through the body.

External force: → Force acts Externally though the body.

⑧. Write the Equation of Equilibrium for 3-dimensions.

Ans:

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M_o = 0$

(9) Define Triangle Rule?

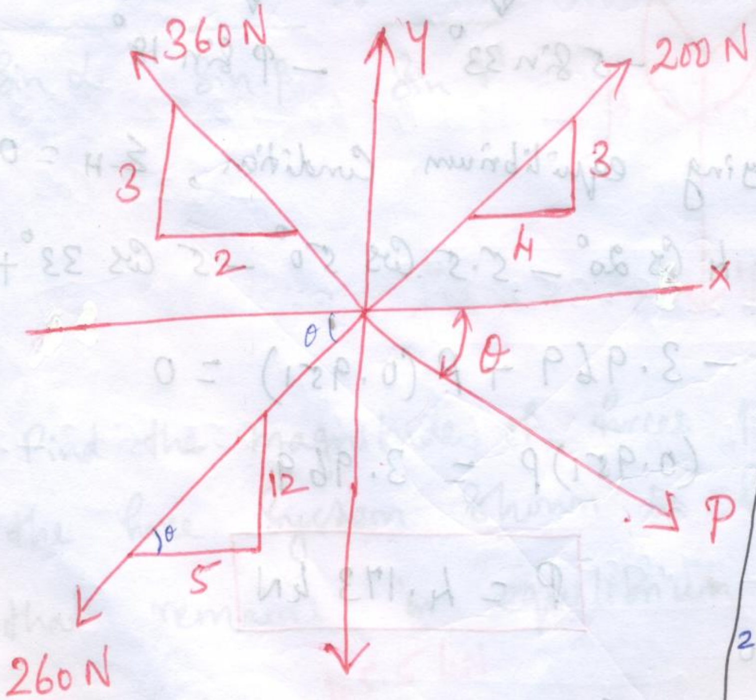
It states "If two vectors acting at a point are represented by the two sides of a triangle, then their resultant is equal to the third side (enclosed side).

(10) Draw the Free Body Diagram for the below diagram.

Ans

— x — x — x —

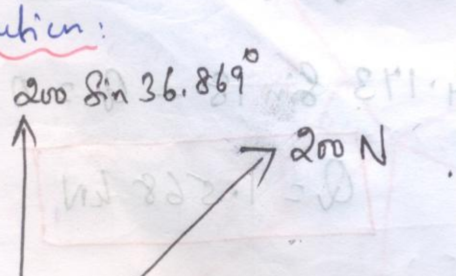
Prob. 11
Q.B. 2(i) The resultant of the force system shown in figure below is 520 N along the negative direction of Y-axis. Determine 'P' and 'θ'.



Note:
For Equilibrium
(3) Conditions
1. For Eq. $\Sigma H = 0$, $\Sigma V = 0 \Rightarrow R = 0$
2. For Hor. $\Sigma V = 0$, $\Sigma H = R$
3. For Ver. $\Sigma H = 0$, $\Sigma V = R$

Solution:

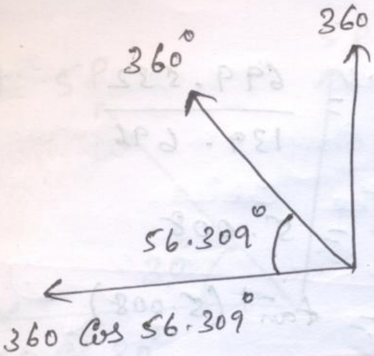
(1)



$\theta = \tan^{-1}(3/4)$

$\boxed{\theta_1 = 36.869^\circ}$

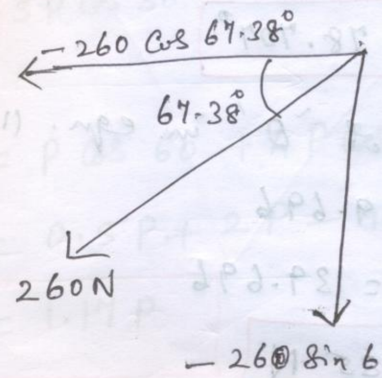
(2)



$\theta = \tan^{-1}(3/2)$

$\boxed{\theta_2 = 56.309^\circ}$

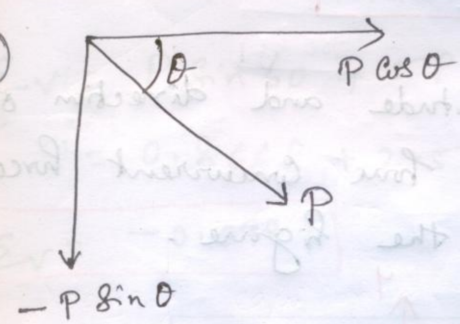
(3)



$\theta = \tan^{-1}(12/5)$

$\boxed{\theta_3 = 67.38^\circ}$

(4)



Resultant in horizontal axis is Zero

$\Sigma H = 0$

$\Sigma H = 200 \cos 36.869 - 360 \cos 56.309 - 260 \cos 67.38^\circ + P \cos\theta = 0$

$-139.696 + P\cos\theta = 0$

$\boxed{P\cos\theta = 139.696}$ —— (1)

Resultant force is vertical downward force,

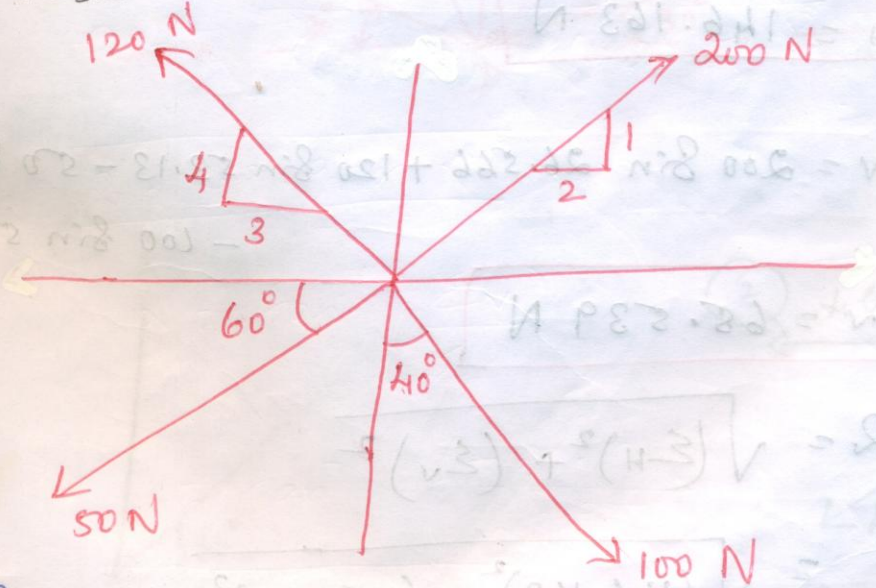
$\Sigma V = -520$

$\Sigma V = 200 \sin 36.869^\circ + 360 \sin 56.309^\circ - 260 \sin 67.38^\circ - P\sin\theta = 0$

$179.532 - P\sin\theta = -520$

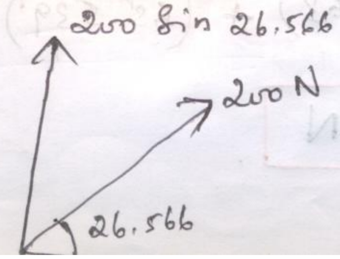
$\boxed{P\sin\theta = 699.532}$ —— (2)

Prob. 6.
QB. 2(ii). A system of 4 forces acting on a body is shown. Determine the magnitude of resultant force and its direction.



Solution:

①



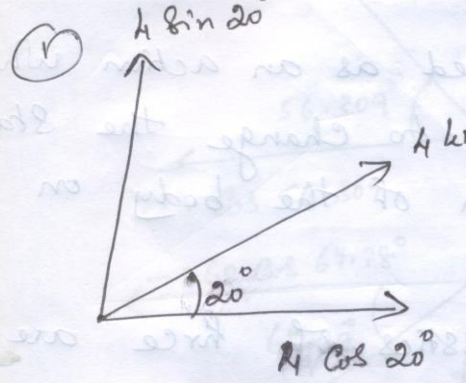
$= {}^{op}/_{ad}$.

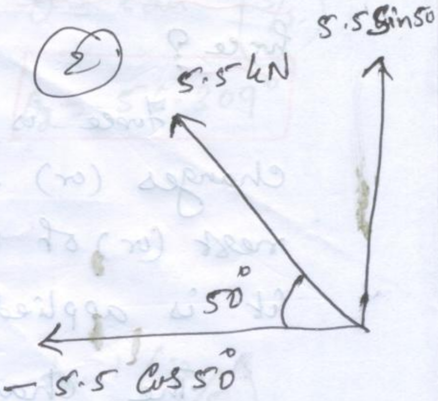
$\tan \theta_1 = (1/2)$

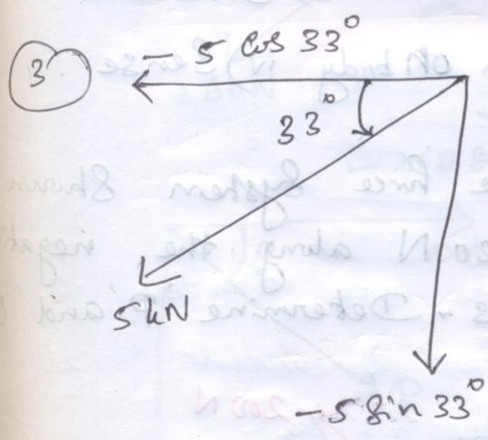
$\theta_1 = \tan^{-1}(1/2)$

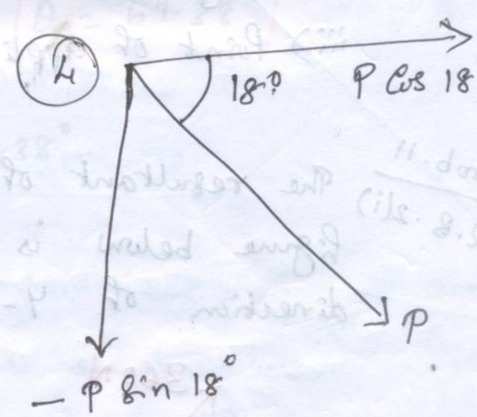
$\boxed{\theta_1 = 26.566°}$

Solution:









Applying equilibrium condition, $\Sigma H = 0$

$$\Sigma H = 4 \cos 20^\circ - 5.5 \cos 50^\circ - 5 \cos 33^\circ + P \cos 18^\circ = 0$$

$$-3.969 + P(0.951) = 0$$

$$(0.951)P = 3.969$$

$$\boxed{P = 4.173 \text{ kN}}$$

$\Sigma V = 0$

$$\Sigma V = 4 \sin 20^\circ + 5.5 \sin 50^\circ - 5 \sin 33^\circ - P \sin 18^\circ - Q = 0$$

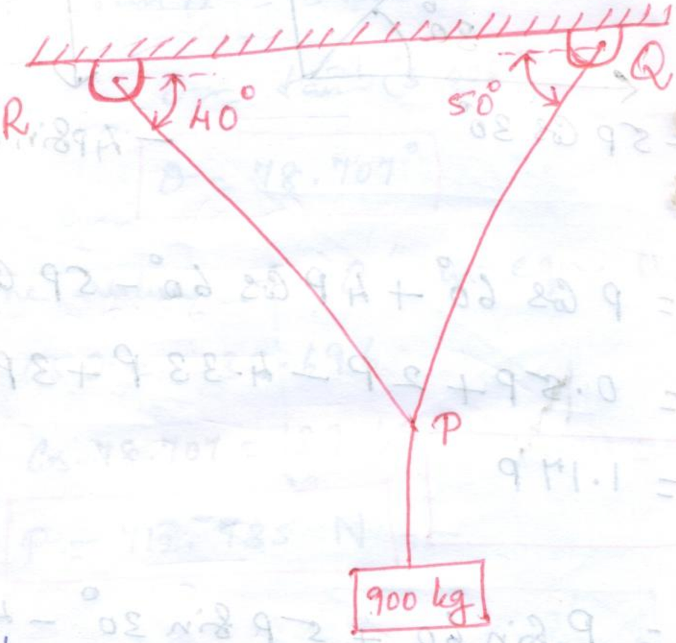
$$2.858 - 4.173 \sin 18 - Q = 0$$

$$\boxed{Q = 1.568 \text{ kN}}$$

Result:

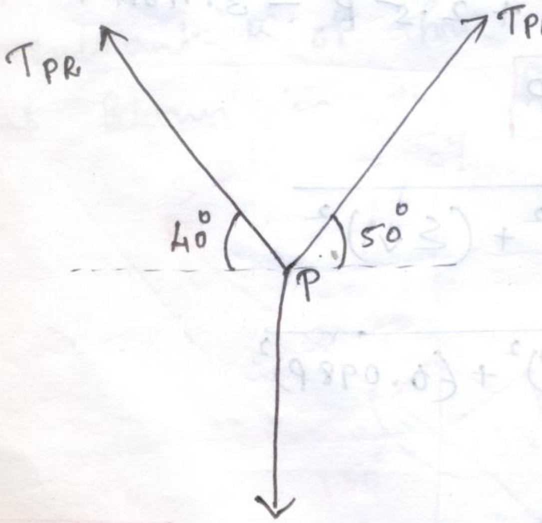
Magnitude of $\vec{P} = 4.173$ kN

Prob. 13 / Q.B. 4(ii). A body of mass 900 kg is supported by two cables as shown in figure below. Find the tension in the cables.



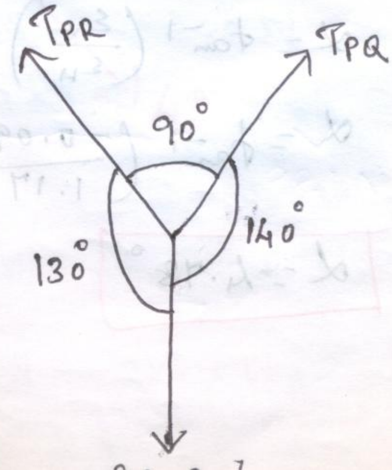
Solution:

Free body diagram



$W = 900 \times 9.81$

$\boxed{W = 8829\ N}$



Applying Lami's Theorem,

$$\frac{T_{PR}}{\sin 140^\circ} = \frac{T_{PQ}}{\sin 130^\circ} = \frac{8829}{\sin 90^\circ}$$

$$\frac{T_{PR}}{\sin 140^\circ} = \frac{8829}{\sin 90^\circ}$$

$$T_{PR} = \frac{8829}{1} \times 0.643$$

$$\boxed{T_{PR} = 5677.047 \text{ N}}$$

$$\frac{T_{PQ}}{\sin 130^\circ} = \frac{8829}{\sin 90^\circ}$$

$$T_{PQ} = \frac{8829}{1} \times 0.766$$

$$\boxed{T_{PQ} = 6763.014 \text{ N}}$$

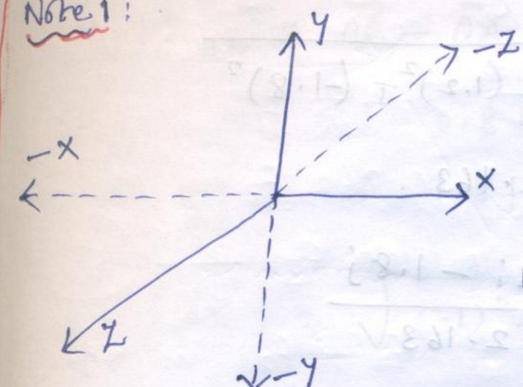
Result:

Tension in Cable PR = 5677.047 N

Tension in Cable PQ = 6763.014 N

Solution:

Note 1:



$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$

$\vec{F} = F(\lambda)$

Where,

$\lambda \rightarrow$ unit vector

Note: 2
Co-ordinates

X-axis $\rightarrow$ $Y=0$, $Z=0$

Y-axis $\rightarrow$ $X=0$, $Z=0$

Z-axis $\rightarrow$ $X=0$, $Y=0$

Let us Consider,

$F_1 = F_{AP}$

$F_2 = F_{AQ}$

$F_3 = F_{AR}$

Coordinate $A\ (0, 1.8, 0)$

$P\ (1.2, 0, 0)$

$Q\ (0, 0, 1.2)$

$R\ (-1, 0, -0.8)$

$W = -2j$

For equilibrium Condition, $\vec{R} = 0$.

$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{W}$

$\vec{F_1} = F_1\ (\lambda_{AP})$

$\lambda_{AP} = \dfrac{\vec{AP}}{|\vec{AP}|}$

$\vec{AP} = \vec{OP} - \vec{OA}$

$= (1.2i) - (1.8j)$

$\vec{AP} = 1.2i - 1.8j$

$$|\overrightarrow{AP}| = \sqrt{i^2 + j^2 + k^2}$$

$$= \sqrt{(1.2)^2 + (-1.8)^2}$$

$$|\overrightarrow{AP}| = 2.163.$$

$$\lambda_{AP} = \frac{1.2i - 1.8j}{2.163}$$

$$= 0.554i - 0.832j$$

$$\overrightarrow{F_1} = F_1(0.554i - 0.832j)$$

$$\boxed{\overrightarrow{F_1} = 0.554F_1 i - 0.832F_1 j}$$

$$\overrightarrow{F_2} = F_2(\lambda_{AQ}) \qquad \lambda_{AQ} = \frac{\overrightarrow{AQ}}{|\overrightarrow{AQ}|}$$

$$\overrightarrow{AQ} = \overrightarrow{OQ} - \overrightarrow{OA}$$

$$= 1.2k - 1.8j$$

$$\overrightarrow{AQ} = -1.8j + 1.2k$$

$$|\overrightarrow{AQ}| = \sqrt{(-1.8)^2 + (1.2)^2}$$

$$|\overrightarrow{AQ}| = 2.163.$$

$$\lambda_{AQ} = -0.832j + 0.554k$$

$$\overrightarrow{F_2} = F_2(-0.832j + 0.554k)$$

$$\boxed{\overrightarrow{F_2} = -0.832F_2 j + 0.554F_2 k}$$

$$\vec{F_3} = F_3 (\lambda_{AR}) \qquad \lambda_{AR} = \frac{\vec{AR}}{|\vec{AR}|}$$

$$\vec{AR} = \vec{OR} - \vec{OA}$$

$$= -i - 0.8k - 1.8j$$

$$\vec{AR} = -i - 1.8j - 0.8k$$

$$|\vec{AR}| = \sqrt{(-1)^2 + (1.8)^2 + (0.8)^2}$$

$$|\vec{AR}| = 2.209$$

$$\lambda_{AR} = -0.452i - 0.814j - 0.362k$$

$$\vec{F_3} = F_3(-0.452i - 0.814j - 0.362k)$$

$$\boxed{\vec{F_3} = -0.452 F_3 i - 0.814 F_3 j - 0.362 F_3 k}$$

$$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{W}$$

$$\vec{R} = i(0.554 F_1 - 0.452 F_3) + j(-2 - 0.832 F_1 - 0.832 F_2 - 0.814 F_3) + k(0.554 F_2 - 0.362 F_3)$$

For equilibrium Condition;

$$\vec{R} = 0$$

$$0.554 F_1 - 0.452 F_3 = 0 \quad \text{--- (1)}$$

$$-2 - 0.832 F_1 - 0.832 F_2 - 0.814 F_3 = 0$$

$$-0.832 F_1 - 0.832 F_2 - 0.814 F_3 = 2 \quad \text{--- (2)}$$

$$0.554 F_2 - 0.362 F_3 = 0 \quad \text{--- (3)}$$

(1) $\Rightarrow$ $0.554 F_1 - 0.452 F_3 = 0$

$$\cancel{+} 0.452 F_3 = \cancel{+} 0.552 F_1$$

$F_1 = 0.816 F_3$ — (4)

(3) $\Rightarrow 0.554 F_2 - 0.362 F_3 = 0$

$0.554 F_2 = 0.362 F_3$

$F_2 = 0.653 F_3$ — (5)

Sub. the value of F_1 & F_2 in eqn. (2)

(2) $\Rightarrow -0.832 F_1 - 0.832 F_2 - 0.814 F_3 = 2$

$-0.832 (0.816 F_3) - 0.832 (0.653 F_3) - 0.814 F_3 = 2$

$-0.679 F_3 - 0.543 F_3 - 0.814 F_3 = 2$

$-2.036 F_3 = 2$

$\boxed{F_3 = -0.982 \text{ kN}}$

Sub. the value of F_3 in eqn. (4)

(4) $\Rightarrow F_1 = 0.816 F_3$

$= 0.816 (-0.982)$

$\boxed{F_1 = -0.801 \text{ kN}}$

Sub. the value of F_3 in (5)

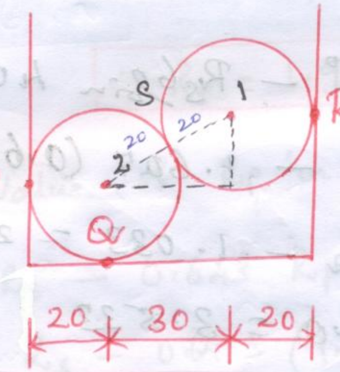
(5) $\Rightarrow F_2 = 0.653 F_3$

$F_2 = 0.653 (-0.982)$

$\boxed{F_2 = -0.641 \text{ kN}}$

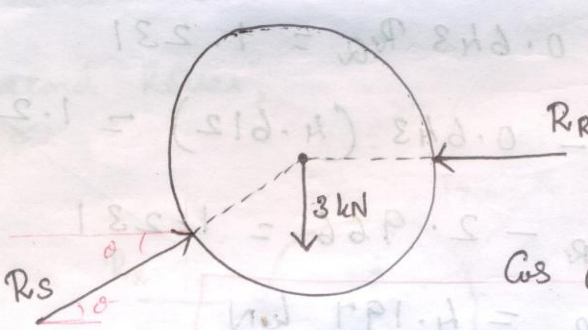
Prob: . Two identical Spheres each of weight 3 kN

Q.B: 7 and the radius 20 cm are kept in a horizontal channel of width 70 cm as Shown in figure. Determine the reactions at the Points of Contact P, Q and R.



Solution:

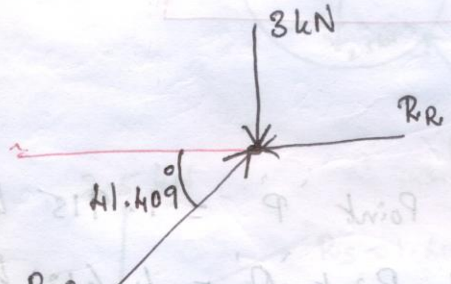
(i) For the first Sphere,



$\cos\theta = \dfrac{\text{Adj. Side}}{\text{Hyp. Side}}$

$\theta = \cos^{-1}\left(\dfrac{30}{40}\right)$

$\boxed{\theta = 41.409^\circ}$



For equilibrium Condition,

$\Sigma H = 0$

$R_S \cos 41.409^\circ - R_R = 0$

$0.75\, R_S - R_R = 0$ —— (1)

$$R_S \sin 41.409 - 3 = 0$$

$$0.661\, R_S = 3$$

$$\boxed{R_S = 4.539 \text{ kN}}$$

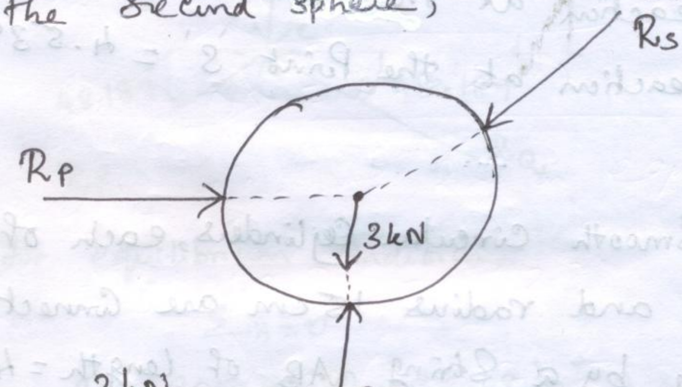
Sub. the value of R_S in eqn. (1')

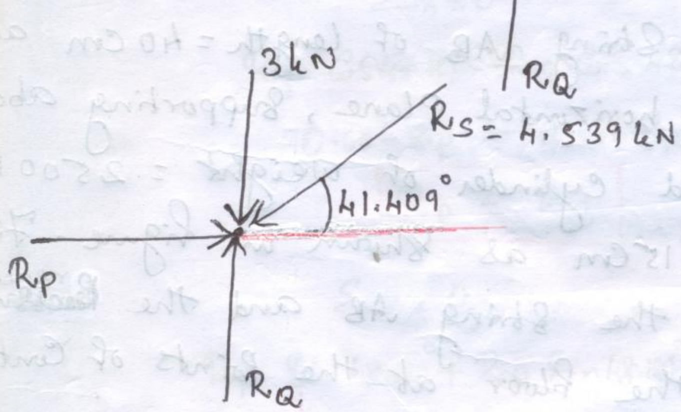
$$(1') \Rightarrow 0.75\, R_S - R_R = 0$$

$$0.75\,(4.539) - R_R = 0$$

$$\boxed{R_R = 3.404 \text{ kN}}$$

ii) For the Second Sphere,





For equilibrium Condition,

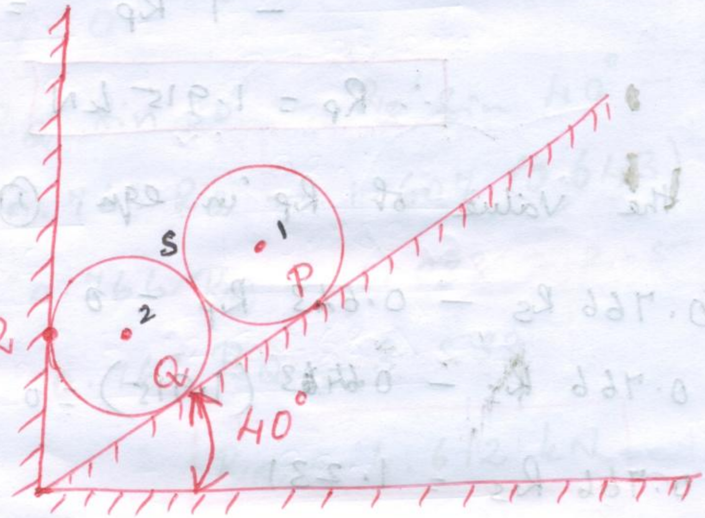
$$\Sigma H = 0$$

$$R_P - R_S \cos 41.409° = 0$$

$$R_P = (4.539)(0.75)$$

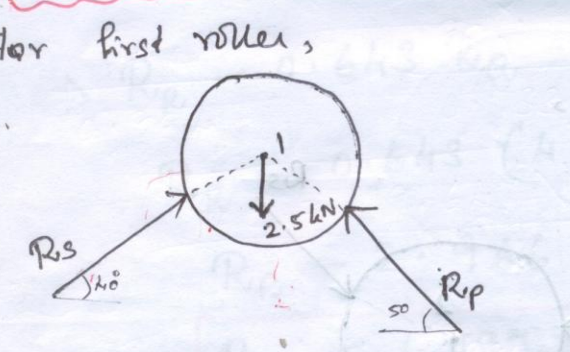
$$\boxed{R_P = 3.404 \text{ kN}}$$

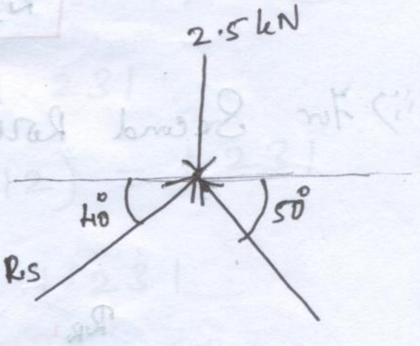
8 Two identical rollers each of weight [illegible] rest in between an inclined wall and a vertical wall as shown in figure. Determine the reactions at the points of contact P, Q and R. Assume the wall surfaces to be smooth.



Solution :

(i) For first roller,





For equilibrium condition,

$\Sigma_H = 0$

$R_S \cos 40^\circ - R_P \cos 50^\circ = 0$

$0.766\, R_S - 0.643\, R_P = 0$ ——— (1)

$\Sigma_V = 0$

$R_S \sin 40^\circ + R_P \sin 50^\circ = 2.5 = 0$

$0.643\, R_S + 0.766\, R_P = 2.5$ ——— (2)

Solve ① & ② by Simultaneous method.

$$① \times 0.643 \Rightarrow 0.493\, R_s - 0.413\, R_p = 0$$

$$② \times 0.766 \Rightarrow \cancel{0.493\, R_s} + 0.587\, R_p = 1.915$$

$$(-) \qquad (-) \qquad (-)$$

$$-1\, R_p = -1.915$$

$$\boxed{\therefore R_p = 1.915 \text{ kN}}$$

Sub. the value of R_p in eqn. ①

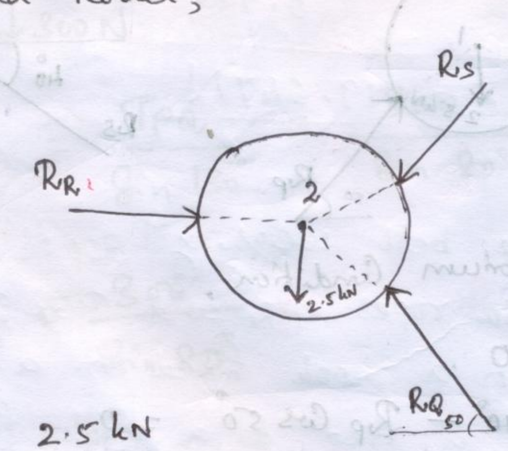
$$① \Rightarrow 0.766\, R_s - 0.643\, R_p = 0$$

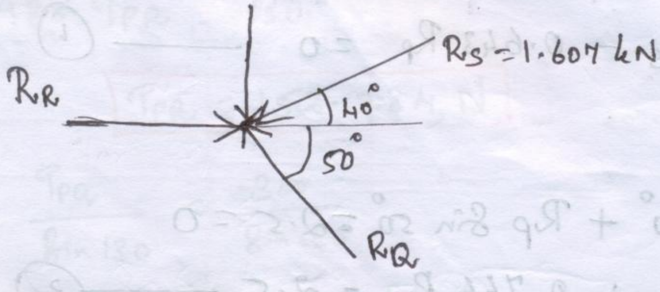
$$0.766\, R_s - 0.643\,(1.915) = 0$$

$$0.766\, R_s = 1.231$$

$$\boxed{R_s = 1.607 \text{ kN}}$$

ii) For Second Roller,





For equilibrium Condition,

$R_R - R_Q \cos 50^\circ - R_S \cos 40^\circ = 0$

$R_R - R_Q (0.643) - 1.607 (0.766) = 0$

$R_R - 0.643 R_Q = 1.231$ ——— (3)

$\Sigma V = 0$

$R_Q \sin 50^\circ - R_S \sin 40^\circ - 2.5 = 0$

$0.766 R_Q - 1.607 (0.643) = 2.5$

$0.766 R_Q - 1.033 = 2.5$

$0.766 R_Q = 3.533$

$$\boxed{R_Q = 4.612 \text{ kN}}$$

Sub. the value of R_Q in eqn. (3)

(3) $\Rightarrow R_R - 0.643 R_Q = 1.231$

$R_R - 0.643 (4.612) = 1.231$

$R_R - 2.966 = 1.231$

$$\boxed{R_R = 4.197 \text{ kN}}$$

Result:

1. The reaction at Point 'P' = 1.915 kN
2. The reaction at Point 'Q' = 4.612 kN
3. The reaction at Point 'R' = 4.197 kN
4. The reaction at Point 'S' = 1.607 kN

UNIT – II

FORCES IN SPACE & FRICTION

① what is simply supported Beam & overhanging Beam?

Simply supported Beam:

A Beam that resting freely on the supports without any fixity at the Ends is called S S B.

overhanging Beam:

A Beam in which one or both Ends are extended beyond the supports.

② write about the different types of supports?

(i). Simple support (or) knife edge
(ii). Roller support
(iii). Hinged support
(iv). Fixed support.

③. Define uniformly Distributed load and Uniformly varying load.

UDL:

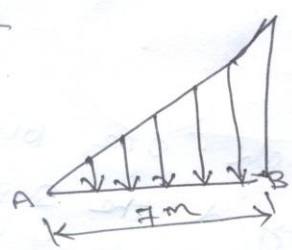
A load which is acting on a beam for Entire length.

UVL:

In UVL, the load is increased from one end to other gradually from zero at one end to maximum at other end.

4. Determine the Equivalent Point load and its location on the Beam for the given figure.

Given:



Point load $P_{EQU} = w \times \frac{L}{2}$
$= 20 \times \frac{7}{2}$
$= 70\,kN$

$\therefore \Sigma V = 0$

$\boxed{R_A + R_B = 70}$

$\boxed{\Sigma M_A = 0}$

$-70 \times 4.667 + R_B \times 7 = 0$

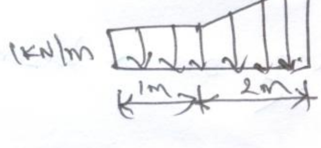
$\therefore 7R_B = 326.69$

$R_B = \frac{326.69}{7}$

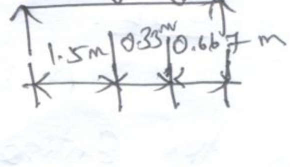
$\boxed{R_B = 46.67\,kN}$

$\therefore R_A = 23.33\,kN.$

5. Determine the Equivalent Point load and its location on the beam for the given figure.



Ans



(6). Define coefficient of friction?

It is defined as ratio of limiting force to the normal component of the reaction of the contact surface.

(7) The coefficient of friction b/w a body inclined surface on which it is resting $1/\sqrt{3}$, for the body to be in equilibrium, find the max. angle made by the surface with respect to horizontal.

Soln:

$$\mu = \tan\theta$$

$$\frac{1}{\sqrt{3}} = \tan\theta$$

$$\theta = \tan^{-1}\left[1/\sqrt{3}\right]$$

$$\boxed{\theta = 29.985^{\circ}}$$

(8). State laws of dry friction.

Dry friction develops b/w dry surfaces (or) unlubricated surfaces of bodies in contact.

(9). State Coloumb's Law of Dry friction.

(i). Magnitude of force of friction equal to applied force.

(ii). Force of friction depends on surface roughness.

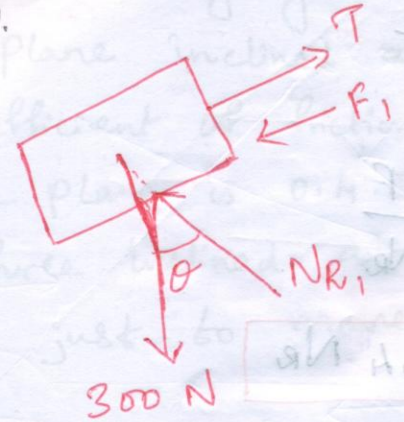
(10). List out the different types of friction?

(i). Static friction

(ii) Dynamic (or) Kinetic friction.

— x — x —

Solution:



For equilibrium condition,

$$\Sigma_H = 0$$

$$T - F_1 - 300 \cos(90 - \theta) = 0$$

$$T - F_1 - 300 \sin\theta = 0$$

$$\mu = F/N_R$$

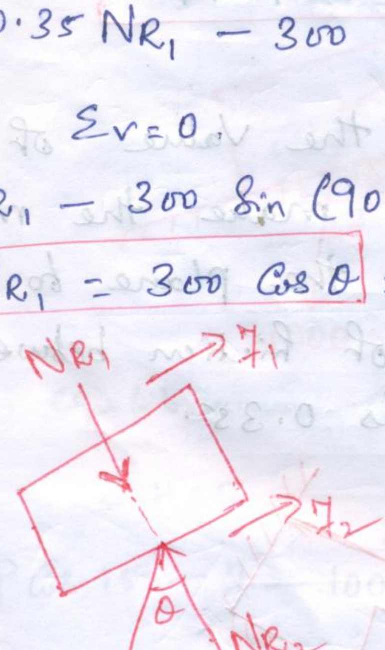
$$F_1 = 0.35 \, N_{R_1}$$

$$T - 0.35 \, N_{R_1} - 300 \sin\theta = 0 \quad \text{--- (1)}$$

$$\Sigma_V = 0.$$

$$N_{R_1} - 300 \sin(90 - \theta) = 0$$

$$\boxed{N_{R_1} = 300 \cos\theta} \quad \text{--- (2)}$$



$$\Sigma_H = 0.$$

$$F_1 + F_2 - 900 \sin\theta = 0$$

$$-N_{R_1} + N_{R_2} - 900\cos\theta = 0.$$

$$N_{R_2} = N_{R_1} + 900\cos\theta$$

$$N_{R_2} = 300\cos\theta + 900\cos\theta$$

$$\boxed{N_{R_2} = 1200\cos\theta} \quad \text{——— (4)}$$

Sub. the value of N_{R_1} & N_{R_2} in eqn. (3)

$$0.35\, N_{R_1} + 0.35\, N_{R_2} = 900\sin\theta.$$

$$(3) \Rightarrow \quad 0.35\, N_{R_1} + 0.35\left(1200\cos\theta\right) = 900\sin\theta$$

$$0.35\left(300\cos\theta\right) + 0.35\left(1200\cos\theta\right) = 900\sin\theta$$

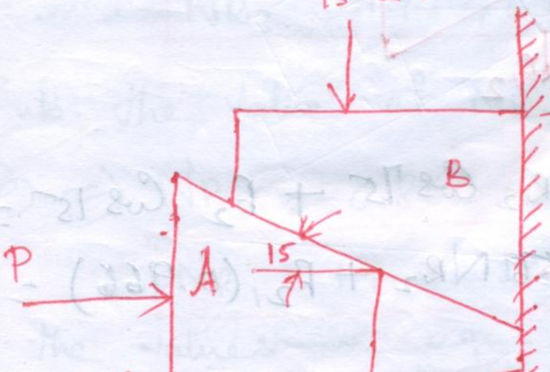
$$105\cos\theta + 420\cos\theta = 900\sin\theta$$

$$525\cos\theta = 900\sin\theta.$$

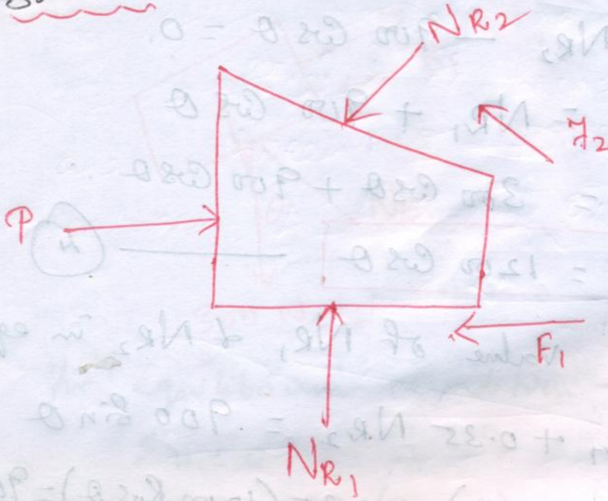
$$\tan\theta = 0.583$$

$$\boxed{\theta = 30.242^\circ}$$

Prob. 6 / Q.B.3 Determine the smallest force 'P' required to lift the 15 kN load shown in figure. The co-efficient of friction between A & C and between B and D is 0.3 and that between A & B is 0.4.



Solution:



$\Sigma H = 0$

$-F_2 \cos 15 - F_1 - N_{R_2} \cos 75 + P = 0$

$-0.966 F_2 - F_1 - 0.259 N_{R_2} + P = 0$

$-0.386 N_{R_2} - 0.3 N_{R_1} - 0.259 N_{R_2} + P = 0$

$-0.645 N_{R_2} - 0.3 N_{R_1} + P = 0$ — (1)

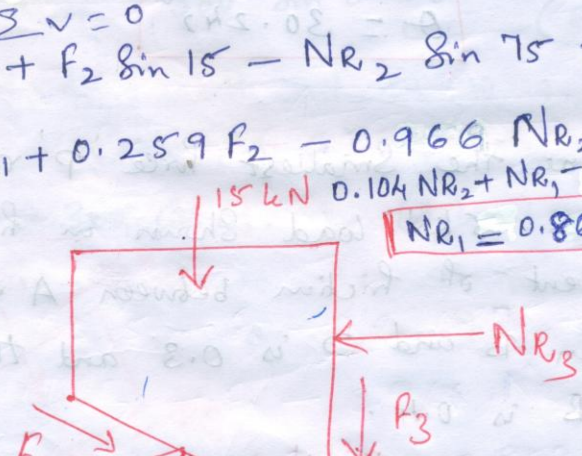
$\Sigma V = 0$

$N_{R_1} + F_2 \sin 15 - N_{R_2} \sin 75 = 0.$

$N_{R_1} + 0.259 F_2 - 0.966 N_{R_2} = 0$

$0.104 N_{R_2} + N_{R_1} - 0.966 N_{R_2} = 0$

$\boxed{N_{R_1} = 0.862 N_{R_2}}$ — (2)



$\Sigma H = 0$

$-N_{R_3} + N_{R_2} \cos 75 + F_2 \cos 15 = 0.$

$-N_{R_3} + 0.259 N_{R_2} + F_2 (0.966) = 0$

$\mu = \frac{F_2}{N_{R_2}}$

$-N_{R_3} + 0.259 N_{R_2} + 0.966 N_{R_2} (0.4) = 0.$

$-N_{R_3} + 0.259 N_{R_2} + 0.386 N_{R_2} = 0$

$-N_{R_3} + 0.645 N_{R_2} = 0$

$\boxed{N_{R_3} = 0.645 N_{R_2}}$ —— (3)

$\Sigma V = 0$

$-15 - F_3 + N_{R_2} \sin 75 - F_2 \sin 15 = 0.$

$0.966 N_{R_2} - 0.259 F_2 - F_3 = 15.$

$0.966 N_{R_2} - 0.259 (0.4 N_{R_2}) - 0.3 N_{R_3} = 15$

$0.966 N_{R_2} - 0.104 N_{R_2} - 0.3 N_{R_3} = 15$

$0.862 N_{R_2} - 0.3 N_{R_3} = 15$ —— (4)

Sub. the value of N_{R_3} in eqn. (4)

(4) $\Rightarrow$ $0.862 N_{R_2} - 0.3 (0.645 N_{R_2}) = 15$

$0.862 N_{R_2} - 0.194 N_{R_2} = 15$

$0.668 N_{R_2} = 15$

$\boxed{N_{R_2} = 22.422 \text{ kN}}$

Sub. the value of N_{R_2} in eqn. (3)

(3) $\Rightarrow$ $N_{R_3} = 0.645 (22.422)$

$\boxed{N_{R_3} = 14.462 \text{ kN}}$

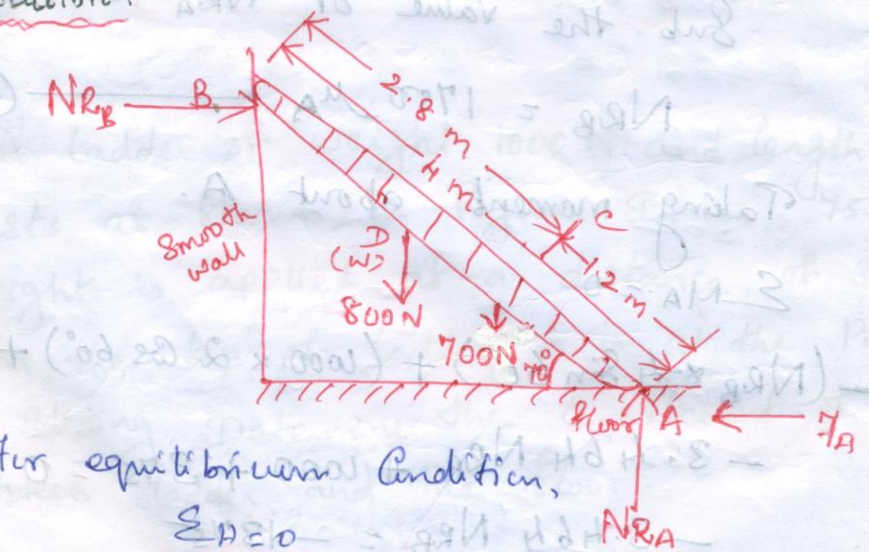
Sub. the value of N_{R_2} in eqn. (2)

(2) $\Rightarrow$ $N_{R_1} = -0.862 \times (22.422)$

$\boxed{N_{R_1} = 19.328 \text{ kN}}$

Prob. 9
Q.B-6 . A ladder 4 m long leans against a smooth vertical wall at an angle 70° with the horizontal. The weight of the ladder is 800 N. When a person weighing 700 N stands at 1.2 m from the bottom of the ladder, the ladder is just about to slide. Calculate the co-efficient of friction between the ladder and the floor. Assume smooth wall.

Solution:



For equilibrium condition,

$$\Sigma H = 0$$

$$N_{RB} - F_A = 0$$

$$N_{RB} = \mu N_{RA} \quad \text{———} \quad ①$$

$\Sigma V = 0$

$N_{RA} - 800 - 700 = 0$

$\boxed{N_{RA} = 1500 \text{ N}}$

Sub. the value of N_{RA} in eqn. (1)

$N_{RB} = 1500 \mu$ ——— (2)

Taking moment about A.

$\Sigma M_A = 0$

$-(N_{RB} \times 4 \sin 70^\circ) + (800 \times 2 \cos 70^\circ) + (700 \times 1.2 \cos 70^\circ) = 0$

$-3.759 N_{RB} + 547.232 + 287.297 = 0$

$-3.759 N_{RB} = -834.529$

$\boxed{N_{RB} = 222.008 \text{ N}}$

Sub. the value of N_{RB} in eqn. (2)

(2) $\Rightarrow N_{RB} = 1500 \mu$

$222.008 = 1500 \mu$

$\boxed{\mu = 0.148}$

∴ The co-efficient of friction between the ladder and the floor is 0.148.

Sub. the value of R_B in eqn. (1)

$$(1) \Rightarrow \quad R_{A(V)} + R_B = 15.171$$

$$R_{A(V)} + 8.895 = 15.171$$

$$\boxed{R_{A(V)} = 6.276 \text{ kN}}$$

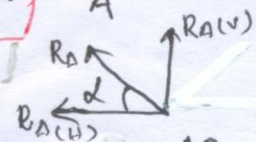
$R_A = \sqrt{(-3.221)^2 + (6.276)^2}$

$$\boxed{R_A = 7.054 \text{ kN}}$$

$\alpha = \tan^{-1}\left(\frac{R_{A(V)}}{R_{A(H)}}\right)$

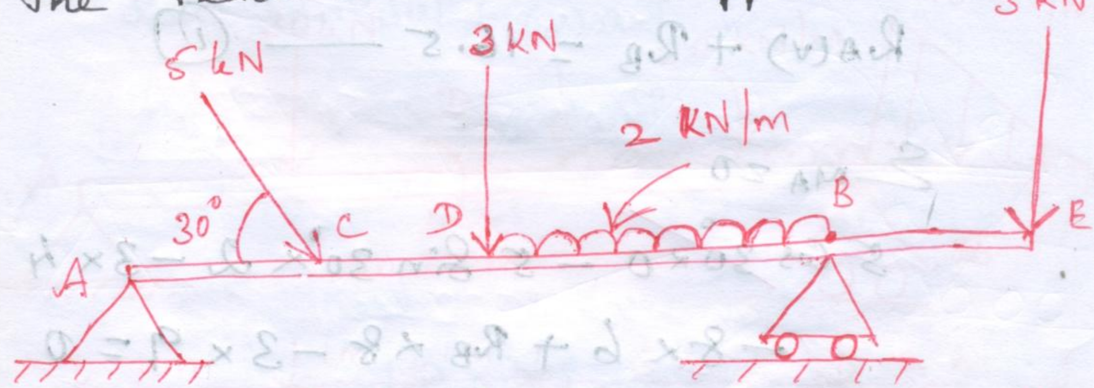
$= \tan^{-1}\frac{6.276}{-3.221}$

$\boxed{\alpha = -62.826^\circ}$

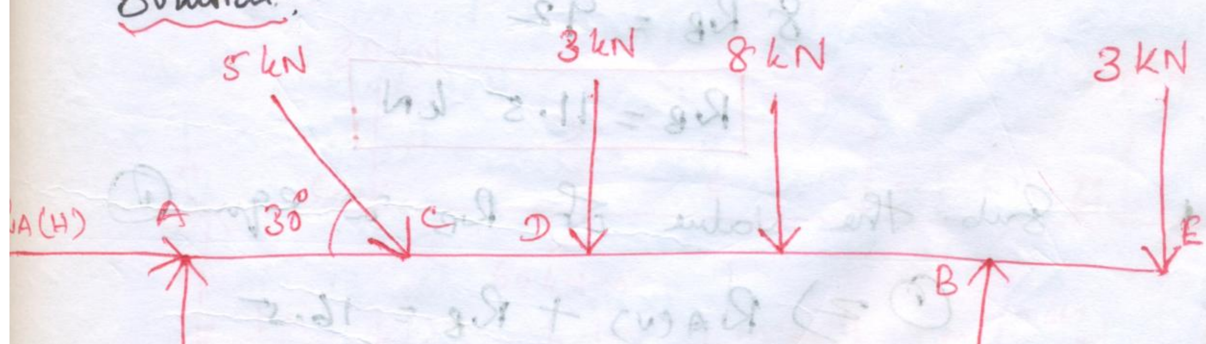


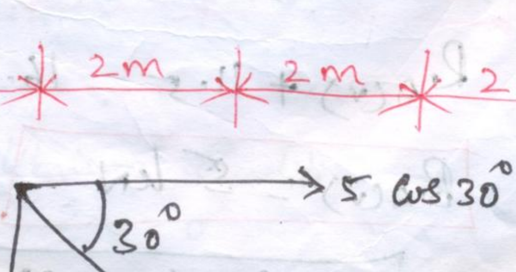
Prob. 8

Q.B. 8: An Overhanging beam carries the loads as shown in figure. Calculate the reactions at the supports.



Solution:





For equilibrium Condition

$\Sigma_H = 0$

$R_{A(H)} + 5\cos 30^\circ = 0$

$\boxed{R_{A(H)} = -4.33\ kN}$

$\Sigma_V = 0$

$R_{A(V)} - 5\sin 30^\circ - 3 - 8 + R_B - 3 = 0$

$R_{A(V)} - 2.5 - 14 + R_B = 0$

$R_{A(V)} + R_B = 16.5$ —— (1)

$\Sigma M_A = 0$

$\cancel{5\cos 30^\circ \times 0} - 5\sin 30^\circ \times 2 - 3\times 4$

$-8\times 6 + R_B \times 8 - 3\times 9 = 0$

$-2.5\times 2 - 12 - 48 + 8R_B - 27 = 0$

$8R_B = 92$

$\boxed{R_B = 11.5\ kN}$

Sub. the value of R_B in eqn. (1)

(1) => $R_{A(V)} + R_B = 16.5$

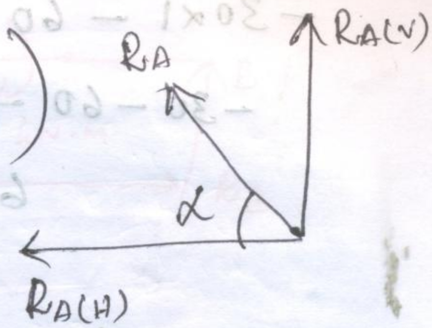
$R_{A(V)} + 11.5 = 16.5$

$\boxed{R_{A(V)} = 5\ kN}$

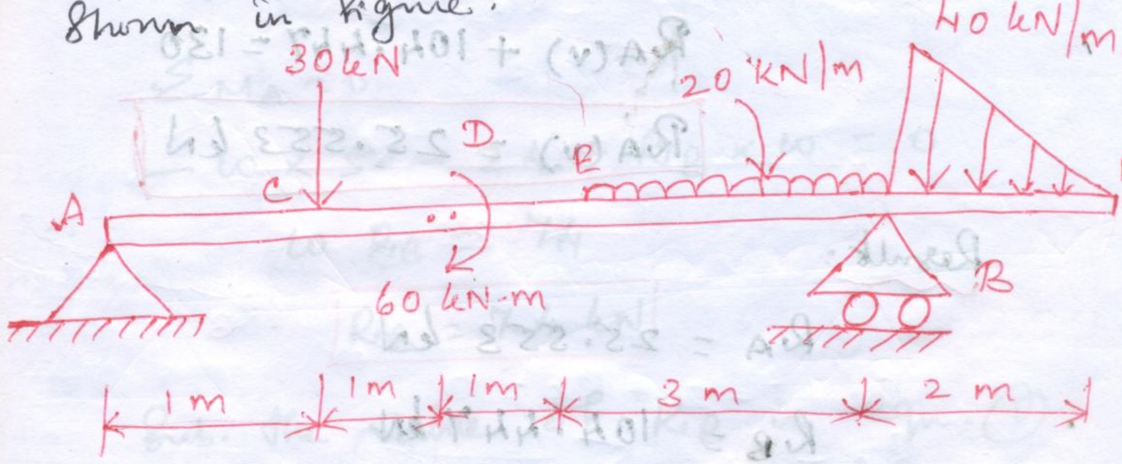
$$\alpha = \tan^{-1}\left(\frac{\quad}{(R_A)_H}\right)$$

$$= \tan^{-1}\left(\frac{5}{-4.33}\right)$$

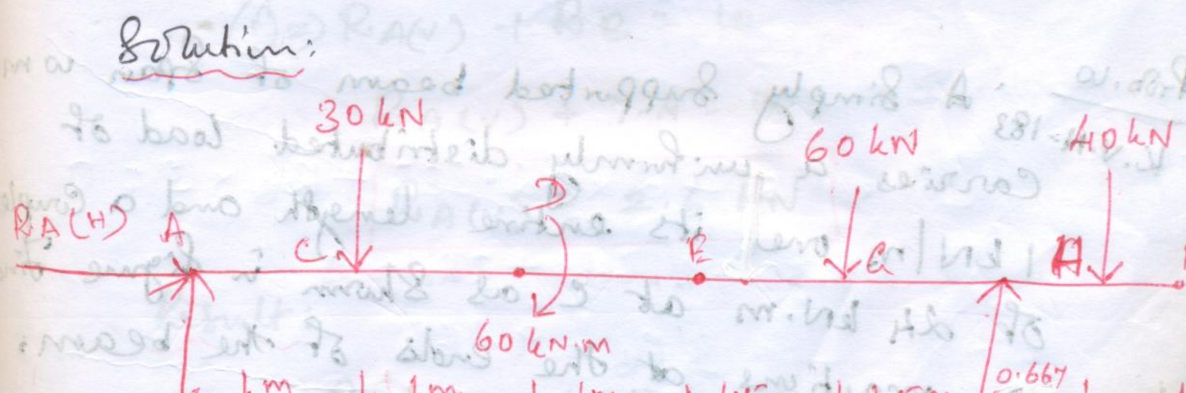
$$\boxed{\alpha = -49.107^\circ}$$



Prob. 9 / Q.B.9. Find the equivalent concentrated loads and the external reactions for the beam shown in figure.



Solution:



For equilibrium condition,

$$\Sigma V = 0$$

$$R_{A(V)} - 30 - 60 - 40 + R_B = 0$$

$$R_{A(V)} + R_B = 130$$

$$\Sigma M_A = 0$$

$$-30\times1 - 60 - 60\times4.5 + R_B\times6 - 40\times6.667 = 0$$

$$-30 - 60 - 270 + 6R_B - 266.68 = 0$$

$$6R_B = 626.68$$

$$\boxed{R_B = 104.447 \text{ kN}}$$

Sub. the value of R_B in eqn. (1)

$$(1) \Rightarrow R_{A(V)} + R_B = 130$$

$$R_{A(V)} + 104.447 = 130$$

$$\boxed{R_{A(V)} = 25.553 \text{ kN}}$$

Result:

$$R_A = 25.553 \text{ kN}$$

$$R_B = 104.447 \text{ kN}$$

UNIT - III

PROPERTIES OF SURFACES & SOLIDS

1. Define centre of Gravity.

It the point on the body through which the whole (or) Entire weight of a body acts on it.

2. Define First moment of an area about the axis

The moment about an area is called first moment of an area.

Moment of length about x-axis is

$$M_L = \int y_i \, dL$$

3. State Pappus - Guldinus theorems.

Theorem I:

The area of a surface of revolution is equal to the product length of the generating curve and the distance travelled by the centroid

Theorem II:

The volume of a body of revolution is equal to the product of the generating area and the distance travelled by the centroid of the area.

(4) What is meant by Moment of Inertia of a Body?

It is defined as the Product of Force and the Square of its Perpendicular distance, then it is known as moment of Inertia.

(5) Explain the term radius of gyration of an area.

The Term of gyration of the Body with respect to the axis xx is defined by the relation

$$I = k^2 M$$

$$k = \sqrt{\frac{I}{M}}$$

Unit: Metre.

(6). State Parallel axis theorem.

It states that "If the moment of Inertia of a Plane area about an axis through its centre of ~~centre~~ gravity be denoted by I_G, then the Moment of Inertia of the area about axis AB Parallel to the first and at a distance h from the centre of gravity is given by

$$\boxed{I_{AB} = I_G + Ah^2}$$

(8) When will the centroid and centre of Mass Coincide

Ans: When Body is symmetrical

(9) Define polar moment of Inertia of an area and state its applications,

It is defined as the moment of Inertia of a lamina for plane about the axis perpendicular to the plane of the section. It is denoted by I_{zz} (or) J.

(10) Determine the second moment of area a triangle with respect to the base.

$$I_{base} = \int_0^h \frac{by}{h}(h-y)^2 \, dy$$

By simplifying

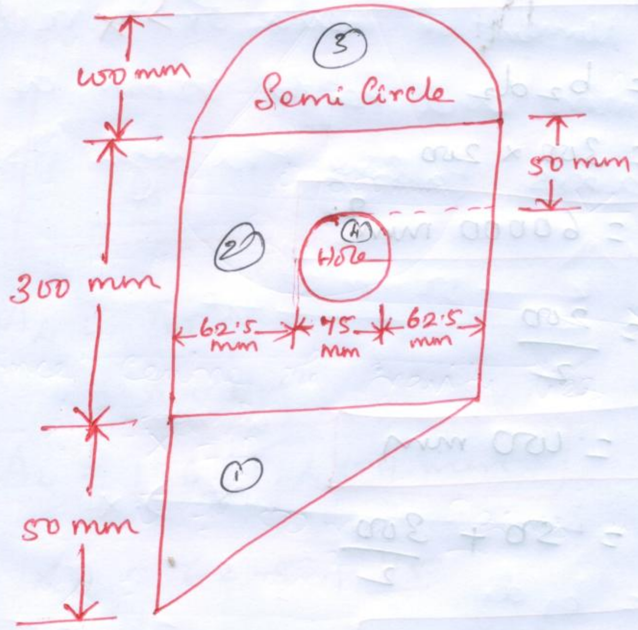
$$\boxed{I_{AB} = \frac{bh^3}{12}}$$

About centroidal axis

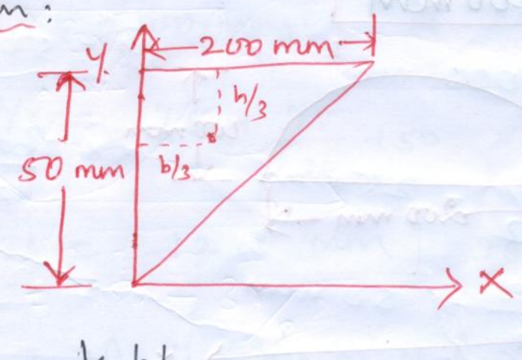
$$I_{xx} = \frac{bh^3}{36}$$

Prob. 5. Calculate the centroid of the figure.

Q. B. 2



Solution:



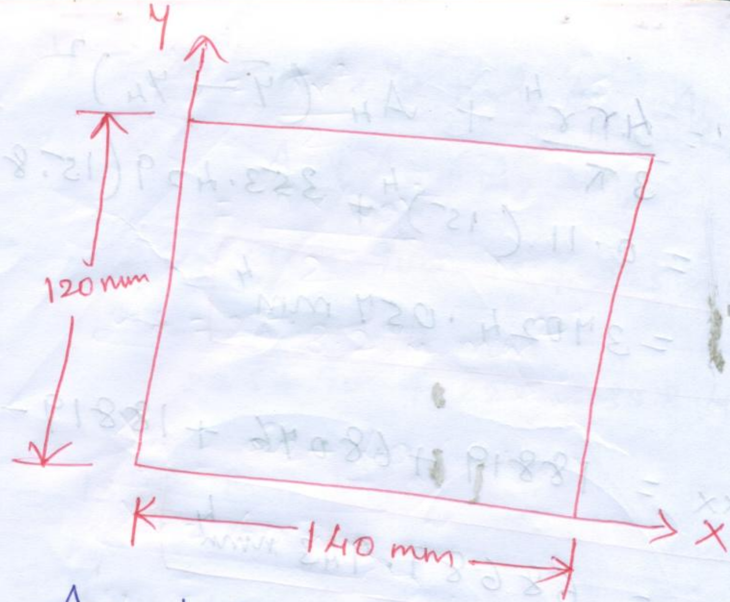
$A_1 = \frac{1}{2} b_1 h_1$

$= \frac{1}{2} \times 200 \times 50$

$\boxed{A_1 = 5000 \text{ mm}^2}$

$X_1 = \frac{200}{}$

$$y_1 = 50 - h/2$$



$$A_1 = b_1 d_1$$

$$= 120 \times 140$$

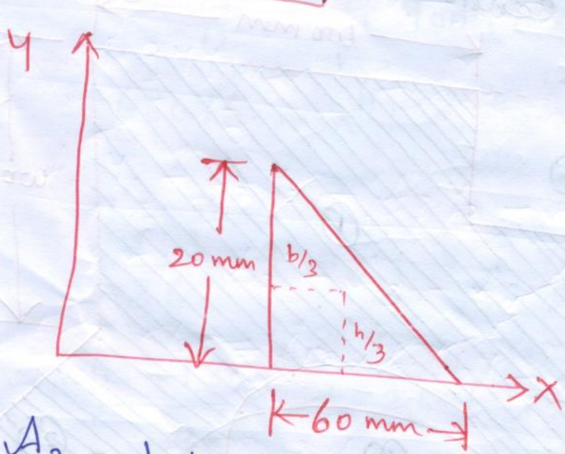
$$\boxed{A_1 = 16{,}800 \text{ mm}^2}$$

$$x_1 = \frac{140}{2}$$

$$\boxed{x_1 = 70 \text{ mm}}$$

$$y_1 = \frac{120}{2}$$

$$\boxed{y_1 = 60 \text{ mm}}$$



$$A_2 = \frac{1}{2} b_2 \times h_2$$

$$= \frac{1}{2} 60 \times 20$$

$$\boxed{A_2 = 600 \text{ mm}^2}$$

$$X_2 = 140 + b/3$$

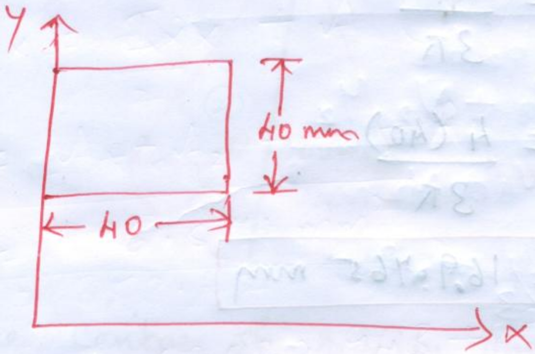
$$= 140 + 60/3$$

$$\boxed{X_2 = 160 \text{ mm}}$$

$$Y_2 = h/3$$

$$= 20/3$$

$$\boxed{Y_2 = 6.667 \text{ mm}}$$



$$A_3 = s^2$$

$$= 40^2$$

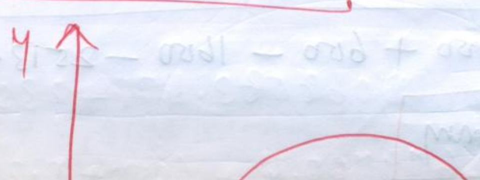
$$\boxed{A_3 = 1600 \text{ mm}^2}$$

$$X_3 = 40/2$$

$$\boxed{X_3 = 20 \text{ mm}}$$

$$Y_3 = 80 + \frac{40}{2}$$

$$\boxed{Y_3 = 100 \text{ mm}}$$



$$A_4 = \frac{\pi r^2}{2}$$

$$= \frac{\pi \times (40)^2}{2}$$

$$\boxed{A_4 = 2513.274 \ mm^2}$$

$$x_4 = 20 + 40 \ (\text{radius})$$

$$\boxed{x_4 = 60 \ mm}$$

$$y_4 = \frac{4r}{3\pi}$$

$$= \frac{4(40)}{3\pi}$$

$$\boxed{y_4 = 16.97 \ mm}$$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 - A_3 x_3 - A_4 x_4}{A_1 + A_2 - A_3 - A_4}$$

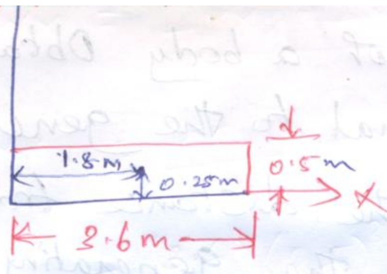
$$= \frac{16800 \times 70 + 600 \times 160 - 1600 \times 20 - 2513.274 \times 60}{16800 + 600 - 1600 - 2513.274}$$

$$\boxed{\bar{x} = 81.977 \ mm} = 81.976 mm$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3 - A_4 y_4}{A_1 + A_2 - A_3 - A_4}$$

$$= \frac{16800 \times 60 + 600 \times 6.667 - 1600 \times 100 - 2513.274 \times 16.976}{16800 + 600 - 1600 - 2513.274}$$

$$\boxed{\bar{y} = 32.012 \ mm}$$



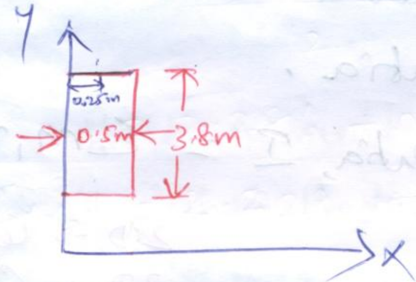
$A_1 = l_1 b_1$

$= 3.6 \times 0.5$

$\boxed{A_1 = 1.8\ m^2}$

$\boxed{X_1 = 1.8\ m}$

$\boxed{Y_1 = 0.25\ m}$



$A_2 = l_2 b_2$

$= 3.8 \times 0.5$

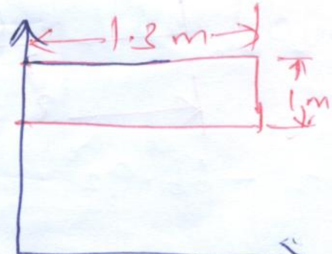
$\boxed{A_2 = 1.9\ m^2}$

$\boxed{X_2 = 0.25\ m}$

$Y_2 = 0.5 + 3.8/2$

$= 0.5 + 1.9$

$\boxed{Y_2 = 2.4\ m}$



$A_3 = 1.3 \times 1$

$\boxed{A_3 = 1.3\ m^2}$

$$x_3 = \frac{1.3}{2}$$

$$\boxed{x_3 = 0.65 \text{ m}}$$

$$y_3 = 0.5 + 3.8 + \tfrac{1}{2}$$

$$\boxed{y_3 = 4.8 \text{ m}}$$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$= \frac{(1.8)(1.8) + (1.9)(0.25) + (1.3)(0.65)}{1.8 + 1.9 + 1.3}$$

$$= \frac{3.24 + 0.475 + 0.845}{5}$$

$$\boxed{\bar{x} = 0.912 \text{ m}}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(1.8)(0.25) + (1.9)(2.4) + (1.3)(4.8)}{1.8 + 1.9 + 1.3}$$

$$= \frac{0.45 + 4.56 + 6.24}{5}$$

$$\boxed{\bar{y} = 2.25 \text{ m}}$$

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

$$I_{xx_1} = \frac{b_1 d_1^3}{12} + A_1 (\bar{y} - y_1)^2$$

$$= \frac{(3.6)(0.5)^3}{12} + 1.8 (2.25 - 0.25)^2$$

$$= 0.0375 + 7.2$$

$$\boxed{I_{xx_1} = 7.238 \ m^4}$$

$$I_{xx_2} = \frac{b_2 d_2^3}{12} + A_2 (\bar{y} - y_2)^2$$

$$= \frac{(0.5)(3.8)^3}{12} + 1.9 (2.25 - 2.4)^2$$

$$= 2.286 + 0.0428$$

$$\boxed{I_{xx_2} = 2.329 \ m^4}$$

$$I_{xx_3} = \frac{b_3 d_3^3}{12} + A_3 (\bar{y} - y_3)^2$$

$$= \frac{(1.3)(1)^3}{12} + 1.3 (2.25 - 4.8)^2$$

$$= 0.108 + 8.453$$

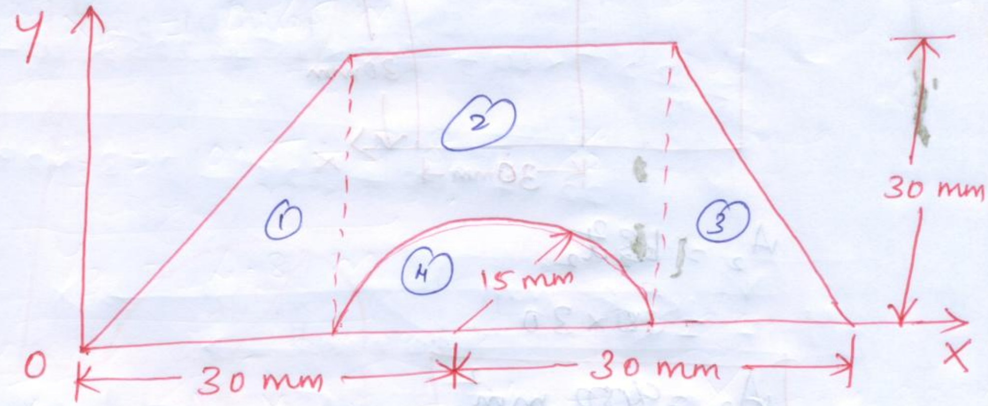
$$\boxed{I_{xx_3} = 8.561 \ m^4}$$

$$\therefore I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

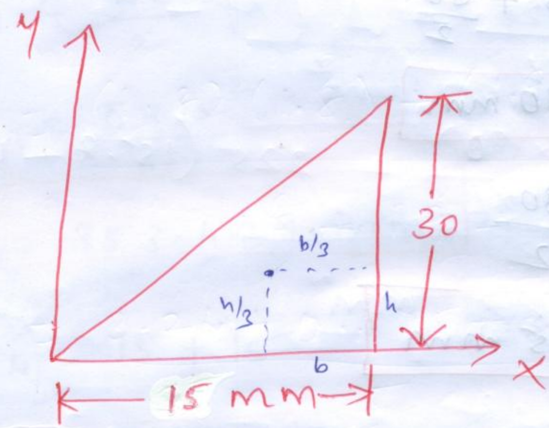
$$= 7.238 + 2.329 + 8.561$$

$$\boxed{\text{Moment of Inertia, } I_{xx} = 18.128 \ m^4}$$

Prob. 2 / Q.B.8 Find the Moment of Inertia for the given section about OX.



Solution:



$A_1 = \frac{1}{2} bh$

$= \frac{1}{2} \times 15 \times 30$

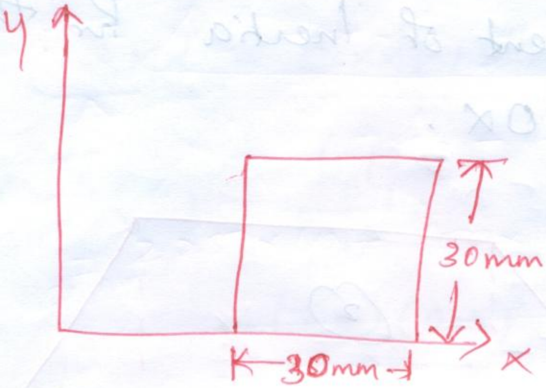
$\boxed{A_1 = 225 \text{ mm}^2}$

$X_1 = 15 - b/3$

$\boxed{X_1 = 10 \text{ mm}}$

$Y_1 = h/3 = \frac{30}{3}$

$\boxed{Y_1 = 10 \text{ mm}}$



$A_2 = S^2$

$= 30 \times 30$

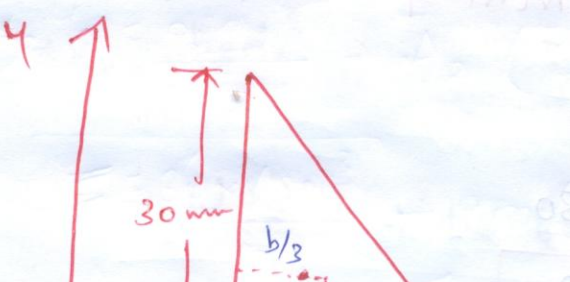
$\boxed{A_2 = 900\ mm^2}$

$X_2 = 15 + \frac{30}{2}$

$\boxed{X_2 = 30\ mm}$

$Y_2 = \frac{30}{2}$

$\boxed{Y_2 = 15\ mm}$



$A_3 = \frac{1}{2}\, b_3 h_3$

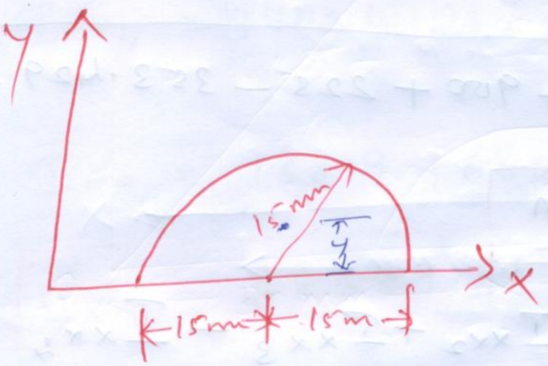
$= \frac{1}{2} \times 15 \times 30$

$\boxed{A_3 = \cancel{8}\,225\ mm}$

$X_3 = 15 + 30 + 15 \quad (b/3$

$$y_3 = \frac{h}{3} = \frac{30}{3}$$

$$\boxed{y_3 = 10 \text{ mm}}$$



$$A_4 = \frac{\pi r^2}{2}$$

$$= \frac{\pi \times 15^2}{2}$$

$$\boxed{A_4 = 353.429 \text{ mm}^2}$$

$$x_4 = 15 + 15$$

$$\boxed{x_4 = 30 \text{ mm}}$$

$$y_4 = \frac{4r}{3\pi}$$

$$= \frac{4 \times 15}{3\pi}$$

$$\boxed{y_4 = 6.366 \text{ mm}}$$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 - A_4 x_4}{A_1 + A_2 + A_3 - A_4}$$

$$= \frac{225 \times 10 + 900 \times 30 + 225 \times 50 - 353.429 \times 30}{225 + 900 + 225 - 353.429}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 - A_4 y_4}{A_1 + A_2 + A_3 - A_4}$$

$$= \frac{225 \times 10 + 900 \times 15 + 225 \times 10 - 353.429 \times 6.366}{225 + 900 + 225 - 353.429}$$

$$\boxed{\bar{y} = 15.8 \text{ mm}}$$

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3} - I_{xx_4}$$

$$I_{xx_1} = \frac{b_1 h_1^3}{36} + A_1 (\bar{y} - y_1)^2$$

$$= \frac{15 \times (30)^3}{36} + 225 (15.8 - 10)^2$$

$$\boxed{I_{xx_1} = 18819 \text{ mm}^4}$$

$$I_{xx_2} = \frac{b_2 d_2^3}{12} + A_2 (\bar{y} - y_2)^2$$

$$= \frac{30 \times (30)^3}{12} + 900 (15.8 - 15)^2$$ $0.11r^4$

$$\boxed{I_{xx_2} = 68076 \text{ mm}^4}$$

$$I_{xx_3} = \frac{b_3 h_3^3}{36} + A_3 (\bar{y} - y_3)^2$$

$$= \frac{15 \times (30)^3}{36} + 225 (15.8 - 10)^2$$

$$\boxed{I_{xx_3} = 18819 \text{ mm}^4}$$

$$I_{xx_4} = 0.11r^4 + A_4(\bar{y} - y_4)^2$$

$$= 0.11(15)^4 + 353.429(15.8 - 6.366)^2$$

$$\boxed{I_{xx_4} = 37024.057\ mm^4}$$

$$I_{xx} = 18819 + 68076 + 18819 - 37024.057$$

$$\boxed{I_{xx} = 68689.943\ mm^4}$$

$$I_{ox} = I_{xx} + A(\bar{y})^2$$

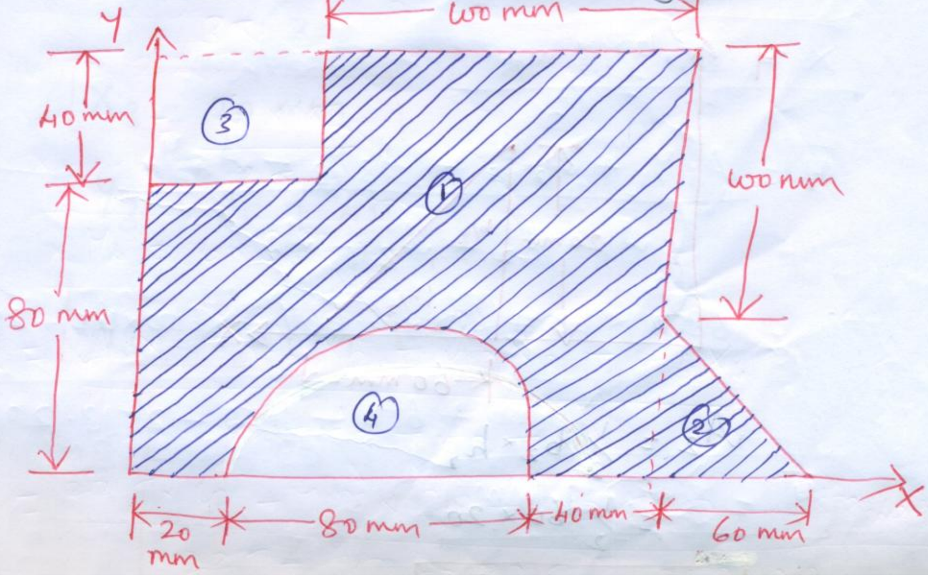
$$A = A_1 + A_2 + A_3 - A_4$$

$$\boxed{A = 996.571\ mm^2}$$

$$\therefore I_{ox} = 68689.943 + 996.571(15.8)^2$$

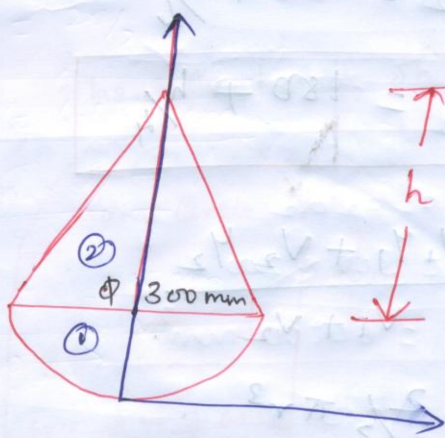
$$\boxed{I_{ox} = 317473.927\ mm^4}$$

Prob. 3 / Q.B.3 Find the centroid of the figure.



Prob. 4 / Q.B-10. A Cone of base diameter 300 mm is fitted to a hemisphere of diameter 300 mm centrally. What should be the height of the cone so that the centroid of the solid combination lies at the junction between the cone and the hemisphere?

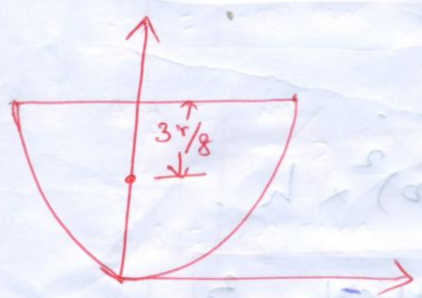
Solution.



Since centre of gravity lies on the 'y' axis

$\therefore \bar{x} = 0$

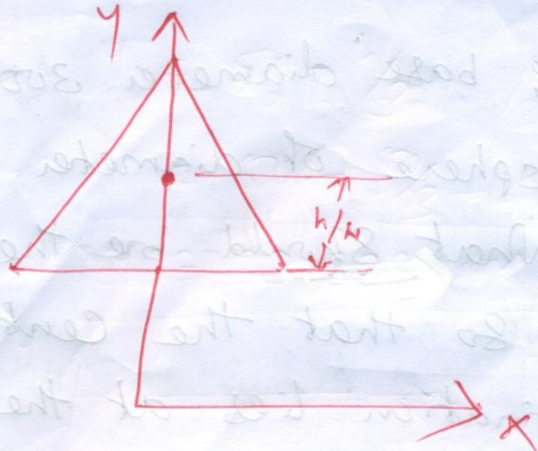
$\bar{y} = 150$ mm [gives in question].



$y_1 = 150 - \frac{3r}{8}$

$= 150 - \frac{3(150)}{8}$

$\boxed{y_1 = 93.75 \text{ mm}}$



$$y_2 = 150 + h/4$$

$$\boxed{y_2 = 180 + h/4}$$

$$\bar{Y} = \frac{V_1 Y_1 + V_2 Y_2}{V_1 + V_2}$$

$$V_1 = 2/3 \pi r^3$$

$$= 2/3 \pi (150)^3$$

$$\boxed{V_1 = 7068583.471 \text{ mm}^3}$$

$$V_2 = 1/3 \pi r^2 h.$$

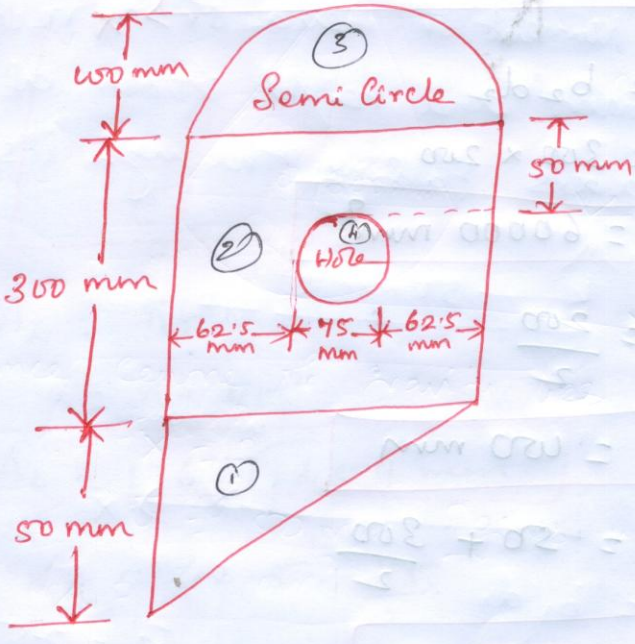
$$V_2 = 1/3 \pi (150)^2 \times h.$$

$$\boxed{V_2 = 23561.945 h \text{ mm}^3}$$

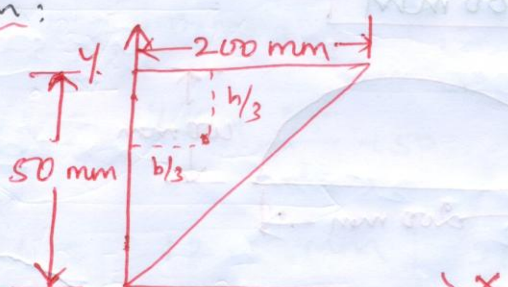
$$150 = \frac{7068583.471 \times 93.75 + (150 + h/4)\ 23561.94}{7068583.471 + 23561.945 h}$$

$$1060287521 + \cancel{3534291.75\,h} = 6626797004$$
$$+ \cancel{3534291.75h} + 5890.486\,h^2$$
$$5890.486\,h^2 = 397607820.6$$
$$h^2 = 67500.003$$
$$\boxed{h = 259.8\ m}$$

Prob. 5 / Q. B-2. Calculate the Centroid of the figure.



Solution:

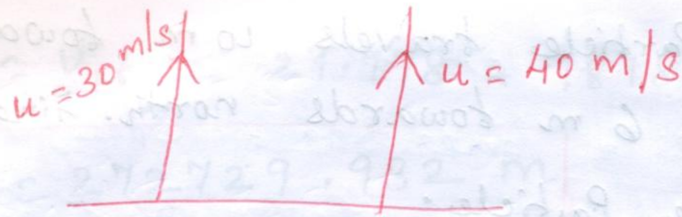


$$A_1 = \frac{1}{2} b_1 h_1$$
$$= \frac{1}{2} \times 200 \times 50$$
$$\boxed{A_1 = 5000\ mm^2}$$
$$X_1 = \frac{200}{3}$$

UNIT-IV

- An Object is thrown vertically upward with a velocity of 30 m/s. Four Seconds later a Second Object is Project vertically upward with a velocity of 40 m/s. Determine (i) the time (after the first object is thrown) when the two Objects will meet each other in air. (ii) the earth at which the two Objects will

Given:



$u_1 = 30\ m/s$

$u_2 = 40\ m/s$ & $t_1 = t_2 + 4$

$a = -g = -9.81\ m/s^2$

$S_1 = u_1 t_1 + \frac{1}{2} a t_1^2$

$= 30 t_1 - \frac{1}{2}(9.81) t_1^2$

$S_1 = 30 t_1 - 4.905 t_1^2$ —— (1)

$S_2 = u_2 t_2 + \frac{1}{2} a t_2^2$

$= 40 t_2 + \frac{1}{2}(-9.81) t_2^2$

$S_2 = 40 t_2 - 4.905 t_2^2$

Here $S_1 = S_2$

$30 t_1 - 4.905 t_1^2 = 40 t_2 - 4.905 t_2^2$

$30 t_1 - 4.905 t_1^2 - 40 t_2 + 4.905 t_2^2 = 0$ — (2)

Sub. $t_2 = t_1 - 4$ in eqn. (2)

$30 t_1 - 4.905 t_1^2 - 40(t_1 - 4) + 4.905(t_1 - 4)^2 = 0$

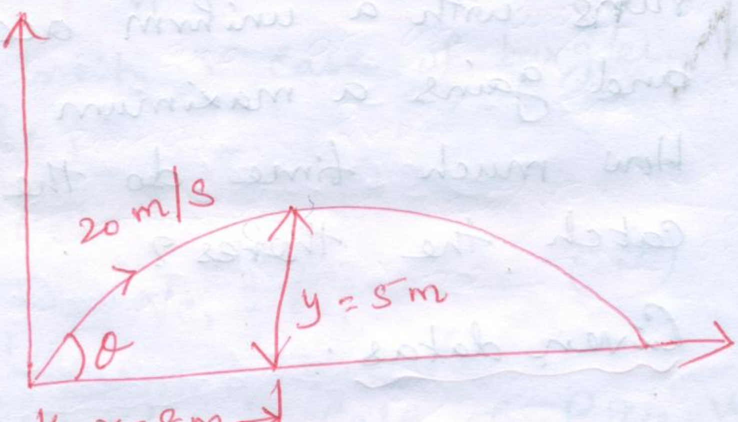
$30 t_1 - 4.905 t_1^2 - 40 t_1 + 160 + 4.905(t_1^2 + 16 - 8t_1)$ $= 0$

$-10 t_1 - \cancel{4.905 t_1^2} + 160 + \cancel{4.905 t_1^2} + 78.48 - 39.24 t_1 = 0$

A Projectile is Projected with an initial velocity of 20 m/s into space at an angle to the horizontal. At a Particular instant, the x & y co-ordinates of a Point on the Path of the Projectile with reference to the Point of Projection are 8 m and 5 m respectively. Determine the angle of Projection of the projectile.

Given:

$u = 20$ m/s



$$y = x \tan\theta - \frac{g (x)^2}{2 u^2 \cos^2\theta}$$

$$5 = 8 \tan\theta - \frac{9.81 (8)^2}{2 (20)^2 \cos^2\theta}$$

$$5 = 8 \tan\theta - \frac{627.84}{800 \cos^2\theta}$$

$$5 = 8 \tan\theta - 0.785 \sec^2\theta$$

$$5 = 8 \tan\theta - 0.785 (1 + \tan^2\theta)$$

$\tan\theta = 0.783$ | $\tan\theta = 9.407$

$\boxed{\theta = 38.061^\circ}$ | $\boxed{\theta = 83.932^\circ}$

Prob.-11. Q.B.H. A burglar car starts from a bank after robbery, with a uniform acceleration of $0.2\ m/s^2$ till it attains a speed of 40 km/hr after which the speed is maintained constant. The police jeep starts one minute after the burglar car from the same bank and runs with a uniform acceleration of $0.4\ m/s^2$ and gains a maximum speed of 60 km/hr. How much time do the police take to catch the thieves?

Given datas:

$$a_1 = 0.2\ m/s^2$$

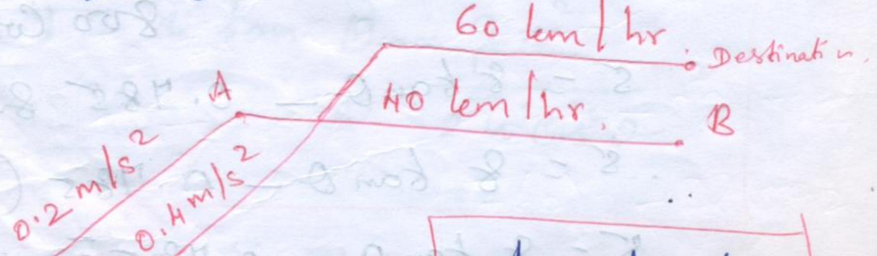
$$v_1 = 40\ km/hr = \frac{40000}{3600} = 11.111\ m/s$$

$$u_1 = 0$$

$$a_2 = 0.4\ m/s^2$$

$$v_2 = 60\ km/hr = \frac{60,000}{3600} = 16.667\ m/s$$

$$u_2 = 0$$



Variable velocity in case of burgler vehicle is

$$v_1 = u_1 + a_1 t_{v_1}$$

$$11.111 = 0 + 0.2 (t_{v_1})$$

$$\boxed{t_{v_1} = 55.555 \text{ sec}}$$

$$v_1^2 = u_1^2 + 2a_1 s_{v_1}$$

$$(11.111)^2 = 0 + 2(0.2) s_{v_1}$$

$$\boxed{s_{v_1} = 308.636 \text{ m}}$$

Constant velocity in case of burgler vehicle is

$$v_{c_1} = \frac{s_{c_1}}{t_{c_1}}$$

$$s_{c_1} = v_{c_1} \times t_{c_1} \quad \text{--- (1)}$$

Variable velocity in case of Police vehicle is

$$v_2 = u_2 + a_2 t_2$$

$$16.667 = 0 + 0.4 (t_2)$$

$$\boxed{t_{v_2} = 41.667 \text{ sec}}$$

$$v_2^2 = u_2^2 + 2a_2 s_{v_2}$$

$$(16.667)^2 = 0 + 2(0.4)(s_{v_2})$$

$$\boxed{s_{v_2} = 347.236 \text{ m}}$$

Constant velocity in case of Police vehicle.

Total displacement of burgler vehicle is

$$S_1 = S_{V_1} + S_{C_1}$$

Total displacement of Police vehicle is

$$S_2 = S_{V_2} + S_{C_2}$$

$S_1 = 308.636 + (V_{C_1} \cdot t_{C_1})$

$= 308.636 + (11.111 \times t_{C_1})$

$= 308.636 + (11.111 \times (t_1 - 55.555))$

$= 308.636 + 11.111\, t_1 - 617.272$

$t_1 = t_{V_1} + t_{C_1}$

$t_{C_1} = t_1 - t_{V_1}$

$t_{C_1} = t_1 - 55.555$

$\boxed{S_1 = 11.111\, t_1 - 308.636}$ —— (3)

$S_2 = S_{V_2} + S_{C_2}$

$= 347.236 + (V_{C_2} \cdot t_{C_2})$

$= 347.236 + (16.667 \times (t_2 - 41.667))$

$= 347.236 + 16.667\, t_2 - 694.464$

$t_2 = t_{V_2} + t_{C_2}$

$t_{C_2} = t_2 - t_{V_2}$

$t_{C_2} = t_2 - 41.667$

$\boxed{S_2 = 16.667\, t_2 - 347.228}$ —— (4)

Here $S_1 = S_2$

$11.111\, t_1 - 308.636 = 16.667\, t_2 - 347.228$

$11.111\, t_1 - 16.667\, t_2 = -38.592$ —— (5)

Here,

$t_1 = t_2 + 60 \Rightarrow t_2 = t_1 - 60$

Sub. the value of t_2 in eqn. (5)

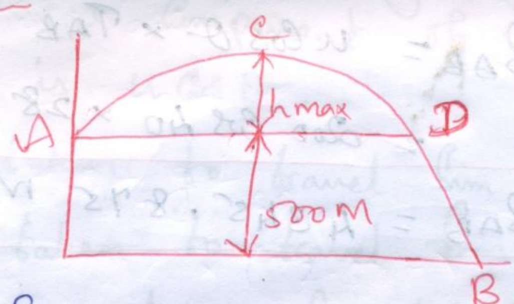
(5) $\Rightarrow$ $11.111\, t_1 - 16.667\, t_2 = -38.592$

$11.111\, t_1 - 16.667(t_1 - 60) = -38.592$

$11.111\, t_1 - 16.667\, t_1 + 1000.02 = -38.592$

Q.B-8 A bullet is fired from the top of a mountain of height 500 m with a velocity of 350 m/s at an angle of elevation of 60°. Determine the (a) Maximum elevation reached by the bullet above the ground (b) Horizontal distance between the Point of firing and the Point where the bullet will strike the ground (c) Magnitude and direction of velocity with which the bullet will strike the ground.

Solution:



(a) $S_1 = 500\,m$

$u = 350\,m/s$

$\theta = 60^\circ$

$$h_{max} = \frac{u^2 \sin^2\theta}{2g}$$

$$= \frac{(350)^2 (\sin 60)^2}{2 \times 9.81}$$

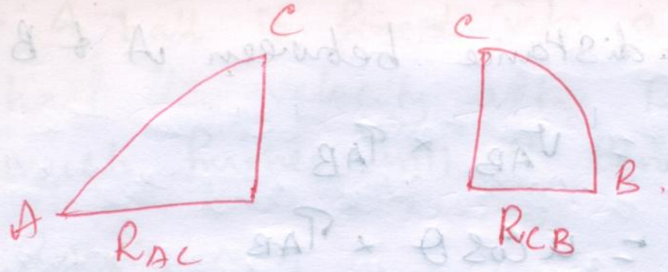
$$\boxed{h_{max} = 4682.722\,m}$$

Maximum elevation reached by the bullet above ground is $h = h_{max} + S_1$

$$= 4682.72 + 500$$

$$\boxed{h = 5182.722\,m}$$

b)



As we are calculating horizontal displacement Horizontal Component of velocity is Constant which is $u \cos\theta$.

Time taken to travel from A to C is t_{AC}.

Path - 1 (A to C)

$$t_{AC} = \frac{u \sin\theta}{g}$$

$$= \frac{380 \sin 60}{9.81}$$

$$\boxed{t_{AC} = 30.898 \text{ sec}}$$

Time take to travel from C to B.

Path - 2 (C to B)

$$s = ut + \frac{1}{2} a t^2$$

$$h = 0(t) + \frac{1}{2} g t^2$$

$$5182.722 = \frac{1}{2}(9.81) t^2$$

$$t^2 = 1056.620$$

$$\boxed{t_{CB} = 32.505 \text{ sec}}$$

Total time taken to travel from A to B is

Horizontal distance between A & B is

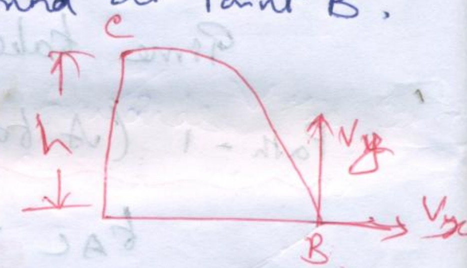
$$S_{AB} = v_{AB} \times T_{AB}$$

$$= u \cos\theta \times T_{AB}$$

$$= 350 \cos 60 \times (63.404)$$

$$\boxed{S_{AB} = 11095.638 \text{ m}}$$

(c) Bullet reaches the ground at Point B.



$V_x = u\cos\theta = 175$ m/s.

V_y is Variable

$V_y = u + at$

$V_y = 0 + (g)t_{CB}$

$V_y = 9.81 \times (32.505)$

$$\boxed{V_y = 318.880 \text{ m/s.}}$$

$$V = \sqrt{V_x^2 + V_y^2}$$

$$= \sqrt{(318.88)^2 + (175)^2}$$

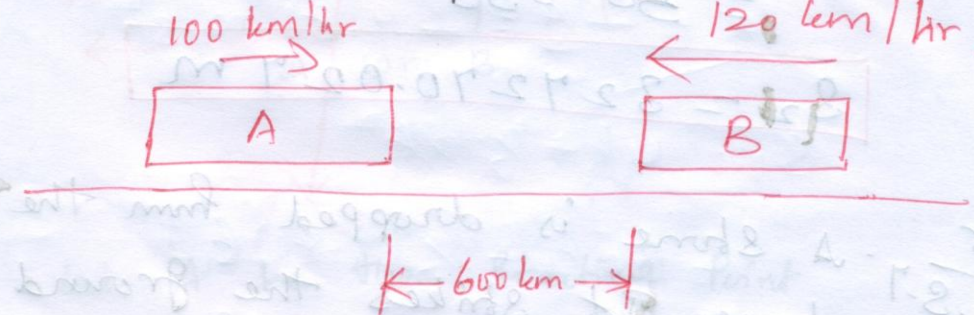
$$\boxed{V = 363.744 \text{ m/s}}$$

$$\alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right)$$

$$\alpha = \tan^{-1}\left(\frac{318.88}{175}\right)$$

$$\boxed{\alpha = 61.242^\circ}$$

3.6 Two vehicles approach each other in opposite lanes of a straight horizontal roadway as shown in figure. Find the time and positions at which the vehicles meet if both continue to move with constant speed.



Given:

$$v_1 = 100 \text{ km/hr} = \frac{100 \times 10^3}{3600} = 27.778 \text{ m/s}$$

$$v_2 = 120 \text{ km/hr} = \frac{120 \times 10^3}{3600} = 33.333 \text{ m/s}.$$

$$\text{Velocity } (v) = \frac{\text{displacement } (S)}{\text{time } (t)}$$

$v_1 = S_1/t_1$ & $v_2 = S_2/t_2$

$S_1 = v_1 t_1$ & $S_2 = v_2 t_2$.

But

$$S_1 + S_2 = 600 \text{ km}$$

$$v_1 t_1 + v_2 t_2 = 600000 \text{ m}$$

$$27.778\, t_1 + 33.333\, t_2 = 600000$$

Here $t_1 = t_2 = t$

$27.778\, t + 33.333\, t$

$S_1 = v_1 t = 27.778 \times 9818.199$ sec

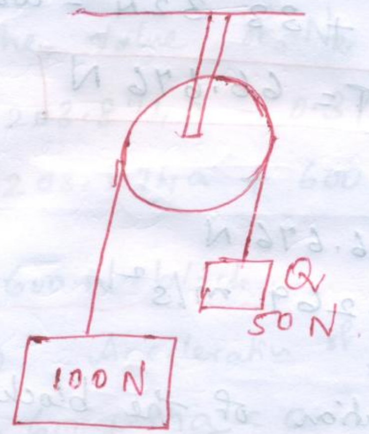
$\boxed{S_1 = 272729.932 \text{ m}}$

$S_2 = v_2 t$

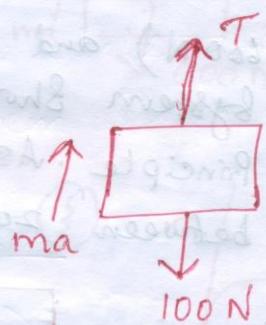
$= 33.333 \times 9818.199$

$\boxed{S_2 = 327270.027 \text{ m}}$

Q.B.2 Find the acceleration of the blocks and tension in the cable for the system shown in figure by D'Alembert's Principle.



i) Consider block 'P'.



For equilibrium condition,

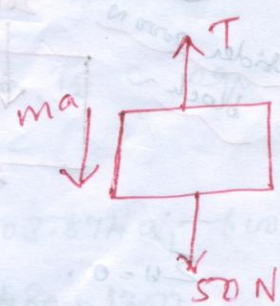
$$\Sigma V = 0$$

$$T + ma - 100 = 0$$

$$T + \left(\frac{100}{9.81}\right) a = 100$$

$$T + 10.194\, a = 100 \quad \text{——} \quad (1)$$

ii) Consider block 'Q'



$$\Sigma V = 0$$

$$T - 50 - ma = 0$$

$$T - \left(\frac{50}{9.81}\right) a = 50$$

$$T - 5.097\, a = 50 \quad \text{——} \quad (2)$$

$(1) \Rightarrow \cancel{T} + 10.194\, a = 100$

$(2) \Rightarrow \cancel{T} - 5.097\, a = 50$

$\quad (-) \quad (+) \quad\quad (-)$

$15.291\, a = 50$

$\Sigma V = 0$

$N_R - 2000 = 0$

$\boxed{N_R = 2000\ N}$

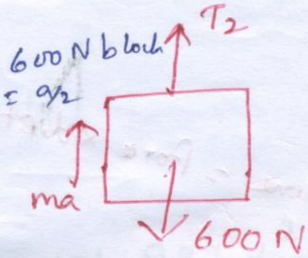
Sub. the value of N_R in equ. (1)

$T_1 - 203.874a - 0.3(2000) = 0$

$T_1 = 203.874a + 600$ —— (2)

Consider 600 N block.

$\Sigma V = 0$ Acceleration of 600 N block $= a/2$

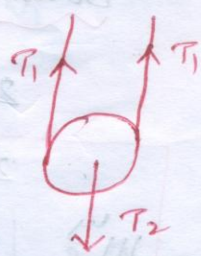


$T_2 - 600 + m\frac{a}{2} = 0$

$T_2 - 600 + \left(\frac{600}{9.81}\right)\frac{a}{2} = 0$

$T_2 - 600 + 30.581\,a = 0$

$T_2 = -30.581\,a + 600$ —— (3)



Consider the Pulley.

$\Sigma V = 0$

$2T_1 - T_2 = 0$

$T_2 = 2T_1$ —— (4)

Sub. the value of T_1 & T_2 in equ. (4)

(4) $\Rightarrow$ $T_2 = 2T_1$

$-30.581a_1 + 600 = 2(203.874a_1 + 600)$

$-30.581a_1 + 600 = 407.748a_1 + 1200$

$-30.581a - 407.748a_1 = 1200 - 600$

$-438.329a_1 = 600$

$\boxed{a_1 = -1.368\ m/s^2}$

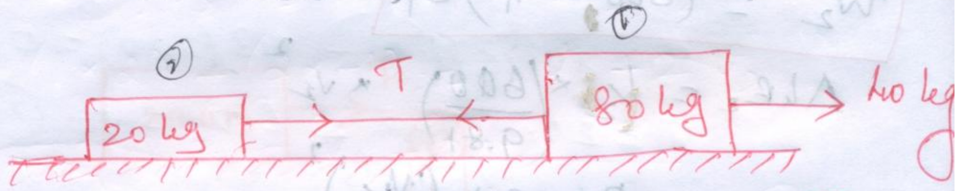
$2a_2 = a_1$,

$a_2 = -0.684\ m/s^2$

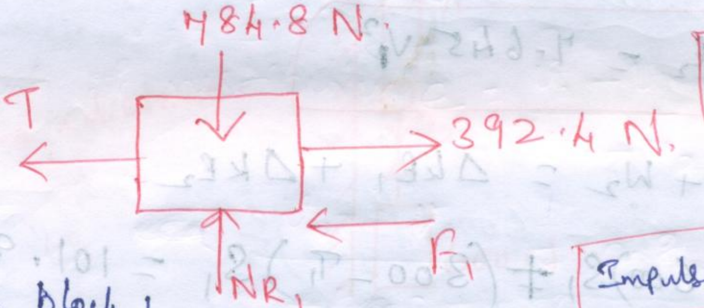
Sub. in (2)

$T_1 = 203.874a + 600$ | Sub. in (3) $T_2 = -30.581\,a + 600$

Prob. 5
Q.B. 6 Find acceleration of the blocks and tension in the cable for the system shown in figure by Impulse Momentum Principle. Assume the Co-efficient of friction between all contact surfaces is 0.35.



Solution:



Direction of motion → (+)

Impulse = force × time

Impulse of block 1

$I_1 = (\Sigma_H) t_1$

$= (392.4 - F_1 - T) t_1$

$= (392.4 - 274.68 - T) t_1$

$\boxed{I_1 = (117.72 - T) t_1}$

$F_1 = \mu . NR_1$

$\Sigma v = 0$

$NR_1 = 784.8 N$

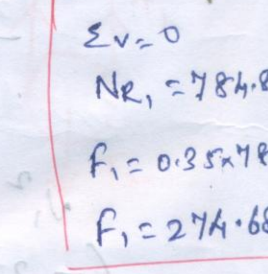
$F_1 = 0.35 \times 784.8$

$F_1 = 274.68$

changes in momentum $\Delta P_1 = M_1 v_1$

$= \left(\frac{784.8}{9.81}\right) v_1$

$\boxed{\Delta P_1 = 80 v_1}$



$v_1 = v_2$
$t_1 = t_2$

Impulse of block 2

$I_2 = (\Sigma_H) t_2$

$= (T - F_2) t_2$

$\boxed{I_2 = (T - 68.67) t_1}$

$F_2 = \mu . NR_2$

$\Sigma v = 0$

$NR_2 = 196.2 N$

$F_2 = 0.35 \times 196.2$

$$I_1 + I_2 = \Delta P_1 + \Delta P_2$$

$$(117.72 - T)\,t_1 + (T - 68.67)\,t_1 = 80 v_1 + 20 v_1$$

$$t_1(117.72 - \cancel{T} + \cancel{T} - 68.67) = 100 v_1$$

$$49.05\, t_1 = 100 v_1$$

$$\boxed{v_1 = 0.4905\, t_1}$$

$$v_1 = u_1 + a_1 t_1$$

$$0.4905\, \cancel{t_1} = 0 + a_1 \cancel{t_1}$$

$$\boxed{a_1 = 0.4905\ m/s^2}$$

For block 1 $\quad I_1 = \Delta P_1$

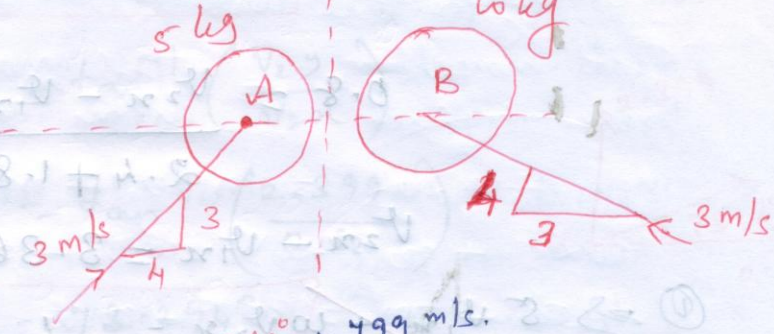
$$(117.72 - T)\,t_1 = 80 v_1$$

$$(117.72 - T)\,\cancel{t_1} = 80(0.4905\, \cancel{t_1})$$

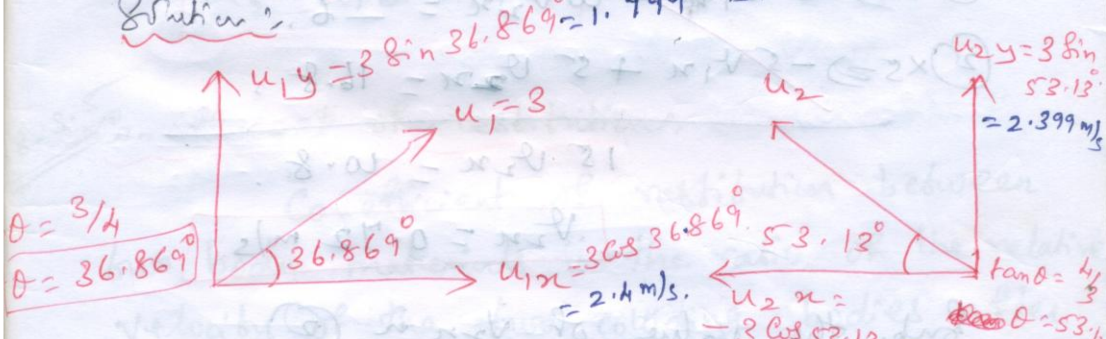
$$117.72 - T = 39.24$$

$$\boxed{T = 78.48\ N}$$

No. 7 / Q.B-9 In the Oblique Central impact shown in figure the Co-efficient of restitution is 0.8. The flat disks shown, slide on a smooth horizontal surface. Determine the final velocity of each disk directly after impact.



Solution:



$u_1 y = v_1 y$

$u_2 y = v_2 y$

[Assumption: vertical component of velocity remains same, it affect only horizontal component]

Given data:

$m_1 = 5$ kg

$m_2 = 10$ kg

$u_1 = 3$ m/s.

$u_2 = 3$ m/s.

$e = 0.8$

According to the Law of Conservation of momentum

Total momentum before impact = Total Momentum after impact.

$m_1 u_{1x} + m_2 u_{2x} = m_1 v_{1x} + m_2 v_{2x}$.

$$12 - 18 = 5 v_{1x} + 10 v_{2x}$$

$$5 v_{1x} + 10 v_{2x} = -6 \quad \text{——— (1)}$$

Co-efficient of restitution $e = \dfrac{v_{2x} - v_{1x}}{u_{1x} - u_{2x}}$

$$0.8 = \frac{v_{2x} - v_{1x}}{2.4 + 1.8}$$

$$v_{2x} - v_{1x} = 3.36 \quad \text{——— (2)}$$

$$(1) \Rightarrow \cancel{5 v_{1x}} + 10 v_{2x} = -6,$$

$$(2) \times 5 \Rightarrow \cancel{-5 v_{1x}} + 5 v_{2x} = 16.8,$$

$$15 v_{2x} = 10.8.$$

$$\boxed{v_{2x} = 0.72 \text{ m/s.}}$$

Sub. the value of v_{2x} in (2)

$$0.72 - v_{1x} = 3.36$$

$$\boxed{v_{1x} = -2.64 \text{ m/s}}$$

$$v_1 = \sqrt{(v_{1x})^2 + (v_{1y})^2}$$

$$= \sqrt{(-2.64)^2 + (1.799)^2}$$

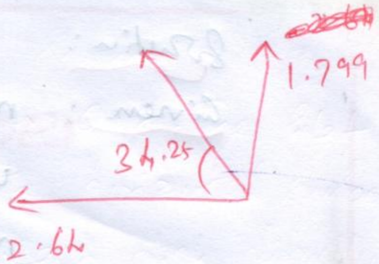
$$\boxed{v_1 = 3.195 \text{ m/s.}}$$

$$v_2 = \sqrt{(v_{2x})^2 + (v_{2y})^2}$$

$$= \sqrt{(0.72)^2 + (2.399)^2}$$

$$\boxed{v_2 = 2.505 \text{ m/s}}$$

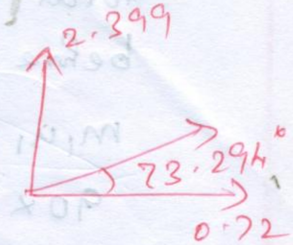
$$\alpha_1 = \tan^{-1}\left(\frac{v_{1y}}{v_{1x}}\right)$$



$$\alpha_1 = \tan^{-1}\left(\frac{1.799}{-2.64}\right)$$

$$\boxed{\alpha_1 = -34.25^\circ}$$

$$\alpha_2 = \tan^{-1}\left(\frac{v_{2y}}{v_{2x}}\right)$$



$$= \tan^{-1}\left(\frac{2.399}{0.72}\right)$$

$$\boxed{\alpha_2 = 73.294^\circ}$$

B.S.5 **Co-efficient of restitution:**

Co-efficient of restitution between two body materials is the ratio of the relative velocity of the two colliding bodies after impact to the relative velocity of the bodies before impact.

$$e = \frac{\text{impulse during the Period of restitution}}{\text{impulse during the Period of deformation}}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

6.8
B.8 A ball of Mass 90 kg moves with a velocity of 5 m/s Collides Centrally with another ball of mass 50 kg moving in opposite direction with a velocity of 10 m/s. If the Co-efficient of restitution as 0.6, Find the velocity of each ball after impact both in magn[illegible]

Given : $m_1 = 90\,kg$

$u_1 = 5\,m/s$

$m_2 = 50\,kg$

$u_2 = -10\,m/s$

According to the law of Conservation of Momentum

Total momentum before impact = Total momentum after impact

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$90\times 5 + 50\times(-10) = 90v_1 + 50v_2$$

$$450 - 500 = 90v_1 + 50v_2$$

$$90v_1 + 50v_2 = -50 \quad \text{—— (1)}$$

Co-efficient of restitution $e = \dfrac{v_2 - v_1}{u_1 - u_2} = 0.6$

$$\frac{v_2 - v_1}{5+10} = 0.6$$

$$v_2 - v_1 = 9 \quad \text{—— (2)}$$

$$(1) \Rightarrow \cancel{90v_1} + 50v_2 = -50$$

$$(2)\times 90 \Rightarrow \cancel{-90v_1} + 90v_2 = 810$$

$$140v_2 = 760$$

$$\boxed{v_2 = 5.429\,m/s}$$

Sub. the value of v_2 in eqn. (2)

$$v_2 - v_1 = 9$$

$$5.429 - v_1 = 9$$

$$-v_1 = 3.571$$

$$\boxed{v_1 = -3.571\,m/s}$$